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Exploration of Spatio-Temporal Environmental Conditions: Harmonized Databases and Analytical Techniques

ECOST-DATA, Bari, Italy, July 3-4, 2025

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Editors

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Preface

This volume collects the scientific contributions presented at the Workshop “Exploration of Spatio-Temporal Environmental Conditions: Harmonized Databases and Analytical Techniques (ECoST-DATA)”, held at the University of Bari on July 3-4, 2025, as Satellite event of DUST 2025. It is a key result of the ECoST-DATA Project, funded by European Union – NextGenerationEU with the Cascade Open Calls published by ALMA MATER STUDIORUM – University of Bologna, inside the Project GRINS within the PNRR – Mission 4, Component 2, Investment 1.3 “Partnership extended to Universities, Research Centers, Firms and research projects funding”, D.D. 341 of 15/03/2022, CUP: J33C22002910001.

The contents are dedicated to advanced tools and techniques for analyzing and predicting environmental variables with a spatial or spatio-temporal structure. In particular, they include theoretical reviews on spatio-temporal covariance modelling as well as innovative approaches for assessing environmental quality and its effects on climate change by integrating georeferenced data from multiple sources.

Among the primary objectives of this collection, it is worth mentioning: transfer advanced analytic skills for the modelling, estimation, and prediction of spatial and spatio-temporal environmental variables; provide practical guidance on integrating heterogeneous georeferenced data while accounting for measurement error and spatial misalignment; support risk assessment by equipping practitioners with methods to quantify exposure and uncertainty relevant to human health; and inform sustainability planning through models that identify spatially and temporally targeted mitigation or adaptation strategies.

As a result, the contributions are intended for researchers and graduate students in spatial statistics, environmental science, and geosciences; data scientists and practitioners working on environmental monitoring, climate impacts, and public-health risk assessment; policy analysts interested in data-driven sustainability planning.

Regarding the outcomes, readers will gain both the theoretical grounding and the practical toolset necessary to build, validate, and deploy spatio-temporal models for environmental variables, produce reliable forecasts, and translate model outputs into actionable insights for health and sustainability interventions.

The volume is organized in three parts.

In the first one, *Random fields and Covariance modelling*, Posa clarifies the relationship between covariance functions and positive definite functions, emphasizing that Crum's theorem is the necessary generalization of Bochner's theorem. De Iaco discusses on some families of spatio-temporal complex covariance functions for vectorial data. It is also described how these functions can be generalized to the spatio-temporal setting, as well as how some other complex-valued covariance families can be constructed as positive mixtures of infinite components or as a convolution of the real part. Hristopulos presents the hybrid spectral approach for generating physically inspired, non-separable covariance kernels from stochastic ordinary differential equations. Pere et al. propose the stationary subspace analysis to decompose a random field into independent stationary and non-stationary subspaces.

In Part II, *Environmental control and integration*, Stein focuses on the use of modern deep learning methods for the analysis and management of large data volumes. His study explores the use of satellite imagery and time series to analyze air quality and monitor environmental changes. A further challenging approach is presented in Gómez-Hernández et al. to identify atmospheric dust sources, using the ensemble smoother with multiple data assimilation methodology. Capasso discusses the optimization of pollution control within an economic growth system, presenting a spatially structured model. This model analyzes an optimal control problem designed to maximize intertemporal utility while minimizing pollution, the disutility of taxation, and intervention costs. Additionally, the study by Dutillo and Di Battista explores the functional structure and variation of Italian carbon emissions from electricity using functional data analysis. Daily and monthly profiles are constructed using Fourier basis functions and analyzed at national and electricity market zone levels.

The volume concludes with Part III, *Environmental Analysis*, where Jalilian et al. apply a flexible generalized additive model framework for surface ozone prediction, using spatio-temporally varying coefficients to correct biases between satellite and in situ data. Gomes et al. investigate the geostatistical characterization of climate extremes dynamics, focusing on drought. A Random Forest-based short-term alert system is presented for predicting drought severity classes at a monthly scale, demonstrating strong predictive performance for local areas and offering a flexible tool for managing drought-related risks. For environmental quality assessment, the work by Masoumi et al. addresses a gap in biodiversity modelling in the Apulia region by comparing the performance of various machine learning models (Random Forests, Support Vector Machines, Extreme Gradient Boosting, Decision Trees,

and Linear Regression) for assessing the effects of environmental variables on the multilevel biodiversity index. Furthermore, in Cappello et al., spatio-temporal blind source separation model is reviewed and employed to model and predict ground-level ozone concentration in a complex multivariate spatio-temporal framework.

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Part I
Random Fields and Covariance Modelling

Spatiotemporal Complex Covariance Functions for Vectorial Data



Sandra De Iaco

Abstract In the literature, complex-valued random fields were initially employed to characterize the spatial evolution of vector data, while the temporal dimension was typically treated separately or modeled using time-dependent complex covariance structures. Nevertheless, as in the real case, extending the complex formalism to a unified spatiotemporal framework and developing new classes of spatiotemporal complex covariance models are of clear interest to researchers, driven in part by the rapid increase in environmental vector observations collected over space and time. This contribution outlines the foundational elements of the complex spatiotemporal random field framework and describes how several spatiotemporal complex-valued covariance families can be derived from the spatial setting as well as from positive mixtures of infinite components or a convolution of the real part.

Keywords Vectorial data · Spatiotemporal covariance · Complex random fields · Complex covariance model

1 Introduction

The theoretical framework of the complex-valued random fields is reasonably considered appropriate for phenomena naturally decomposed into magnitude and direction, like wind, sea currents, and electric fields. In particular, [34] provided an early exposition for one-dimensional domains, while key spatial studies in this context include the contributions of [15, 18, 19, 32]. Some developments in parametric complex-domain covariance models appear in [26, 27] and some fitting

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and computational issues can be found in [12]. Moreover the R package *cgeostat* [9] supports full analyses of 2D vector data.

The growing applied importance of complex space-time analysis drives demand for spatiotemporal tools. Many works treat spatiotemporal modeling for real scalar data [6, 8, 25, 29, 30, 33], but complex-domain extensions are scarce. Moreover, [2, 3] presented time-varying complex covariance functions whose parameters evolve over time, yet these do not model the joint space-time dependence, because they are not functions of temporal lags. In addition, [4] proposed a spatiotemporal complex random field characterization using Bochner's theorem. Indeed, the construction of wide families of spatiotemporal complex covariance models, for second-order stationary spatiotemporal complex-valued random fields, can be based on different techniques. The first two approaches, described in [11], propose an extension, to the space-time domain, of existing complex covariance models given by Lajaunie and Béjaoui [19], who built the imaginary component of a continuous complex covariance model on the basis of the real component through Radon-Nikodym's theorem and by De Iaco et al. [15], which started from the spectral representation of positive definite functions. Note that the latter includes by definition a periodic factor, whose effect can be damped through the product with a real monotonically decaying covariance model, while for the former the periodic component can be optionally incorporated in the model. The other approach allows for building a new family of complex covariance models through positive mixtures [10]. In particular, this family can be used to describe nonnegative real part or real and imaginary parts with or without a periodic component. This method generalizes the convolution-based class and enables flexible construction of spatiotemporal complex covariance families by defining real and imaginary parts using known space-time positive definite functions.

In this chapter, Sect. 2 introduces the concept of complex-valued space-time random fields, the second-order stationarity assumption, and the space-time complex covariance characterization. Section 3 provides an overview of spatiotemporal complex covariance families, with some examples, and discusses some advantages and further generalizations, while Sect. 4 presents some conclusions.

2 Theory of Spatio-temporal Complex Random Fields

A complex-valued random field in space-time is defined as follows:

$$Z(\mathbf{w}) = U(\mathbf{w}) + i V(\mathbf{w}), \quad \mathbf{w} = (\mathbf{w}_s, w_t) \in \mathbb{R}^N \times \mathbb{R}, \quad (1)$$

where U and V are two real-valued spatiotemporal random fields, defined on $\mathbb{R}^N \times \mathbb{R}$, and $i = \sqrt{-1}$ is the imaginary number.

Assuming that Z has finite variance, the first-order moment $\mathbb{E}[Z(\mathbf{w})]$ of Z is finite and is defined through the following complex-valued deterministic function:

$$m(\mathbf{w}) = m_U(\mathbf{w}) + i m_V(\mathbf{w}),$$

where m_U and m_V are the expected values of U and V , respectively, whereas the covariance function $R(\mathbf{w}, \mathbf{w}')$ is finite and defined as

$$R(\mathbf{w}, \mathbf{w}') = \mathbb{E} \left[(Z(\mathbf{w}) - m(\mathbf{w}))(\overline{Z(\mathbf{w}') - m(\mathbf{w}')}) \right] = \quad (2)$$

$$= R_U(\mathbf{w}, \mathbf{w}') + R_V(\mathbf{w}, \mathbf{w}') + i [R_{VU}(\mathbf{w}, \mathbf{w}') - R_{UV}(\mathbf{w}, \mathbf{w}')], \quad \mathbf{w}, \mathbf{w}' \in \mathbb{R}^N \times \mathbb{R},$$

where R_U and R_V denote, respectively, the spatiotemporal covariance functions associated with the real part U and the imaginary part V of Z , whereas R_{VU} and R_{UV} represent the related cross-covariances. As usually done, the complex conjugation is denoted with an overline in Eq. (2).

The complex formalism differs conceptually from multivariate Geostatistics; indeed, it is recalled when data admit a complex representation. Vector measurements like wind or ocean currents are naturally handled as complex-valued fields. The key modeling benefit is that the complex covariance fully characterizes the two-dimensional correlation, where the real part captures direct covariances and the imaginary part captures cross-covariances, so only the sum of direct covariances and the difference of cross-covariances need to be modeled. This advantage grows for spatiotemporal data. Further details on this aspect can be found in [12, 32].

2.1 Second-Order Stationarity Hypothesis

Given a random field as in (1), it is second-order stationary in space-time, if its complex-valued expected value is constant, that is, $\mathbb{E}[Z(\mathbf{w})] = m = m_U + i m_V$, where the expected values of U and V are constant for any point of the domain and the space-time covariance function R depends, for any pair of points $\mathbf{w} = (\mathbf{w}_s, w_t)$ and $\mathbf{w}' = (\mathbf{w}'_s, w'_t)$, on the lag separation vector $\mathbf{h}_s = \mathbf{w}'_s - \mathbf{w}_s \in \mathbb{R}^N$, $h_t = w_t - w'_t \in \mathbb{R}$, that is,

$$R(\mathbf{h}_s, h_t) = R^{re}(\mathbf{h}_s, h_t) + i R^{im}(\mathbf{h}_s, h_t), \quad (3)$$

where the corresponding real part, indicated with R^{re} , is given as follows:

$$R^{re}(\mathbf{h}_s, h_t) = R_U(\mathbf{h}_s, h_t) + R_V(\mathbf{h}_s, h_t), \quad (4)$$

and the imaginary part, denoted with R^{im} , is defined as

$$R^{im}(\mathbf{h}_s, h_t) = R_{VU}(\mathbf{h}_s, h_t) - R_{UV}(\mathbf{h}_s, h_t). \quad (5)$$

Note that, by definition, the even function in Eq. (4) is also a space-time covariance function on the basis of the convexity property [5]; on the other hand, the odd function in Eq. (5) is not a space-time covariance function.

For a second-order stationary complex-valued random field Z , the space-time covariance function R satisfies the following properties, i.e.:

- It is nonnegative at zero, $R(\mathbf{0}, 0) \geq 0$.
- It is symmetric, $R(-\mathbf{h}_s, -h_t) = \overline{R(\mathbf{h}_s, h_t)}$, that is, the covariance matrix is Hermitian.
- It is bounded, $|R(\mathbf{h}_s, h_t)| \leq R(\mathbf{0}, 0)$.

Moreover, any complex covariance function must be nonnegative definite, i.e.,

$$\sum_{i=1}^k \sum_{j=1}^k R(\mathbf{w}_i - \mathbf{w}_j) a_i \overline{a_j} \geq 0, \quad (6)$$

$$\forall \mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_k \in \mathbb{R}^N \times \mathbb{R}, \quad \forall a_1, \dots, a_k \in \mathbb{C}.$$

2.2 Bochner's Theorem and Space-Time Complex Covariance

On the basis of the full characterization given for any continuous complex covariance function by Bochner's theorem [1], the definition of a space-time complex covariance function is provided as follows:

$$R(\mathbf{h}_s, h_t) = \int_{\mathbb{R}^N \times \mathbb{R}} \cos(\mathbf{h}_s \cdot \mathbf{q}_s + h_t q_t) dF(\mathbf{q}_s, q_t) + i \int_{\mathbb{R}^N \times \mathbb{R}} \sin(\mathbf{h}_s \cdot \mathbf{q}_s + h_t q_t) dF(\mathbf{q}_s, q_t), \quad (7)$$

where $\mathbf{h}_s \cdot \mathbf{q}_s$ denotes the inner product $\sum_{i=1}^N h_i q_i$ and F , called the *space-time spectral distribution function* of R , is a bounded, real-valued function and satisfies the following condition:

$$\int_A dF(\mathbf{q}_s, q_t) \geq 0, \quad \text{for any measurable } A \subseteq \mathbb{R}^N \times \mathbb{R}.$$

Thus, the space-time covariance function of Z can be written as the sum of an even part and an odd one. If the spectral distribution function F is absolutely continuous, the space-time spectral density f exists and is a bounded and continuous nonnegative function. In particular, if the spectral distribution F is symmetric about the origin of $\mathbb{R}^N \times \mathbb{R}$ or if a spectral density function f is an even function, then the covariance function is real-valued. If the spectral density is separable, i.e., $f(\mathbf{q}_s, q_t) = f_1(\mathbf{q}_s) f_2(q_t)$, then the space-time complex covariance function

is separable:

$$R(\mathbf{h}_s, h_t) = \int_{\mathbb{R}^N} e^{i(\mathbf{h}_s \cdot \mathbf{q}_s)} f_1(\mathbf{q}_s) d\mathbf{q}_s \int_{\mathbb{R}} e^{ih_t q_t} f_2(q_t) dq_t = R_1(\mathbf{h}_s) R_2(h_t), \quad (8)$$

where R_1 and R_2 are spatial and temporal complex covariance functions, respectively. Thus, two different subclasses can be derived by assuming that one of the spectral densities, f_1 or f_2 , is symmetric. Note that, only for the real case, the covariance function R is an even function, since the imaginary part of (3) is zero.

3 Some Complex Families from Space to Space-Time

Spatiotemporal complex covariance families on $\mathbb{R}^n \times \mathbb{R}$ can be derived by extending spatial models on $\mathbb{R}^N$. In the following, some classes, obtained by using the Radon-Nikodym theorem [19] or on the basis of spectral-density translation [15] as well as through positive mixtures, have been presented, as discussed in [4, 11].

3.1 Convolution of the Real Part

One of the earliest contributions to complex covariance models on spatial domains was by Lajaunie and Béjaoui [19], who constructed the imaginary part directly from the real part. Given the real component $R^{re}(\mathbf{h}_s, h_t)$, the imaginary part follows from the Radon-Nikodym theorem, as explained below:

$$i R^{im}(\mathbf{h}_s, h_t) = (F * R^{re})(\mathbf{h}_s, h_t), \quad (9)$$

where $*$ represents the convolution operation and F is a complex distribution whose Fourier transform is a real odd function f with values in the interval $[-1, 1]$. Thus, the following class of complex spatiotemporal covariance model can be obtained

$$R(\mathbf{h}_s, h_t) = R^{re}(\mathbf{h}_s, h_t) + (F * R^{re})(\mathbf{h}_s, h_t). \quad (10)$$

The imaginary components $R^{im}(\mathbf{h}_s, h_t)$, linked to a real continuous covariances $R^{re}(\mathbf{h}_s, h_t)$ by Eq. (10), are called compatible imaginary parts. A parametric family of such compatible imaginary components can be constructed as follows:

$$R^{im}(\mathbf{h}_s, h_t) = 0.5 \int [R^{re}(\mathbf{h}_s - \boldsymbol{\tau}_s, h_t - \tau_t) - R^{re}(\mathbf{h}_s + \boldsymbol{\tau}_s, h_t + \tau_t)] \mu(d\boldsymbol{\tau}_s, d\tau_t), \quad (11)$$

where μ is a real bounded measure such that $|\mu(\omega_s, \omega_t)| < 1$ with $f(\omega_s, \omega_t) = -\int \sin(\omega_s \cdot \boldsymbol{\tau}_s, \omega_t \tau_t) \mu(d\boldsymbol{\tau}_s, d\tau_t)$. In other terms, the imaginary parts of this class

can be got through the expected value of the difference $[R^{re}(\mathbf{h}_s - \boldsymbol{\tau}_s, h_t - \tau_t) - R^{re}(\mathbf{h}_s + \boldsymbol{\tau}_s, h_t + \tau_t)]\mu(d\boldsymbol{\tau}_s, d\tau_t)$ with respect to $\boldsymbol{\tau}_s$ and τ_t , i.e.,

$$R^{im}(\mathbf{h}_s, h_t) = 0.5\mathbb{E}_{\boldsymbol{\tau}_s, \tau_t}[R^{re}(\mathbf{h}_s - \boldsymbol{\tau}_s, h_t - \tau_t) - R^{re}(\mathbf{h}_s + \boldsymbol{\tau}_s, h_t + \tau_t)]. \quad (12)$$

As a special case of (12), it is worth introducing the following simple complex covariance model in space-time:

$$R(\mathbf{h}_s, h_t) = R^{re}(\mathbf{h}_s, h_t) + 0.5i[R^{re}(\mathbf{h}_s - \boldsymbol{\tau}_s, h_t - \tau_t) - R^{re}(\mathbf{h}_s + \boldsymbol{\tau}_s, h_t + \tau_t)], \quad (13)$$

where the imaginary component is determined through the difference of the real-valued covariance function R^{re} translated backward and forward of the vector $(\boldsymbol{\tau}_s, \tau_t)$. An example of a class of compatible imaginary parts associated with the Gneiting class of real-valued covariance functions has been proposed below.

Example 1 Suppose that the real component of Eq.(13) is described by the following class of Gneiting covariance models:

$$R_{Gn}^{re}(\mathbf{h}_s, h_t) = \frac{1}{(a|h_t|^{2\alpha} + 1)} \exp - \frac{b\|\mathbf{h}_s\|^{2\gamma}}{(a|h_t|^{2\alpha} + 1)^{\gamma\beta}}, \quad (14)$$

with $a, b \in]0, +\infty]$, $\alpha, \gamma, \beta \in]0, 1]$; then the corresponding compatible imaginary part can be obtained on the basis of the abovementioned difference, that is,

$$\begin{aligned} R_{Gn}^{im}(\mathbf{h}_s, h_t) &= \frac{0.5}{(a|h_t - \tau_t|^{2\alpha} + 1)} \exp \frac{-b\|\mathbf{h}_s - \boldsymbol{\tau}_s\|^{2\gamma}}{(a|h_t - \tau_t|^{2\alpha} + 1)^{\gamma\beta}} + \\ &- \frac{0.5}{(a|h_t + \tau_t|^{2\alpha} + 1)} \exp \frac{-b\|\mathbf{h}_s + \boldsymbol{\tau}_s\|^{2\gamma}}{(a|h_t + \tau_t|^{2\alpha} + 1)^{\gamma\beta}}. \end{aligned} \quad (15)$$

Figure 1 provides a graphical representation of the spatial and temporal marginals of the complex model obtained from Eqs. (14) and (15), that is, the real marginals in space and time

$$R_{Gn}^{re}(\mathbf{h}_s, 0) = \frac{1}{(a|h_t|^{2\alpha} + 1)}, \quad R_{Gn}^{re}(\mathbf{0}, h_t) = \exp[-b\|\mathbf{h}_s\|^{2\gamma}], \quad (16)$$

and the imaginary marginals in space and time

$$R_{Gn}^{im}(\mathbf{h}_s, 0) = 0.5/a' \left\{ \exp[-b/a'^{\gamma\beta}\|\mathbf{h}_s - \boldsymbol{\tau}_s\|^{2\gamma}] - \exp[-b/a'^{\gamma\beta}\|\mathbf{h}_s + \boldsymbol{\tau}_s\|^{2\gamma}] \right\}, \quad (17)$$

$$R_{Gn}^{im}(\mathbf{0}, h_t) = \frac{0.5}{(a|h_t - \tau_t|^{2\alpha} + 1)} \exp[-b'/(a|h_t - \tau_t|^{2\alpha} + 1)^{\gamma\beta}] +$$

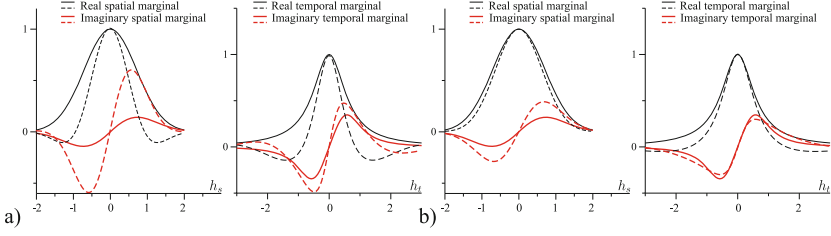


Fig. 1 Real and imaginary marginals of the complex models based on: the construction given in (13), written in terms of the Gneiting class in Eq. (14), with $\alpha = \gamma = 1$, $a = 2.5$, $b = 1$, $\beta = 1$, and $\tau_s = (0.2, 0.2)$, $\tau_t = 0.5$ (continuous line) and the construction in (24), with $\tilde{R}$ defined as in Eq. (14) with the same parameter values $\alpha = \gamma = 1$, $a = 2.5$, $b = 1$, $\beta = 1$ (dashed line), and (a) $\mathbf{c}_s = (1.5, 1.5)$, $c_t = 1.8$, (b) $\mathbf{c}_s = (0.5, 0.5)$, $c_t = 1$

$$- \frac{0.5}{(a|h_t + \tau_t|^{2\alpha} + 1)} \exp[-b'/(a|h_t + \tau_t|^{2\alpha} + 1)^{\gamma\beta}], \quad (18)$$

where $a' = (a|\tau_t|^{2\alpha} + 1)$, $b' = b\|\tau_s\|^{2\gamma}$.

It is clear that any periodic component can be included, for example, through the sum or the product of the class in Eq. (13) with the basic complex covariance function, that is, $R(\mathbf{h}_s, h_t) = \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) + i \sin(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)$, or just with the real part of the same basic complex covariance function.

3.2 Construction Based on Asymmetric Spectral Density

Starting from Bochner's characterization, a first attempt to construct complex covariance models for the spatial case was proposed by De Iaco et al. [15], through a translation of an even spectral density function. In [4], the above approach has been suitably generalized to build a class of complex-valued spatiotemporal covariance models. Indeed, given a spectral representation of a real-valued covariance function $\tilde{R}(\mathbf{h}_s, h_t)$ and the corresponding even spectral density function $f(\mathbf{q}_s, q_t)$, then the translated function $f(\mathbf{q}_s - \mathbf{c}_s, q_t - c_t)$, which is a not-even spectral density function $\forall \mathbf{c} = (\mathbf{c}_s, c_t) \in \mathbb{R}^N \times \mathbb{R}$, $\mathbf{c} = (\mathbf{c}_s, c_t) \neq \mathbf{0}$, is used to define the following class of space-time complex covariance models:

$$\begin{aligned} R(\mathbf{h}_s, h_t; \mathbf{c}) &= \int_{\mathbb{R}^N \times \mathbb{R}} \exp[i(\mathbf{h}_s \cdot \mathbf{q}_s + h_t q_t)] f(\mathbf{q}_s - \mathbf{c}_s, q_t - c_t) d\mathbf{q}_s dq_t = \\ &= \exp[i(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)] \tilde{R}(\mathbf{h}_s, h_t). \end{aligned} \quad (19)$$

For modeling purposes, the above class can be better made explicit as follows:

$$R(\mathbf{h}_s, h_t; \mathbf{c}) = \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) \tilde{R}(\mathbf{h}_s, h_t) + i \sin(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) \tilde{R}(\mathbf{h}_s, h_t), \quad (20)$$

where $\tilde{R}(\mathbf{h}_s, h_t)$ is a real space-time covariance function and $\mathbf{c} = (\mathbf{c}_s, c_t) \in \mathbb{R}^N \times \mathbb{R}$ represents the shifting factor.

Looking simultaneously to Eqs. (3) and (20), it is worth pointing out that the real part $\cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) \tilde{R}(\mathbf{h}_s, h_t)$ describes the sum of the covariances $R_U(\mathbf{h}_s, h_t)$ and $R_V(\mathbf{h}_s, h_t)$, while the imaginary part $\sin(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) \tilde{R}(\mathbf{h}_s, h_t)$ is a model for the difference of the two cross-covariances R_{VU} and R_{UV} .

Note that the spatial and temporal complex marginal covariance models are

$$R(\mathbf{h}_s, 0; \mathbf{c}) = \cos(\mathbf{h}_s \cdot \mathbf{c}_s) \tilde{R}(\mathbf{h}_s, 0) + i \sin(\mathbf{h}_s \cdot \mathbf{c}_s) \tilde{R}(\mathbf{h}_s, 0), \quad (21)$$

$$R(\mathbf{0}, h_t; \mathbf{c}) = \cos(h_t c_t) \tilde{R}(\mathbf{0}, h_t) + i \sin(h_t c_t) \tilde{R}(\mathbf{0}, h_t), \quad (22)$$

which can be very useful under a computational point of view, by following the same idea given in [9, 12]. A discussion on the effect of the shifting factor can be found in [16].

Note that it is common to adopt real-valued spatiotemporal covariance models which are isotropic only in space; thus the following space-time complex covariance can be introduced as a special case of (20):

$$R(\mathbf{h}_s, h_t; \mathbf{c}) = \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) \tilde{R}(\|\mathbf{h}_s\|, h_t) + i \sin(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) \tilde{R}(\|\mathbf{h}_s\|, h_t). \quad (23)$$

In the following examples, some spatiotemporal models on $\mathbb{R}^N \times \mathbb{R}$, with $N = 2$ and vector coordinate $(\mathbf{h}_s, h_t) = (h_x, h_y, h_t)^T$, are presented.

Example 2 Let $\tilde{R}$ be the class of Gneiting covariance models in Eq. (14); then given the complex class in (23), the following space-time complex covariance model can be built:

$$\begin{aligned} R(\mathbf{h}_s, h_t) = & \frac{\cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{(a|h_t|^{2\alpha} + 1)} \exp\left[-\frac{b\|\mathbf{h}_s\|^{2\gamma}}{(a|h_t|^{2\alpha} + 1)^{\gamma\beta}}\right] + \\ & + i \frac{\sin(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{(a|h_t|^{2\alpha} + 1)} \exp\left[-\frac{b\|\mathbf{h}_s\|^{2\gamma}}{(a|h_t|^{2\alpha} + 1)^{\gamma\beta}}\right]. \end{aligned} \quad (24)$$

Note that the real-valued covariance component $\tilde{R}(\mathbf{h}_s, h_t)$ is assumed to be isotropic in space. Moreover, the spatial and temporal complex marginal covariance models are defined as follows:

$$R(\mathbf{h}_s, 0) = \cos(\mathbf{h}_s \cdot \mathbf{c}_s) \exp[-b\|\mathbf{h}_s\|^{2\gamma}] + i \sin(\mathbf{h}_s \cdot \mathbf{c}_s) \exp[-b\|\mathbf{h}_s\|^{2\gamma}], \quad (25)$$

$$R(\mathbf{0}, h_t) = \frac{\cos(h_t c_t)}{(a|h_t|^{2\alpha} + 1)} + i \frac{\sin(h_t c_t)}{(a|h_t|^{2\alpha} + 1)}. \quad (26)$$

Figure 1 provides a graphical representation of the real and imaginary parts of the spatial and temporal marginals of the complex model in (24), together with the

marginals of the model in Eq. (13) for comparison purposes. Indeed, both models are based on the Gneiting class defined in Eq. (14), with the same values for the parameters. It is clear the effect of the factors containing the shifting vector $\mathbf{c}$; however, the $\tilde{R}$ produces a damping effect for both the real and imaginary components of the spatial and temporal marginals. The damped periodicity might be further mitigated by using lower values for the shifting factor, as in Fig. 1b.

3.3 Infinite Positive Power Mixture

As will be clarified, other families of complex-valued spatiotemporal covariance models can be generated through a combination of an infinite number of terms, as in (20), weighted with a power of a constant in the interval]0, 1[[10].

Theorem 1 *Let $\tilde{R}(\mathbf{h}_s, h_t)$ be a real-valued covariance function and $a \in]0, 1[$; then*

$$\begin{aligned} R(\mathbf{h}_s, h_t; a, \mathbf{c}) = & \frac{1 - a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{1 - 2a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) + a^2} \tilde{R}(\mathbf{h}_s, h_t) \\ & + i \frac{a \sin(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{1 - 2a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) + a^2} \tilde{R}(\mathbf{h}_s, h_t) \end{aligned} \quad (27)$$

is a class of complex-valued covariance models.

One of the advantages of this class is related to the minimum and the maximum of the factors

$$\frac{1 - a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{1 - 2a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) + a^2}, \quad (28)$$

$$\frac{a \sin(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{1 - 2a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) + a^2}, \quad (29)$$

which are different for the real and the imaginary parts, that is, $[1/(1+a), 1/(1-a)]$ for (28) and $[-a/(1-a^2), a/(1-a^2)]$ for (29).

Moreover, apart from the amplitude of the periodic component of this class, which can be smoothed by fixing low values of the parameter a , it is worth pointing out the flexibility of the class (27), with respect to (20), in describing real and imaginary parts which are not affected by the presence of periodic components, as shown in the following example.

Example 3 Let $\tilde{R}$ be described by the integrated class, obtained by integrating the stable model with the gamma function, that is,

$$\tilde{R}(\mathbf{h}_s, h_t) = \frac{1}{(b_t |h_t|^{2\alpha} + b_s \|\mathbf{h}_s\|^{2\gamma} + 1)}, \quad (30)$$

with $b_s, b_t \in]0, +\infty]$ and $\alpha = \gamma \in]0, 1]$; then from (27), the obtained complex spatiotemporal covariance function is given below:

$$R(\mathbf{h}_s, h_t; \boldsymbol{\theta}) = R^{re}(\mathbf{h}_s, h_t; \boldsymbol{\theta}) + i R^{im}(\mathbf{h}_s, h_t; \boldsymbol{\theta}), \quad (31)$$

where the real and imaginary parts are, respectively,

$$R^{re}(\mathbf{h}_s, h_t; \boldsymbol{\theta}) = \frac{1 - a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{(1 - 2a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) + a^2)(b_t |h_t|^{2\alpha} + b_s \|\mathbf{h}_s\|^{2\gamma} + 1)}, \quad (32)$$

$$R^{im}(\mathbf{h}_s, h_t; \boldsymbol{\theta}) = \frac{a \sin(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{(1 - 2a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) + a^2)(b_t |h_t|^{2\alpha} + b_s \|\mathbf{h}_s\|^{2\gamma} + 1)}, \quad (33)$$

with $\boldsymbol{\theta} = (a, b_s, b_t, \mathbf{c}, \alpha, \gamma)$. Figure 2a, b provides a graphical comparison between the spatial and temporal marginals of the classes (20) and (31), where the parameters related to the shifting factors in space and time are assumed to be the same for the two classes. The amplitude of the periodic component of the class (31) is smoothed by fixing a value of the parameter a less than 0.5. Moreover, Fig. 2c, d illustrates the flexibility of the class (31), with respect to (20), in describing real and imaginary parts which are not characterized by periodic components. It is clear that the class (31) is able to describe random fields whose correlation decays monotonously, while

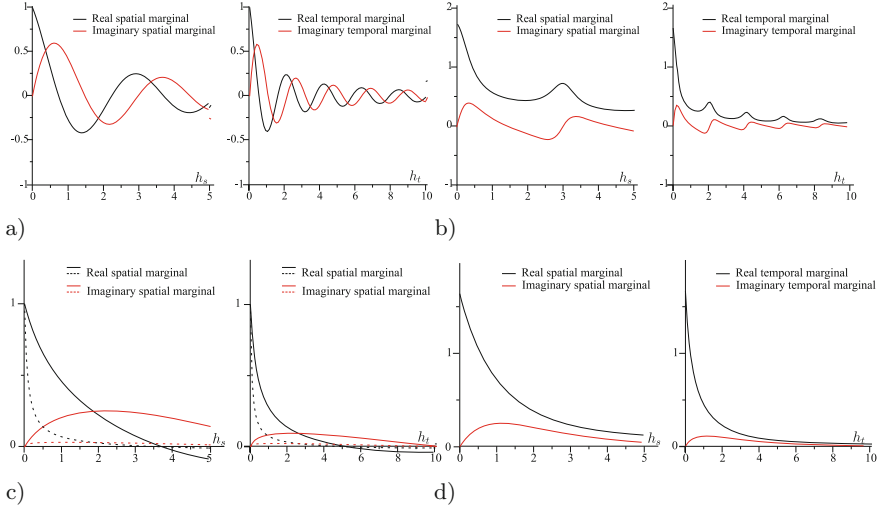


Fig. 2 Real and imaginary parts of the spatial and temporal marginals of the complex models, based on the construction given in (a) (20) with $(\mathbf{c}_s, c_t) = (1.5, 1.5, 3)$, (b) (31), with $(\mathbf{c}_s, c_t) = (1.5, 1.5, 3)$ and $a = 0.4$, where $\tilde{R}$ is defined in Eq. (30), with $(b_s, b_t, \alpha, \gamma) = (1, 0.5, 0.5, 0.5)$, (c) (20), with $(\mathbf{c}_s, c_t) = (0.3, 0.3, 0.3)$ and $\tilde{R}$ as in Eq. (30) with $(b_s, b_t, \alpha, \gamma) = (1, 2.5, 0.5, 0.5)$ (continuous line) or $\tilde{R}$ as in Eq. (30) with $(b_s, b_t, \alpha, \gamma) = (12, 12, 0.5, 0.5)$ (dashed line), and (d) (31), with $(\mathbf{c}_s, c_t) = (0.3, 0.3, 0.3)$, $a = 0.4$, and $(b_s, b_t, \alpha, \gamma) = (1, 2.5, 0.5, 0.5)$

for the class (20) the effect of the periodic component can be left out by keeping low the $\tilde{R}$ parameters associated with the ranges, as shown by the dashed lines in Fig. 2a.

3.4 Advantages and Further Generalizations

The potentiality of the proposed approaches is related to the flexibility of building wide families of spatiotemporal complex-valued covariance functions, by easily defining the real and imaginary parts exploiting what is already known on real-valued positive definite functions in space-time. Indeed, the real component used in the convolution-based class, given in Eq. (13), or the real-valued covariance factor in the construction based on asymmetric spectral density, as in (20), as well as the real-valued covariance factor in the construction through a positive mixture, as in (27), can be explicated by selecting a model within a wide range of separable and non-separable spatiotemporal classes of covariance models available in the literature [7, 13, 14, 17, 20–24, 28, 29, 31]. Moreover, the classes proposed in this section can be further used to generalize the convolution-based class in Eq. (13), as specified below.

Let $R^{re}(\mathbf{h}_s, h_t)$ be a real-valued covariance function and $a \in]0, 1[$; then

$$R(\mathbf{h}_s, h_t; a, \mathbf{c}) = \frac{1 - a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{1 - 2a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) + a^2} R^{re}(\mathbf{h}_s, h_t) + \\ + 0.5i \frac{1 - a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t)}{1 - 2a \cos(\mathbf{h}_s \cdot \mathbf{c}_s + h_t c_t) + a^2} \\ [R^{re}(\mathbf{h}_s - \boldsymbol{\tau}_s, h_t - \tau_t) - R^{re}(\mathbf{h}_s + \boldsymbol{\tau}_s, h_t + \tau_t)] \quad (34)$$

is a class of complex-valued covariance models.

The convolution-based class and its generalization, the class based on asymmetric spectral density and the class obtained through a positive mixture cover an extensive range of complex correlation structures, whose real part (not necessarily nonnegative) and imaginary part do/do not exhibit a periodic component. In particular, the convolution-based class can be easily applied to describe complex covariance surfaces with a nonnegative real part, the class based on asymmetric spectral density is marked by a periodic factor for both the real and imaginary components, whose effect is often damped through the product with a real monotonically decaying covariance model. Thus, they can be typically used for modeling complex covariance surfaces, whose real part can assume negative values. Differently from the other two classes, the family of the mixture models presents a real part which is nonnegative unless the model used for the real-valued covariance factor assumes negative values; moreover, the amplitude and the frequency of the periodic factor can be conveniently fixed according to the empirical behavior of the real and imaginary components, or if necessary, the effect of the periodic factor can be left out, without producing a contextual flattening of the imaginary part as happens for the class based on translation of a symmetric spectral density. Some details on the

estimation and modeling aspects and some applications on oceanographic data can be found in [10, 11].

4 Conclusions

In this contribution, the formalism of complex-valued random fields was introduced in a space-time domain and some classes of spatiotemporal complex covariance models were presented. However, these advances of the complex-valued random fields in space-time can be considered a good starting point for future works, focused on further theoretical developments and practical aspects which can also unleash the potential in environmental applications.

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