Published for SISSA by Springer

Received: *December 4, 2025*
Accepted: *February 8, 2026*
Published: *March 27, 2026*

2-loop free energy of M2 brane in $\mathrm{AdS}_7 \times S^4$ and surface defect anomaly in (2,0) theory

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Abstract: $\frac{1}{2}$-BPS surface operator viewed as a conformal defect in rank N 6d (2,0) theory is expected to have a holographic description in terms of a probe M2 brane wrapped on AdS_3 in the $\mathrm{AdS}_7 \times S^4$ M-theory background. The M2 brane has the effective tension $\mathrm{T}_2 = \frac{2}{\pi}N$ so that the large tension expansion corresponds to the $1/N$ expansion. The value of the defect conformal anomaly coefficient in $\mathrm{SU}(N)$ (2,0) theory was previously argued to be $\mathrm{b}= 12N - 9 - 3N^{-1}$. Semiclassically quantizing M2 brane it was found in arXiv:2004.04562 that the first two terms in b are indeed reproduced by the classical and 1-loop corrections to the M2 free energy. Here we address the question if the 2-loop term in the M2 brane free energy reproduces the N^{-1} term in b. Remarkably, despite the general non-renormalizability of the standard BST M2 brane action we find that the 2-loop correction to the free energy of the AdS_3 M2 brane in $\mathrm{AdS}_7 \times S^4$ is UV finite (modulo power divergences that can be removed by an analytic regularization). Moreover, the 2-loop correction vanishes in the dimensional and ζ-function regularizations. This result appears to be in disagreement with the non-vanishing of the coefficient of the N^{-1} term in the expected expression for the anomaly coefficient b. We discuss possible resolutions of this puzzle, including the one that the M2 brane probe computation may be capturing the surface defect anomaly in the $\mathrm{U}(N)$ rather than the $\mathrm{SU}(N)$ boundary 6d CFT.

Keywords: $1/N$ Expansion, AdS-CFT Correspondence, M-Theory

ArXiv ePrint: 2511.22306

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https://doi.org/10.1007/JHEP03(2026)257

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Contents

1 Introduction

It was suggested in [1] that considering a quantum M2 brane wrapped on $\mathrm{AdS}_3 \subset \mathrm{AdS}_7$ in $\mathrm{AdS}_7 \times S^4$ background one may capture not only the leading in N but also subleading coefficients in the defect b-anomaly coefficient of the S^2 conformal defect in the boundary theory

$$\mathrm{b} = 12N - 9 + \mathcal{O}(N^{-1})\,. \tag{1.1}$$

Here N is the number of M5 branes forming the $\mathrm{AdS}_7 \times S^4$ background (rank of the (2,0) boundary CFT). It is related to the effective dimensionless M2 brane tension as $\mathrm{T}_2 = \frac{2}{\pi}N$. The first term in (1.1) corresponds to the classical M2 action contribution while the second one is the 1-loop contribution to the AdS_3 M2 free energy F [1].[1]

The general expression for the b-anomaly corresponding to a $\frac{1}{2}$-BPS surface defect operator in (2,0) theory which corresponds to an $\mathrm{SU}(N)$ representation with the Young tableau having

[1]For a general discussion of the leading brane-probe action contribution to a boundary defect anomaly see [2]. For an arbitrary 2-surface a defect operator has three anomaly coefficients, each multiplying a particular conformally invariant integral on the surface related to its topology, extrinsic curvature and background Weyl tensor [3].

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a large number of boxes is given by [4–6]

$$\mathrm{b} = 24(\rho, \lambda) + 3(\lambda, \lambda)\,, \tag{1.2}$$

where ρ is the Weyl vector of SU(N) and λ is the highest weight of the SU(N) representation. If one formally assumes that this relation is valid for a finite number of boxes then for a surface operator in the fundamental representation (with $(\rho, \lambda) = \frac{1}{2}(N-1),\ (\lambda, \lambda) = 1 - N^{-1}$) which should be described by a single M2 brane probe one finds[2]

$$\mathrm{b} = 12N\left(1 + \frac{1}{4}N^{-1}\right)(1 - N^{-1}) = 12N - 9 - 3N^{-1}\,. \tag{1.3}$$

The first two terms here match the ones in (1.1) while the N^{-1} term should then correspond to the 2-loop M2 brane contribution.

The aim of the present work is to compute the 2-loop term in the M2 brane free energy to check if it reproduces the $-3N^{-1}$ term in (1.3). A similar 2-loop computation of the free energy of the GS string with AdS_2 minimal surface in $\mathrm{AdS}_5 \times S^5$ that is expected to be related to the strong-coupling expansion of the circular Wilson loop in $\mathcal{N} = 4$ SYM theory (cf. [10]) will be discussed in [11].

Let us first review the general setup in [1]. We are going to consider an M2 brane probe in the $\mathrm{AdS}_7 \times S^4$ background

$$ds^2 = L^2(ds^2_{\mathrm{AdS}_7} + \mathrm{r}^2 ds^2_{S^4})\,, \qquad L^3 = 8\pi N \ell_p^3\,, \qquad \mathrm{r} = \frac{1}{2}\,, \tag{1.4}$$

$$\mathcal{F}_4 = dC_3 = \frac{3}{8}L^3 \mathrm{vol}_{S^4}\,, \qquad \int_{S^4} \mathrm{vol}_{S^4} = \mathrm{vol}(S^4) = \frac{8\pi^2}{3}\,. \tag{1.5}$$

The BST M2 brane action [12, 13] contains the bosonic part (the standard Dirac-Nambu term $S_1(X)$ and a WZ-type term $S_2(X)$ of coupling to the 3-form C_3) and also the fermionic part $S_f(X, \theta)$

$$S = S_b + S_f\,, \quad S_b = S^{(1)} + S^{(2)}\,, \quad S^{(1)} = T_2 \int d^3\sigma \sqrt{h}\,, \qquad h_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N G_{MN}(X)\,, \tag{1.6}$$

$$S^{(2)} = iT_2 \int d^3\sigma\, \frac{1}{3!}\epsilon^{\mu\nu\lambda} C_{MNK}(X)\, \partial_\mu X^M \partial_\nu X^N \partial_\lambda X^K\,, \qquad T_2 = \frac{1}{(2\pi)^2 \ell_p^3}\,. \tag{1.7}$$

The explicit form of the M2 brane action in $\mathrm{AdS}_7 \times S^4$ can be found from [14, 15].

The world-volume geometry of an M2 brane ending on a 2-sphere at the boundary of AdS_7 is described by the (Euclidean) AdS_3 metric. The tree level contribution to the free energy $F = -\log Z$ is given by the classical value of the M2 brane action which is proportional to

[2]The expression in (1.3) and thus (1.3) as an exact result in N could still be viewed as conjecture. However, in [6] a similar expression for the d_2 anomaly coefficient was derived as an exact result from a superconformal index computation. It also follows from the 5d Wilson Loop localization computation as in [7]. Given that b and d_2 appear on an equal footing in the spherical entanglement entropy [5, 8] one may expect that the expression for b should also be exact. Indeed, the same expression for b was found on the dual CFT side in [9] using 't Hooft anomaly considerations.

the regularized volume of the induced AdS_3 metric (cf. [16])[3]

$$F_0 = \mathrm{T}_2 \ \mathrm{vol}(\mathrm{AdS}_3) = -2\pi \mathrm{T}_2 \ \log \bar{\Lambda} \,, \qquad \mathrm{vol}(\mathrm{AdS}_3) = -2\pi \log \bar{\Lambda}, \qquad \mathrm{T}_2 = L^3 T_2 = \frac{2}{\pi} N \,. \tag{1.8}$$

Since AdS_3 is a homogeneous space, all quantum corrections to F will be also proportional to $\mathrm{vol}(\mathrm{AdS}_3)$, i.e.

$$F = F_0 + F_1 + F_2 + \ldots = f(\mathrm{T}_2)\, \mathrm{vol}(\mathrm{AdS}_3) \equiv -\frac{1}{3} \mathrm{b} \ \log \bar{\Lambda} \,, \tag{1.9}$$

$$f(\mathrm{T}_2) = \mathrm{T}_2 f_0 + f_1 + (\mathrm{T}_2)^{-1} f_2 + \ldots \,, \qquad \mathrm{b} = 6\pi f = N \, \mathrm{b}_0 + \mathrm{b}_1 + N^{-1} \mathrm{b}_2 + \ldots \,. \tag{1.10}$$

Since $\bar{\Lambda}$ plays the role of a UV cutoff in the boundary theory, b may be interpreted as an S^2 defect conformal anomaly in the (2,0) theory.

Fixing the static gauge and expanding the M2 action to quadratic order in fluctuations near AdS_3 background one finds that the spectrum of the resulting AdS_3 fields consisting of 4 bosons x^i with $m_b^2 = 3$, 4 bosons y^a with $m = 0$ and 8 fermions θ with $m_f = \frac{3}{2}$. This spectrum of transverse fluctuations of the M2 brane subject to the standard Dirichlet b.c. is in direct correspondence with a protected supermultiplet (that includes the displacement operator) of operator insertions on the defect surface [1]. The resulting 1-loop correction F_1 in (1.9) is given by

$$F_1 = \frac{1}{2} \Big[4 \log \det(-\nabla^2 + 3) + 4 \log \det(-\nabla^2) - 8 \log \det \Delta_{1/2} \Big] = f_1 \, \mathrm{vol}(\mathrm{AdS}_3) \,. \tag{1.11}$$

There are no 1-loop log divergences in 3d so that f_1 is finite when computed using the standard ζ-function regularization.[4] As a result, one finds that [1]

$$f_1 = -\frac{3}{2\pi} \,, \qquad \text{i.e.} \qquad \mathrm{b}_1 = -9 \,. \tag{1.12}$$

Combining (1.8) and (1.12) we get the first two terms in (1.1), (1.3).

To find 2-loop correction to the free energy (1.9) requires expanding the M2 brane action (1.6) near AdS_3 surface to quartic order in the fluctuation fields (x^i, y^a, θ). Remarkably, like for the M2 brane in static gauge in flat target space case [19, 20] (and also as for the GS string expanded near AdS_2 in $\mathrm{AdS}_5 \times S^5$ [11]) there exists a natural κ-symmetry gauge in which there are no cubic couplings in the action. The M2 Lagrangian has then the following symbolic form[5]

$$\begin{aligned} L = (\partial x)^2 + m_b^2 x^2 + (\partial y)^2 + \bar{\theta}(\nabla\!\!\!/ + m_f)\theta + \mathrm{T}_2^{-1} \Big[(\partial x)^4 + x^2 (\partial x)^2 + x^4 + (\partial x)^2 (\partial y)^2 \\ + (\partial y)^4 + y^2 (\partial y)^2 + (\partial x \partial x + x^2)(\theta \nabla \theta + \theta^2) + \ldots + \theta\theta\theta\nabla\theta + \theta\nabla\theta\theta\nabla\theta + \ldots \Big] + \mathcal{O}(\mathrm{T}_2^{-2}). \end{aligned} \tag{1.13}$$

[3] Here $\bar{\Lambda} = \Lambda_{\mathrm{IR}} a$ where Λ_{IR} is an IR cutoff in AdS_3 and a is the radius of the boundary S^2. In general, a regularized volume of a global AdS_{p+1} space with S^p as its boundary is log IR divergent for even p (discarding power divergences, see, e.g., [17]): $\mathrm{vol}(\mathrm{AdS}_{p+1}) = \frac{2(-\pi)^{p/2}}{\Gamma(1+p/2)} \log \bar{\Lambda}$.

[4] Using heat-kernel cutoff, one finds that the leading cubic divergence cancels out due to supersymmetric balance of degrees of freedom. Linear divergence does not automatically cancel but is absent in an analytic regularization like the ζ-function one. Similar linear divergence was regularized away using ζ-function in a different 1-loop M2 brane computation in [18].

[5] Here all indices are contracted with the induced AdS_3 metric and derivatives are 3d covariant so there is a manifest AdS_3 symmetry with fermions θ treated effectively as a set of 8 Majorana 3d fermions. We rescaled the fluctuations by $\sqrt{\mathrm{T}_2}$.

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As a result, the relevant 2-loop diagrams are just the bubble "OO" ones, i.e. are given by products of (derivatives of) two boson, one boson and one fermion and two fermion propagators in AdS_3 at coinciding points.

Since in 3d the propagators have no log divergences, the 2-loop correction F_2 may then have only power divergences and thus will be finite assuming one uses an analytic regularization like dimensional regularization where one replaces AdS_3 with AdS_{d+1} with $d = 2 - 2\varepsilon$. Below we will use its dimensional reduction version by treating fermions (and related Dirac matrices) not as $d+1$ but as 3-dimensional ones. We will find that then the 2-loop coefficient in (1.10) is given by

$$(f_2)_{\rm dred} = 12\,\frac{d-2}{d+1}\mathrm{G}_x^2 + 12\,\frac{(d-2)(d+7)}{(d+1)^2}\,\mathrm{G}_x\,\mathrm{G}_\theta + 24\,\frac{(d-2)^2}{(d+1)^2}\,\mathrm{G}_\theta^2\,, \tag{1.14}$$

where G_x and G_θ are the coincident-point limits of the massive scalar ($m_b^2 = 3$) and massive fermion ($m_f = \frac{3}{2}$) propagators respectively. These are finite for $\varepsilon = \frac{2-d}{2} \to 0$

$$\mathrm{G}_x = -\frac{1}{2\pi} + \mathcal{O}(\varepsilon)\,, \qquad\qquad \mathrm{G}_\theta = \frac{1}{2\pi} + \mathcal{O}(\varepsilon)\ . \tag{1.15}$$

Since f_2 in (1.14) is proportional to $d-2$, it thus vanishes in the $\varepsilon \to 0$ limit, i.e.[6]

$$(\mathrm{b}_2)_{\rm dred} = \frac{3\pi^2}{2}(f_2)_{\rm dred} = 0\ . \tag{1.16}$$

This contradicts the prediction in (1.3) which implies that b_2 in (1.10) should be equal to -3.

One may wonder if this mismatch is due to a "wrong" choice of regularization. In general, the world-volume UV regularization should be the one which is consistent with underlying symmetries of the problem and thus hopefully with the suggested AdS/CFT interpretation of the M2 brane free energy as capturing the value of the surface defect anomaly. The dimensional reduction regularization appears, in fact, to be a natural choice as it should preserve the world-volume supersymmetry that is present in the M2 brane action expanded near a supersymmetric minimal surface: this 3d supersymmetry is a residue of the target space supersymmetry after fixing a κ-symmetry gauge [12, 13, 21, 22]. Still, it could be that some other analytic regularization is a more adequate one. Below we will explore this option and conclude that the same vanishing result is found also in the same ζ-function regularization that was used at the 1-loop level in [1]. This strongly suggests that the vanishing of f_2 is indeed a robust conclusion.

One may also suspect that a reason for the disagreement between (1.3) and (1.16) may be related to the fact that the formally non-renormalizable BST M2 brane action should be supplemented by higher derivative counterterms [20]: these may then also contribute to the 2-loop value of the free energy evaluated on the AdS_3 surface. Alternatively, there may be some profound reason for this disagreement that has to do with how the AdS/CFT correspondence is to be implemented in the context of M2 brane partition functions (cf. [23, 24]).

[6] The same vanishing result for f_2 is found also in the straightforward dimensional regularization as near 3d G_x and G_θ do not have poles in $\frac{1}{\varepsilon}$.

Finally, the simplest explanation for how to reconcile our vanishing 2-loop result with the general expression for the b-coefficient in (1.2) was suggested to us by J. van Muiden. One may conjecture that the quantum M2 brane free energy should correspond to a surface defect in the U(N) rather than SU(N) boundary 6d theory. While the standard argument [25] about decoupling of the U(1) subgroup may apply to bulk observables (captured by quantum M2 branes in the bulk of $\mathrm{AdS}_7 \times S^4$ and, in particular, by 11d supergravity modes) the boundary observables described by M2 brane ending on AdS_7 may correspond to defect operators in the U(N) (2,0) theory. Assuming that the general relation for b in (1.2) may be formally applied to the U(N) case[7] with the fundamental representation corresponding to $(\rho, \lambda) = \frac{1}{2}(N-1)$, $(\lambda, \lambda) = 1$ (with scalar product suitably generalized to non-simple U(N) case) one finds instead of (1.3)[8]

$$\mathrm{b} = \left[24(\rho, \lambda) + 3(\lambda, \lambda)\right]\Big|_{\mathrm{U}(N), \square} = 12N - 9\,. \tag{1.17}$$

The rest of this paper is organized as follows. In section 2 we will find the 2-loop contribution to free energy (1.9) from the 4+4 transverse bosonic fluctuations in the static gauge. The fermionic 2-loop contribution will be computed in section 3. The total result for the 2-loop coefficient f_2 will be presented in section 4. In section 5 we will discuss several possible explanations for the above disagreement.

In appendix A we shall summarize the expressions for the bosonic and fermionic Green's functions and their derivatives in AdS_{d+1} and specify them to the case of dimensional reduction regularization in $d = 2 - 2\varepsilon$ and the ζ-function regularization in $d = 2$. In appendix B we will review the supercoset construction of the M2 brane action in $\mathrm{AdS}_7 \times S^4$ and discuss its expansion in powers of fermions. In appendix C we will summarize the expressions for the quadratic and quartic fermionic correlators at coincident points that are used in section 3.

2 2-loop contribution from bosons

2.1 Expansion of bosonic part of M2 brane action

Let us recall the expansion of the bosonic part of the M2 brane action in $\mathrm{AdS}_7 \times S^4$ in the static gauge. As in [1, 27], for generality, let us consider a p-brane in $\mathrm{AdS}_{D+1} \times S^n$ with world volume ending along a p-dimensional surface at the boundary. Let us choose the following AdS_{p+1}-adapted parametrization of AdS_{D+1} (with radius 1)

$$ds^2_{D+1} = \frac{(1 + \frac{1}{4}x^2)^2}{(1 - \frac{1}{4}x^2)^2} ds^2_{p+1} + \frac{dx^i dx^i}{(1 - \frac{1}{4}x^2)^2}\,, \tag{2.1}$$

[7]Note that in [4] this relation was abstracted from their supergravity result *assuming* it corresponds to the SU(N) case. For a n-symmetric SU(N) representation (in the context of [4] N is the number of M5 branes and n is the number of M2 branes) one has from (1.2): $\mathrm{b} = 12nN - 3n(4-n) - 3n^2/N$. The supergravity discussion in [4] is actually applicable for $n, N \gg 1$ with n and N being of the same order. In this limit the last $-3n^2/N$ term in b is subleading and thus one cannot actually distinguish between the SU(N) and U(N) cases (see also footnote 8).

[8]The absence of the $1/N$ correction to the defect anomaly coefficients b and d_2 (that have similar structure) in the case of a general (n, m) representation of U(N) is implied by the matrix model computation of the 5d Wilson loop expectation value related to the d_2 coefficient [7], see appendix H and in particular eq. (H.22) in [26] for details.

where $i=1,\ldots,D-p$. In the static gauge where the p-brane world volume coordinates σ^μ are identified with the AdS_{p+1} ones the induced metric is that of the AdS_{p+1}

$$ds^2_{p+1} \equiv g_{\mu\nu}(\sigma)\, d\sigma^\mu d\sigma^\nu = ds^2_{\mathrm{AdS}_{p+1}} \, . \tag{2.2}$$

Then the volume $S^{(1)}$ part of the brane action in (1.6) takes the form

$$S^{(1)} = \mathrm{T}_p \int d^{p+1}\sigma \sqrt{\det\left[\frac{(1+\frac14 x^2)^2}{(1-\frac14 x^2)^2} g_{\mu\nu}(\sigma) + \frac{\partial_\mu x^i \partial_\nu x^i}{(1-\frac14 x^2)^2} + \frac{\partial_\mu y^a \partial_\nu y^a}{(1+\frac{1}{4\mathrm{r}^2} y^2)^2}\right]} \equiv \int d^{p+1}x\sqrt{g}\; L. \tag{2.3}$$

Here y^a are coordinates of S^n and r is its radius in units of the radius L of AdS_{D+1} (cf. (1.4)). L is absorbed into the dimensionless effective tension $\mathrm{T}_p = L^{p+1} T_p$, cf. (1.8).

Expanding (2.3) in powers of the fluctuations x^i and y^a we get [1][9]

$$L = L_{2\mathrm{b}} + \mathrm{T}_p^{-1} L_{4\mathrm{b}} + \ldots\, , \qquad\qquad L_{4\mathrm{b}} = L_{4x} + L_{2x,2y} + L_{4y}\, , \tag{2.4}$$

$$L_{2\mathrm{b}} = \frac12\left[\partial^\mu x^i \partial_\mu x^i + (p+1)\, x^i x^i\right] + \frac12 \partial^\mu y^a \partial_\mu y^a\, , \tag{2.5}$$

$$L_{4x} = \frac18 (\partial^\mu x^i \partial_\mu x^i)^2 - \frac14 (\partial^\mu x^i \partial_\mu x^j)\,(\partial^\nu x^i \partial_\nu x^j) + \frac14 p\, x^i x^i\, \partial^\mu x^j \partial_\mu x^j + \frac18 (p+1)^2 x^i x^i\, x^j x^j\, , \tag{2.6}$$

$$L_{2x,2y} = \frac14 (\partial^\mu x^i \partial_\mu x^i)\,(\partial^\nu y^a \partial_\nu y^a) - \frac12 (\partial^\mu x^i \partial_\mu y^a)\,(\partial^\nu x^i \partial_\nu y^a) + \frac14 (p-1) x^i x^i\, \partial^\mu y^a \partial_\mu y^a\, , \tag{2.7}$$

$$L_{4y} = \frac18 (\partial^\mu y^a \partial_\mu y^a)^2 - \frac14 (\partial^\mu y^a \partial_\mu y^b)\,(\partial^\nu y^a \partial_\nu y^b) - \frac{1}{4\mathrm{r}^2} y^b y^b\, \partial^\mu y^a \partial_\mu y^a\, . \tag{2.8}$$

The case of the AdS_2 string in $\mathrm{AdS}_5 \times S^5$ considered in [11, 27] corresponds to $p=1,\ D=4,\ n=5,\ \mathrm{r}=1$ while in the present case of AdS_3 M2 brane in $\mathrm{AdS}_7 \times S^4$ (cf. (1.4))

$$p=2\, , \qquad D=6\, , \qquad n=4\, , \qquad \mathrm{r}=\frac12\, . \tag{2.9}$$

We thus get 4 massive transverse AdS_7 fluctuation fields x^i (with $m^2=3$) and 4 massless S^4 fields y^a propagating in the induced AdS_3 geometry.

The WZ term in (1.7) may be written as [1] ($Y^m Y^m = 1, \quad m=1,\ldots,5$)

$$\begin{aligned} S_2 = iT_2 \int C_3 = iT_2 \int \mathcal{F}_4 &= \frac{i}{64}\mathrm{T}_2 \int d^4\sigma\; \epsilon_{mnpqu}\, \epsilon^{\mu\nu\lambda\rho}\, Y^m \partial_\mu Y^n \partial_\nu Y^p \partial_\lambda Y^q \partial_\rho Y^u \\ &= \frac{i}{4}\mathrm{T}_2 \int d^3\sigma\, \epsilon^{\mu\nu\lambda}\, \epsilon_{abcd}\, y^a \partial_\mu y^b \partial_\nu y^c \partial_\lambda y^d + \mathcal{O}(y^5)\, , \end{aligned} \tag{2.10}$$

where $Y^5 = \frac{1-y^2}{1+y^2}\, ,\ \ Y^a = \frac{2y^a}{1+y^2}$. The quartic term in (2.10) (see also (B.36)) will not contribute to the 2-loop free energy F_2 in (1.9) due to the symmetry of the resulting contractions $\langle y^a y^b\rangle \sim \delta^{ab}$. As a result, the bosonic contribution to F_2 will come only from (2.5)–(2.8) and thus will have the form which is universal in p.

[9]Here μ, ν indices are contracted by AdS_{p+1} metric $g_{\mu\nu}$ and we rescaled fluctuations by the square root of the tension.

2.2 Expectation value of quartic bosonic terms

Defining the Euclidean M2 brane partition function as $Z = e^{-F} = \int [dx\, dy\, d\theta]\ e^{-S}$ where S is the action in (1.6) the 2-loop contribution F_2 to free energy in (1.9) may be written as an expectation value of the quartic term in the expansion of the action

$$S_4 = \mathrm{T}_2^{-1} \int d^3\sigma \sqrt{g}\, L_4 \,, \qquad\qquad L_4 = L_{4\mathrm{b}} + L_{4\mathrm{bf}} + L_{4\mathrm{f}} \,, \tag{2.11}$$

$$F_2 = \mathrm{T}_2^{-1} \int d^3\sigma \sqrt{g}\, \langle L_4 \rangle = \mathrm{T}_2^{-1}\, \mathrm{vol}(\mathrm{AdS}_3)\, \langle \mathrm{L}_4 \rangle \,, \qquad f_2 = \langle L_4 \rangle \,, \tag{2.12}$$

where we used that since the M2 brane action written in terms of the AdS_3 metric has constant coefficients and that AdS_3 is a homogeneous space the AdS_3 volume factor factorizes (cf. (1.9), (1.10)). $L_{4\mathrm{b}}$ in (2.11) stands for the quartic bosonic term in (2.4) while the terms involving fermions (cf. (1.13)) will be discussed in section 3 below.

Then from (2.4) we find for the purely bosonic contribution

$$\begin{aligned} f_{2,4\mathrm{b}} = \langle L_{4\mathrm{b}} \rangle =& \frac{1}{8}(p^2-1)N_x^2 \tilde{\mathrm{G}}_x^2 - \frac{1}{4}(p+1)^2 N_x \tilde{\mathrm{G}}_x^2 \\ &+ \frac{1}{4}p(p+1)N_x^2 \mathrm{G}_x \tilde{\mathrm{G}}_x + \frac{1}{8}(p+1)^2 N_x(N_x+2)\mathrm{G}_x^2 \\ &+ \frac{1}{4}(p^2-1)N_x N_y \tilde{\mathrm{G}}_x \tilde{\mathrm{G}}_y + \frac{1}{4}(p^2-1)N_x N_y \mathrm{G}_x \tilde{\mathrm{G}}_y \\ &+ \frac{1}{8}(p^2-1)N_y^2 \tilde{\mathrm{G}}_y^2 - \frac{1}{4}(p+1)^2 N_y \tilde{\mathrm{G}}_y^2 - \frac{1}{4r^2}(p+1)N_y^2 \mathrm{G}_y \tilde{\mathrm{G}}_y \ . \end{aligned} \tag{2.13}$$

Here $N_x = D - p$ and $N_y = n$ are the numbers of the corresponding fluctuations around AdS_{p+1} in $\mathrm{AdS}_{D+1} \times S^n$, i.e. in the M2 brane case (2.9)

$$p = 2: \qquad N_x = 4, \qquad N_y = 4 \ . \tag{2.14}$$

$\mathrm{G}_{x,y}$ and $\tilde{\mathrm{G}}_{x,y}$ are the coincident limits of the corresponding scalar Green's functions in AdS_{p+1}

$$\begin{aligned} \langle x^i(\sigma) x^j(\sigma') \rangle &= \delta^{ij} G_x(\sigma,\sigma') \,, & G_x(\sigma,\sigma) &= \mathrm{G}_x \,, \\ \partial_\mu \partial'_\nu G_x(\sigma,\sigma') &= g_{\mu\nu} \tilde{G}_x(\sigma,\sigma'), & \tilde{G}_x(\sigma,\sigma) &= \tilde{\mathrm{G}}_x \,, \end{aligned} \tag{2.15}$$

and similarly for $\langle y^a(\sigma) y^b(\sigma') \rangle = \delta^{ab} G_y(\sigma,\sigma')$, etc.

Note that (2.13) written for general p is not the same as the result found using dimensional regularization near a particular value of p: in the latter case we are first to specify p (i.e. fix the coefficients in the fluctuation Lagrangian in (2.4)–(2.8), in particular the mass of x^i fluctuations as $m^2 = p+1$) and then replace AdS_{p+1} by AdS_{d+1} with $d = p - 2\varepsilon$, i.e. extend the indices of derivatives and $g_{\mu\nu}$ to $d+1$ values. Then $g_{\mu\nu} g^{\mu\nu} = d+1$ giving a generalization of (2.13)

$$\begin{aligned} f_{2,4\mathrm{b}} =& \frac{1}{8}(d^2-1)N_x^2 \tilde{\mathrm{G}}_x^2 - \frac{1}{4}(d+1)^2 N_x \tilde{\mathrm{G}}_x^2 \\ &+ \frac{1}{4}p(d+1)N_x^2 \mathrm{G}_x \tilde{\mathrm{G}}_x + \frac{1}{8}(p+1)^2 N_x(N_x+2)\mathrm{G}_x^2 \\ &+ \frac{1}{4}(d^2-1)N_x N_y \tilde{\mathrm{G}}_x \tilde{\mathrm{G}}_y + \frac{1}{4}(d+1)(p-1)N_x N_y \mathrm{G}_x \tilde{\mathrm{G}}_y \\ &+ \frac{1}{8}(d^2-1)N_y^2 \tilde{\mathrm{G}}_y^2 - \frac{1}{4}(d+1)^2 N_y \tilde{\mathrm{G}}_y^2 - \frac{1}{4r^2}(d+1)N_y^2 \mathrm{G}_y \tilde{\mathrm{G}}_y \ . \end{aligned} \tag{2.16}$$

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Introducing the notation for the "equation of motion" $(-\nabla^2 + m^2)G|_{\sigma=\sigma'}$ or "$\delta(0)$" combinations

$$\hat{\mathrm{G}}_x \equiv \tilde{\mathrm{G}}_x + \frac{p+1}{d+1}\mathrm{G}_x\,, \qquad \hat{\mathrm{G}}_y \equiv \tilde{\mathrm{G}}_y\,, \tag{2.17}$$

we may represent (2.16) as

$$\begin{aligned} f_{2,4\mathrm{b}} = \frac{(d-p)(p+1)}{4(d+1)} N_x^2\, \mathrm{G}_x^2 + \hat{\mathrm{G}}_x \bigg\{ & \Big[\frac{1}{8}(d^2-1)N_x^2 - \frac{1}{4}(d+1)^2 N_x\Big]\hat{\mathrm{G}}_x \\ & + \Big[\frac{1}{4}(2p-d+1)N_x^2 + \frac{1}{2}(d+1)(p+1)N_x\Big]\mathrm{G}_x \bigg\} \\ + \hat{\mathrm{G}}_y \bigg\{ & \frac{1}{4}(d^2-1)N_x N_y \hat{\mathrm{G}}_x + \frac{1}{2}(p-d)N_x N_y \mathrm{G}_x \\ & + \Big[\frac{1}{8}(d^2-1)N_y^2 - \frac{1}{4}(d+1)^2 N_y\Big]\hat{\mathrm{G}}_y - \frac{1}{4\mathrm{r}^2}(d+1)N_y^2 \mathrm{G}_y \bigg\}. \end{aligned} \tag{2.18}$$

Thus the total expression for any p is given by the sum of the first G_x^2 term with the two extra contributions proportional to $\hat{\mathrm{G}}_x$ and $\hat{\mathrm{G}}_y$, i.e. to the "$\delta(0)$" terms.

Using that in the dimensional regularization where $d = p - 2\varepsilon$ (see appendix A.1)

$$\hat{\mathrm{G}}_x = 0\,, \qquad \hat{\mathrm{G}}_y = 0\,, \tag{2.19}$$

we conclude that (2.18) reduces to

$$(f_{2,4\mathrm{b}})_{\mathrm{dreg}} = \frac{(d-p)(p+1)}{4(d+1)} N_x^2\, \mathrm{G}_x^2 = -\frac{\varepsilon\,(p+1)}{4(p+1-2\varepsilon)} N_x^2\, \mathrm{G}_x^2\,. \tag{2.20}$$

For $p = 2$, i.e. in $d+1 = 3-2\varepsilon$ dimensions, G_x has no pole in ε (see (1.15) and appendix A). We thus conclude that (2.20) vanishes in the limit $d \to p$. The same conclusion applies to other $p > 1$ brane cases.[10]

The expressions for the fermion contributions discussed below will have similar form: they will also be proportional to $d - p = -2\varepsilon$ and thus will vanish for $d \to p = 2$ (cf. (1.14)).

3 2-loop contribution from fermions

3.1 Expansion of the fermionic part of the M2 brane action

The explicit form of the M2 brane action [12, 13] in $\mathrm{AdS}_7 \times S^4$ may be found following [14, 15] (see also [28, 29]). We review its structure in appendix B and discuss the expansion to quartic order in the fermion field θ.

Let us first introduce the notation. Starting with the 11d Majorana spinor θ we will analytically continue to the Euclidean signature, i.e. consider the 32×32 Dirac matrices that satisfy the Clifford algebra ($A = 0,\dots,10$)

$$\{\Gamma_A, \Gamma_B\} = 2\delta_{AB}\,, \qquad \Gamma_A = (\Gamma_{\hat\alpha}, \Gamma_i, \Gamma_a)\,, \quad \hat\alpha = 0,1,2\,, \quad i = 3,\dots,6\,, \quad a = 7,\dots,10\,. \tag{3.1}$$

[10] Note that a similar general expressions (2.18), (2.20) is found also in the case of the AdS_2 string in $\mathrm{AdS}_5 \times S^5$ [11] but there G_x contains a pole and thus the analog of (2.20) is UV divergent.

To account for the Wick rotation, we assume that the charge conjugation matrix C satisfies the same properties as in the Lorentzian case: $C^T = -C$, $\Gamma_A = -C^{-1}\Gamma_A^T C$. Let us define the matrices Γ_* and Γ which satisfy

$$
\begin{aligned}
&\Gamma_* \equiv \Gamma_7\Gamma_8\Gamma_9\Gamma_{10}, && \Gamma_*^2 = \mathbb{I}, && \Gamma_* = C^{-1}\Gamma_*^T C,\\
&[\Gamma_*, \Gamma_{\hat\alpha}] = [\Gamma_*, \Gamma_i] = 0, && \{\Gamma_*, \Gamma_a\} = 0, && \qquad (3.2)\\
&\Gamma_a\Gamma_b\Gamma_c\Gamma_d = \epsilon_{abcd}\Gamma_*, && \Gamma^a\Gamma_* = \frac{1}{3!}\epsilon^{abcd}\Gamma_{bcd}, &&
\end{aligned}
$$

$$
\begin{aligned}
&\Gamma \equiv i\Gamma_0\Gamma_1\Gamma_2, && \Gamma^2 = \mathbb{I}\,, && \Gamma = C^{-1}\Gamma^T C,\\
&[\Gamma, \Gamma_{\hat\alpha}] = 0, && \{\Gamma, \Gamma_i\} = \{\Gamma, \Gamma_a\} = 0, && [\Gamma, \Gamma_*] = 0, \qquad (3.3)\\
&\Gamma^{\hat\alpha}\Gamma = \frac{i}{2!}\epsilon^{\hat\alpha\hat\beta\hat\gamma}\Gamma_{\hat\beta\hat\gamma}, && \epsilon^{012} = 1. &&
\end{aligned}
$$

We shall choose the following κ-symmetry gauge that complements the bosonic static gauge in the same way as in the flat target space case in [19, 20]

$$
\mathrm{P}\,\theta = 0\,, \qquad \mathrm{P} \equiv \frac{1}{2}(\mathbb{I} + \Gamma)\,, \qquad \mathrm{P}^2 = \mathrm{P}\,. \tag{3.4}
$$

Remarkably, like in the flat space case, the resulting expansion of the M2 brane action (see appendix B.1) will then have no cubic boson-fermion-fermion coupling terms. This substantially simplifies the computation of the corresponding 2-loop free energy.

The quadratic fermionic term in the action can be expressed in terms of the analog of the massive AdS_3 Dirac operator with $m_f = \frac{3}{2}$ [1, 30] defined by the following generalized covariant derivative[11]

$$
\mathfrak{D}_\alpha = \nabla_\alpha + \frac{1}{2}\Gamma_*\Gamma_\alpha, \qquad \Gamma_\alpha \equiv e_\alpha^{\hat\alpha}\Gamma_{\hat\alpha}\,, \qquad \not{\mathfrak{D}} = \not\nabla + \frac{3}{2}\Gamma_*\,. \tag{3.5}
$$

Here $e_\alpha^{\hat\alpha}$ is the AdS_3 3-bein ($g_{\alpha\beta} = e_\alpha^{\hat\alpha}e_\beta^{\hat\beta}\delta_{\hat\alpha\hat\beta}$) and ∇_α is the AdS_3 spinor covariant derivative. Defining

$$
h_{\alpha\beta}^{(2b)} = h_{\alpha\beta}^{(2x)} + h_{\alpha\beta}^{(2y)}, \qquad h_{\alpha\beta}^{(2\theta)} = \bar\theta\Gamma_\alpha\mathfrak{D}_\beta\theta, \tag{3.6}
$$

$$
h_{\alpha\beta}^{(2x)} = \partial_\alpha x^i\partial_\beta x^i + x^2 g_{\alpha\beta}, \qquad h_{\alpha\beta}^{(2y)} = \partial_\alpha y^a\partial_\beta y^a, \tag{3.7}
$$

one finds (see (B.35)–(B.38)) that the quadratic and quartic terms in the expansion of the M2 brane Lagrangian that supplement the bosonic terms in (2.4) can be represented as (here we specify to the M2 brane case of $p = 2$, $\mathrm{r} = \frac{1}{2}$)

$$
L = L_2 + \mathrm{T}_2^{-1}(L_{4\mathrm{b}} + L_{2\mathrm{b},2\mathrm{f}} + L_{4\mathrm{f}}) + \dots\,, \qquad L_2 = \frac{1}{2}g^{\alpha\beta}(h_{\alpha\beta}^{(2b)} + h_{\alpha\beta}^{(2\theta)}), \tag{3.8}
$$

$$
L_{2\mathrm{b},2\mathrm{f}} = \frac{1}{8}g^{\alpha\beta}(h_{\alpha\beta}^{(2x)} - 2h_{\alpha\beta}^{(2y)})\bar\theta\Gamma_*\theta + \frac{3}{8}x^2 g^{\alpha\beta}h_{\alpha\beta}^{(2\theta)} + \frac{1}{8}g^{\alpha\beta}g^{\gamma\delta}h_{\alpha\beta}^{(2b)}\,h_{\gamma\delta}^{(2\theta)} - \frac{1}{4}h^{(2b)\alpha\beta}h_{\alpha\beta}^{(2\theta)} + \dots\,, \tag{3.9}
$$

$$
L_{4\mathrm{f}} = \frac{1}{96}g^{\alpha\beta}\bar\theta\Gamma_\alpha\mathcal{M}_+^2\mathfrak{D}_\beta\theta - \frac{1}{16}g^{\alpha\delta}g^{\beta\gamma}h_{\alpha\beta}^{(2\theta)}h_{\gamma\delta}^{(2\theta)} + \frac{1}{16}(g^{\alpha\beta}h_{\alpha\beta}^{(2\theta)})^2\,. \tag{3.10}
$$

[11] One can formally view the 16-component fermion θ (remaining after the gauge fixing (3.4)) as a collection of 8 2-component 3d Majorana spinors.

We defined the following fermionic matrix (see (B.29))

$$\mathcal{M}_{+}^{2}=\Gamma_*\Gamma_\alpha\theta\ \bar\theta\Gamma^\alpha-\frac{1}{2}(\Gamma_{\alpha\beta}\theta\ \bar\theta\Gamma^{\alpha\beta}+\Gamma_{ij}\theta\ \bar\theta\Gamma^{ij}-2\Gamma_{ab}\theta\ \bar\theta\Gamma^{ab})\Gamma_*,\qquad [\Gamma,\mathcal{M}_{+}^{2}]=0\ . \tag{3.11}$$

In (3.9) dots stand for terms in (B.37) that will not contribute to the 2-loop free energy (they vanish upon use of $\langle x^i x^j\rangle\sim\delta^{ij}$, $\langle y^a y^b\rangle\sim\delta^{ab}$, $\langle x^i y^a\rangle=0$) so we will omit them in what follows.

Explicitly, we get from (3.9), (3.10)

$$\begin{aligned}L_{\rm 2b,2f}=&\frac{1}{8}(\partial_\alpha x^i\partial^\alpha x^i+3x^2)\,(\bar\theta\Gamma_*\theta+\bar\theta\not{\mathfrak{D}}\theta)+\frac{1}{8}\partial_\alpha y^a\partial^\alpha y^a(\bar\theta\not{\mathfrak{D}}\theta-2\bar\theta\Gamma_*\theta)\\&+\frac{1}{8}x^2\bar\theta\not{\mathfrak{D}}\theta-\frac{1}{4}(\partial^\alpha x^i\partial^\beta x^i+\partial^\alpha y^a\partial^\beta y^a)\,\bar\theta\Gamma_\alpha\mathfrak{D}_\beta\theta\,,\end{aligned} \tag{3.12}$$

$$\begin{aligned}L_{\rm 4f}=&\frac{1}{96}\bar\theta\Gamma^\alpha\Gamma_\beta\Gamma_*\theta\ \bar\theta\Gamma^\beta\mathfrak{D}_\alpha\theta-\frac{1}{192}\bar\theta\Gamma^\alpha\Gamma_{\beta\gamma}\theta\ \bar\theta\Gamma^{\beta\gamma}\Gamma_*\mathfrak{D}_\alpha\theta-\frac{1}{192}\bar\theta\Gamma^\alpha\Gamma_{ij}\theta\ \bar\theta\Gamma^{ij}\Gamma_*\mathfrak{D}_\alpha\theta\\&+\frac{1}{96}\bar\theta\Gamma^\alpha\Gamma_{ab}\theta\ \bar\theta\Gamma^{ab}\Gamma_*\mathfrak{D}_\alpha\theta-\frac{1}{16}\bar\theta\Gamma^\alpha\mathfrak{D}^\beta\theta\ \bar\theta\Gamma_\beta\mathfrak{D}_\alpha\theta+\frac{1}{16}(\bar\theta\not{\mathfrak{D}}\theta)^2\ .\end{aligned} \tag{3.13}$$

Anticipating the use of dimensional reduction regularization here the indices are contracted using $g_{\alpha\beta}$ of AdS_{d+1} with $d=2-2\varepsilon$ unless they are contracted with Γ_α matrices that restrict them to AdS_3 (Dirac algebra for Γ_α is assumed to be done in 3d).

3.2 Expectation values of the fermionic terms

To find the 2-loop contributions of (3.12) and (3.13) let us define the basic fermionic correlators at coincident points that complement the bosonic ones in (2.15), (2.17)

$$\langle\theta\,\bar\theta\rangle=-\Gamma_*\mathrm{G}_\theta,\qquad\langle\mathfrak{D}_\alpha\theta\,\bar\theta\rangle=-\Gamma_\alpha\hat{\mathrm{G}}_\theta,\qquad\langle\mathfrak{D}_\alpha\theta\,\mathfrak{D}_\beta\theta\rangle=(\tilde{\mathrm{G}}_\theta\,\Gamma_{\alpha\beta}+\bar{\mathrm{G}}_\theta\,g_{\alpha\beta})\,\Gamma_*\,C^{-1}\,, \tag{3.14}$$

where factors of gauge-fixing projector (3.4) are implicit. In dimensional reduction regularization (see appendices A.2 and C)

$$\hat{\mathrm{G}}_\theta=\frac{d-2}{2(d+1)}\mathrm{G}_\theta,\qquad\tilde{\mathrm{G}}_\theta=\hat{\mathrm{G}}_\theta\,,\qquad\bar{\mathrm{G}}_\theta=-\frac{1}{2}(d+2)\hat{\mathrm{G}}_\theta\ . \tag{3.15}$$

Note that these vanish in the strict $d=2$ limit.

As a result, we get from (3.12)

$$\begin{aligned}\langle L_{\rm 2b,2f}\rangle=&\frac{1}{8}(d+1)N_xN_\theta\hat{\mathrm{G}}_x\,(3\hat{\mathrm{G}}_\theta+\mathrm{G}_\theta)+\frac{1}{8}N_yN_\theta(d+1)\hat{\mathrm{G}}_y\,(3\hat{\mathrm{G}}_\theta-2\mathrm{G}_\theta)\\&+\left[\frac{3}{8}+\frac{9}{4(d+1)}\right]N_xN_\theta\mathrm{G}_x\hat{\mathrm{G}}_\theta-\frac{3}{4}N_\theta(N_x\hat{\mathrm{G}}_x+N_y\hat{\mathrm{G}}_y)\,\hat{\mathrm{G}}_\theta\ .\end{aligned} \tag{3.16}$$

Similarly, the expectation value of (3.13) is found not to depend on $\bar{\mathrm{G}}_\theta$ and is given by (see appendix C)

$$\langle L_{\rm 4f}\rangle=\frac{1}{4}N_\theta(\hat{\mathrm{G}}_\theta-3\tilde{\mathrm{G}}_\theta)\,\mathrm{G}_\theta\,+\frac{1}{32}N_\theta^2\,\hat{\mathrm{G}}_\theta(12\hat{\mathrm{G}}_\theta+\mathrm{G}_\theta)\,. \tag{3.17}$$

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4 Total 2-loop contribution

Combining (3.16) and (3.17) with the bosonic contribution in (2.18) (setting there $p = 2$, $\mathrm{r} = \frac{1}{2}$) we get the total result for the 2-loop coefficient f_2 in (1.10) in a general regularization.

In dimensional reduction regularization where $d = 2 - 2\varepsilon$ we get from (3.16) and (3.17) (see (2.17), (3.15); cf. (2.18), (2.20))

$$\langle L_{2\mathrm{b},2\mathrm{f}}\rangle_{\mathrm{dred}} = \frac{3(d-2)(d+7)}{16(d+1)^2}\, N_x N_\theta\, \mathrm{G}_x \mathrm{G}_\theta\,, \tag{4.1}$$

$$\langle L_{4f}\rangle_{\mathrm{dred}} = -\frac{(d-2)}{4(d+1)} N_\theta \Big[1 - \frac{7d-11}{16(d+1)} N_\theta\Big]\, \mathrm{G}_\theta^2. \tag{4.2}$$

where

$$N_x = N_y = 4\,, \qquad N_\theta = 16\,. \tag{4.3}$$

Like the bosonic contribution (2.20) the fermionic contributions to f_2 in (4.1) and (4.2) are all proportional to $d-2$. Summing up (2.20), (4.1), (4.2) and using (4.3) we find that the total 2-loop contribution in dimensional reduction regularization is given by[12]

$$(f_2)_{\mathrm{dred}} = 12\,\frac{d-2}{d+1}\mathrm{G}_x^2 + 12\,\frac{(d-2)(d+7)}{(d+1)^2}\,\mathrm{G}_x\mathrm{G}_\theta + 24\,\frac{(d-2)^2}{(d+1)^2}\,\mathrm{G}_\theta^2\,. \tag{4.4}$$

It thus vanishes in the $d \to 2$ limit as G_x and G_θ do not have poles near $d = 2$ (see (1.15) and appendix A)

$$\begin{aligned} \text{dred}: \qquad & \mathrm{G}_x = -\frac{1}{2\pi} + \mathcal{O}(\varepsilon)\,, \quad \hat{\mathrm{G}}_x = 0\,, \qquad \mathrm{G}_y = -\frac{1}{4\pi} + \mathcal{O}(\varepsilon)\,, \quad \hat{\mathrm{G}}_y = 0\,, \\ & \mathrm{G}_\theta = \frac{1}{2\pi} + \mathcal{O}(\varepsilon)\,, \quad \hat{\mathrm{G}}_\theta = \tilde{\mathrm{G}}_\theta = \mathcal{O}(\varepsilon)\,. \end{aligned} \tag{4.5}$$

The same conclusion is reached also in the ζ-function regularization (see appendix A.3) where the entries in (2.18), (3.16), (3.17) are the same as in the dimensional regularization for $d = 2$

$$\zeta-\text{reg}: \quad \mathrm{G}_x = -\frac{1}{2\pi}\,, \quad \hat{\mathrm{G}}_x = 0\,, \quad \mathrm{G}_y = -\frac{1}{4\pi}\,, \quad \hat{\mathrm{G}}_y = 0\,, \quad \mathrm{G}_\theta = \frac{1}{2\pi}\,, \quad \hat{\mathrm{G}}_\theta = \tilde{\mathrm{G}}_\theta = 0\,, \tag{4.6}$$

so that

$$(f_2)_{\zeta-\mathrm{reg}} = (f_2)_{\mathrm{dred}} = 0\,. \tag{4.7}$$

Note that the three terms in (4.4) vanish separately in $d = 2$. The vanishing of the bosonic contribution in (2.18), (2.20) appears to be a consequence of the special structure of the action in (2.3)–(2.8) leading to the result expressible in terms of the "$\delta(0)$" constants $\hat{\mathrm{G}}_x, \hat{\mathrm{G}}_y$ (cf. (2.15), (2.17)) that vanish for $d = 2$ in both dimensional and ζ-function regularizations. Then a similar vanishing of the mixed boson-fermion and the fermion contributions in (4.4) may be attributed to the supersymmetry of the M2 brane action.

Let us note also that contribution of a priori possible ultralocal measure in the M2 brane path integral vanishes as "$\delta(0)$" terms are zero in the above regularizations. The measure may

[12] Similar expression is found in the standard dimensional regularization where instead of $\hat{\mathrm{G}}_\theta = \frac{d-2}{2(d+1)}\mathrm{G}_\theta$ in (3.15) we have $\hat{\mathrm{G}}_\theta = 0$.

need to be accounted for in a generic cutoff regularization like the heat kernel one in which the 2-loop correction to free energy may contain power divergences (see appendix A.3).[13]

5 Concluding remarks

To summarize, in this paper we computed the 2-loop correction to the free energy of the M2 brane in $\mathrm{AdS}_7 \times S^4$ expanded near AdS_3 minimal surface. Despite general non-renormalizability of the M2 brane theory defined by the BST action (as demonstrated at 2 loops via flat-space S-matrix computation in [20]) we observed that the 2-loop AdS_3 M2 free energy is free from logarithmic UV divergences and, in fact, vanishes in either dimensional or ζ-function regularization.

Our result implies the vanishing of the $1/N$ correction to the M2 brane free energy. This appears to be in disagreement with the expected [4–6, 9] value (1.3) of the defect anomaly coefficient b in the $\mathrm{SU}(N)$ (2,0) theory.

One may suspect that this disagreement is indicating that the "defect AdS/CFT" [27] or quantum M2 brane probe description of the boundary defect CFT as proposed in [1] may be breaking down at higher orders in the semiclassical M2 brane expansion.[14] If true that would also imply that one will fail to reproduce subleading terms in the boundary defect correlators using perturbation theory in the AdS_3 M2 brane world volume theory.[15]

One reason could be that since the M2 brane theory based on BST action is not UV finite starting from 2 loops when expanded near a generic world-volume background [20] it may require to be supplemented by certain higher-derivative counterterms. A 2-loop counterterm (proportional to T_2^{-1}) evaluated on the AdS_3 background may then produce a non-zero $1/N$ correction to the free energy. Unfortunately, there is no known principle (like integrability in 2d string case) that could fix the structure of such counterterms.

As was already mentioned in the Introduction, the simplest possibility is to interpret our result as indicating that the quantum M2 brane probe corresponds to a surface defect in the $\mathrm{U}(N)$ rather than $\mathrm{SU}(N)$ boundary (2,0) theory where the b-coefficient should be given by (1.17), i.e. should have no $1/N$ correction.

A further insight into this issue may come from performing similar 2-loop computations in the case of M2 brane wrapped on either $\mathrm{AdS}_2 \times S^1$ [18] or S^3/Z_k [36] in $\mathrm{AdS}_4 \times S^7/\mathrm{Z}_k$ background defining M-theory dual of the ABJM theory. An analogy with the case discussed above suggests that in the static gauge the expansion of the M2 brane action will also have

[13] One might worry about additional 2-loop contributions to the free energy arising from possible 1-loop finite counterterms introduced to impose renormalization conditions on the 2-point functions $\langle x^i x^j \rangle$ and $\langle \theta\,\bar{\theta} \rangle$. A natural requirement is that the values of the masses remain at their tree-level values, so as to keep the conformal dimensions of the fields dual to the transverse fluctuations and fermions equal to their original values. These fluctuations belong to the protected multiplet of operator insertions on the surface defect in the $(2,0)$ theory which includes the displacement operator. It turns out, in fact, that the resulting counterterm contributions carry the explicit $d-2$ factors as in (4.4) and therefore vanish for $d=2$.

[14] One may also wonder if N of the M-theory background may get shifted like that happens in the $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ [31]. However, such shift is not expected in the maximally supersymmetric $\mathrm{AdS}_7 \times S^4$ case.

[15] Corrections to 2-point correlation functions of defect operators in (2,0) theory were discussed in [32–35].

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no cubic couplings and as a result the 2-loop M2 free energy will again vanish in an analytic regularization.[16] That may then be consistent with the ensemble conjecture in [23, 24].

Another example could be the 2-loop correction to the free energy of an M2 brane wrapped on $S^1 \times S^2$ in $\mathrm{AdS}_7 \times S^4$ where the expression for the (2,0) superconformal index suggests that it should vanish [37]. The methods of the present paper should have a straightforward generalization to these cases.

Acknowledgments

We thank N. Drukker, S. Giombi and J. van Muiden for useful discussions. We are grateful to J. van Muiden for comments on the draft and an important suggestion. MB is supported by the INFN grant GAST. SAK acknowledges the support of the President's PhD Scholarship of Imperial College London. The work of AAT is supported by the STFC grant ST/T000791/1.

A Regularized Green's functions and coincident limits

A.1 Scalar in AdS_{d+1}

For a scalar field in Euclidean AdS_{d+1} space subject to Dirichlet boundary conditions one has (see, e.g., [38])

$$S = \frac{1}{2}\int d^{d+1}\sigma\,\sqrt{g}\,(\nabla_\mu\phi\nabla^\mu\phi + m^2\phi^2), \tag{A.1}$$

$$\langle\phi(\sigma)\,\phi(\sigma')\rangle = G_\phi(\sigma,\sigma')\ , \qquad (-\nabla^2 + m^2)G_\phi(\sigma,\sigma') = \delta(\sigma,\sigma')\ , \tag{A.2}$$

$$G_\phi(\sigma,\sigma') = \frac{\Gamma(\Delta)}{2^\Delta\pi^{d/2}(2\Delta-d)\Gamma(\Delta-\frac{d}{2})}\frac{1}{(u+2)^\Delta}{}_2F_1\Big(\Delta,\Delta-\frac{d}{2}+\frac{1}{2},2\Delta-d+1,\frac{2}{u+2}\Big), \tag{A.3}$$

$$\Delta = \frac{d}{2}+\sqrt{\frac{d^2}{4}+m^2}, \qquad m^2 = \Delta(\Delta-d)\,, \qquad u = \frac{(z-z')^2+(w^v-w'^v)^2}{2zz'}\ . \tag{A.4}$$

Here u is the chordal distance which in (A.4) is specified to the Poincare coordinates, $ds^2_{\mathrm{AdS}_{d+1}} = g_{\mu\nu}(\sigma)d\sigma^\mu d\sigma^\nu = \frac{1}{z^2}(dz^2+dw^v dw^v)$.

To evaluate $G_\phi(\sigma,\sigma')$ in the coincident point limit $u\to 0$ we use that

$${}_2F_1(a,b,c;1) = \frac{\Gamma(c)\Gamma(c-b-a)}{\Gamma(c-b)\Gamma(c-a)}, \qquad \Re(c-a-b)>0, \quad c\neq 0,-1,-2,\dots, \tag{A.5}$$

and analytically continue in d to extend this relation beyond the above conditions on a,b,c parameters. We then find that

$$G_\phi(\sigma,\sigma) \equiv \mathrm{G}_\phi = \frac{1}{(4\pi)^{(d+1)/2}}\frac{\Gamma(\frac{1}{2}-\frac{d}{2})\,\Gamma(\Delta)}{\Gamma(1-d+\Delta)}\,, \qquad \partial_\mu G_\phi(\sigma,\sigma')\Big|_{\sigma=\sigma'} = 0\ . \tag{A.6}$$

[16] In particular, for $k=1$ case the computation of free energy of "instanton" M2 wrapped on S^3 in $\mathrm{AdS}_4\times S^7$ may be closely related to the one in the present paper by a certain analytic continuation (possibly modulo zero and negative modes due to instability of M2 on $S^3\subset S^7$ [36]) and thus the result may again vanish.

Considering the second derivative of G_ϕ we conclude that in dimensional regularization

$$\partial_\mu\partial'_\nu G_\phi(\sigma,\sigma')\Big|_{\sigma=\sigma'} \equiv g_{\mu\nu}\tilde{\mathrm{G}}_\phi\,, \qquad \tilde{\mathrm{G}}_\phi = \frac{1}{2(4\pi)^{(d+1)/2}}\frac{\Gamma(-\frac{1}{2}-\frac{d}{2})\Gamma(\Delta+1)}{\Gamma(-d+\Delta)}\,, \tag{A.7}$$

$$\tilde{\mathrm{G}}_\phi = -\frac{\Delta(\Delta-d)}{d+1}\mathrm{G}_\phi = -\frac{m^2}{d+1}\mathrm{G}_\phi\,, \quad \hat{\mathrm{G}}_\phi \equiv \tilde{\mathrm{G}}_\phi + \frac{m^2}{d+1}\mathrm{G}_\phi = 0\,. \tag{A.8}$$

We assume that the mass m is fixed, i.e. does not depend on d. In the case of the scalar x^i in (2.5) with $m^2 = 3$ in the $p = 2$ case (corresponding to $\Delta = 3$ for $d = p$) one finds setting $d = 2 - 2\varepsilon$, $\varepsilon \to 0$

$$\mathrm{G}_x = -\frac{1}{2\pi} + \frac{3-4\ell}{8\pi}\varepsilon + \ldots\,, \qquad \tilde{\mathrm{G}}_x = \frac{1}{2\pi} + \frac{1+4\ell}{8\pi}\varepsilon + \ldots\,, \qquad \hat{\mathrm{G}}_x = 0\,, \qquad \ell \equiv \log(\pi e^{\gamma_{\mathrm{E}}})\,. \tag{A.9}$$

For the massless fluctuations y^a we get

$$\mathrm{G}_y = -\frac{1}{4\pi} - \frac{\ell}{4\pi}\varepsilon + \cdots\,, \qquad \hat{\mathrm{G}}_y = 0\,. \tag{A.10}$$

A.2 Spinor in AdS$_{d+1}$

Let us consider a Euclidean Dirac operator with a generalized mass term proportional to a matrix $\bar{\gamma}$ commuting with Dirac matrices[17]

$$\not{\mathfrak{D}} = \not\nabla + m\,\bar{\gamma}\,, \qquad \not\nabla = \gamma^\alpha\nabla_\alpha\,, \qquad \bar{\gamma}^2 = 1, \qquad [\gamma^\alpha,\bar{\gamma}] = 0\,. \tag{A.11}$$

The corresponding Green's function is (see, e.g., [39, 40]; cf. (A.3))

$$\begin{aligned} G(\sigma,\sigma') &= \frac{1}{2^{m+(d+3)/2}\pi^{d/2}}\frac{\Gamma(m+\frac{d+1}{2})}{\Gamma(m+\frac{1}{2})}\frac{1}{(u+2)^{m+(d+1)/2}}\Big[\bar{\gamma}(\not\sigma\gamma_0+\gamma_0\not\sigma')\,\mathrm{F}_1(u)+(\not\sigma-\not\sigma')\,\mathrm{F}_2(u)\Big],\\ \mathrm{F}_1(u) &= {}_2F_1\left(m+\frac{d+1}{2},m,2m+1,\frac{2}{u+2}\right), \qquad \mathrm{F}_2(u) = {}_2F_1\left(m+\frac{d+1}{2},m+1,2m+1,\frac{2}{u+2}\right). \end{aligned} \tag{A.12}$$

Similar expression will apply in the case of the fermionic variable θ in the M2 brane action where (cf. (3.5), (3.14))

$$\gamma_\alpha \to \Gamma_\alpha, \qquad \bar{\gamma} \to \Gamma_*\,, \qquad m \to m_f = \frac{3}{2}\,. \tag{A.13}$$

In this case we find

$$G_\theta(\sigma,\sigma) \equiv -\mathrm{G}_\theta\,\Gamma_*\,, \qquad \mathrm{G}_\theta = -\frac{1}{2^{d+1}\pi^{(d+1)/2}}\frac{\Gamma(\frac{1-d}{2})\Gamma(\frac{1+d}{2}+m_f)}{\Gamma(\frac{1-d}{2}+m_f)}\,, \tag{A.14}$$

$$\nabla_\alpha G_\theta(\sigma,\sigma')\Big|_{\sigma=\sigma'} = \frac{m_f}{d+1}\Gamma_\alpha\mathrm{G}_\theta\,. \tag{A.15}$$

Thus in dimensional regularization where $d = 2-2\varepsilon$ and $\Gamma_\alpha\Gamma^\alpha = d+1$

$$\hat{\mathrm{G}}_\theta \equiv -(d+1)^{-1}(\not\nabla + m_f\Gamma_*)G_\theta(\sigma,\sigma')\Big|_{\sigma=\sigma'} = 0\,. \tag{A.16}$$

[17] Below we assume for definiteness that $m > 0$.

The expansion of G_θ for $\varepsilon \to 0$ and $m = \frac{3}{2}$ is given by (cf. (A.9))

$$\mathrm{G}_\theta = \frac{1}{2\pi} + \frac{1+2\ell}{4\pi}\,\varepsilon + \ldots\ . \tag{A.17}$$

In dimensional reduction regularization that we used in section 3 the Dirac algebra is assumed to be done in 3 dimensions so that $\Gamma^\alpha\Gamma_\alpha = 3$. Defining $\mathfrak{D}_\alpha$ as in (3.5)

$$\mathfrak{D}_\alpha \equiv \nabla_\alpha + \frac{1}{2}\Gamma_*\Gamma_\alpha = \nabla_\alpha + \frac{1}{3}m_f\Gamma_*\Gamma_\alpha\,, \tag{A.18}$$

we then get (cf. (3.14), (3.15))

$$\mathfrak{D}_\alpha G_\theta(\sigma,\sigma')\Big|_{\sigma=\sigma'} = -\Gamma_\alpha\hat{\mathrm{G}}_\theta\,, \qquad\qquad \not{\mathfrak{D}}G_\theta(\sigma,\sigma')\Big|_{\sigma=\sigma'} = -3\,\hat{\mathrm{G}}_\theta\,, \tag{A.19}$$

$$\hat{\mathrm{G}}_\theta = \frac{d-2}{3(d+1)}\,m_f\,\mathrm{G}_\theta = \mathcal{O}(\varepsilon)\ . \tag{A.20}$$

A.3 ζ-function and heat kernel regularization

In the spectral ζ-function regularization we fix $d=2$, i.e. consider all fields defined in Euclidean $\mathrm{AdS}_3 = \mathbb{H}^3$. Given a differential operator Δ with a positive discrete spectrum (with no zero modes), the corresponding Green's function and heat kernel are defined as $(\delta(\sigma,\sigma') = \frac{1}{\sqrt{g}}\delta(\sigma-\sigma'))$

$$\Delta\,G(\sigma,\sigma') = \delta(\sigma,\sigma'), \qquad K(t;\sigma,\sigma') = \langle\sigma|e^{-t\Delta}|\sigma'\rangle, \qquad \langle\sigma|\sigma'\rangle = \delta(\sigma,\sigma')\ . \tag{A.21}$$

For a complete set of eigenfunctions $f_n(\sigma)$ satisfying

$$\Delta f_n(\sigma) = \lambda_n f_n(\sigma), \qquad \sum_n f_n(\sigma)f_n^\dagger(\sigma') = \delta(\sigma,\sigma'), \qquad \int d^3\sigma\sqrt{g}\ f_n^\dagger(\sigma)f_n(\sigma) = \delta_{nm}, \tag{A.22}$$

the corresponding spectral ζ-function is defined as

$$\zeta(s;\sigma,\sigma') = \sum_n \frac{f_n(\sigma)f_n^\dagger(\sigma')}{\lambda_n^s} = \frac{1}{\Gamma(s)}\int_0^\infty dt\,t^{s-1}\,K(t;\sigma,\sigma'). \tag{A.23}$$

This gives

$$G(\sigma,\sigma') = \zeta(1;\sigma,\sigma'), \qquad\qquad \Delta\,G(\sigma,\sigma') = \delta(\sigma,\sigma') = \zeta(0;\sigma,\sigma'). \tag{A.24}$$

If the spectrum is continuous, i.e. $\lambda_n \to \lambda(\nu)$, one may define the spectral measure $\mu(\nu)$ as

$$\sum_n \to \mathcal{N}\int_0^\infty d\nu\,\mu(\nu). \tag{A.25}$$

For a massive scalar in $\mathbb{H}^3$ one finds

$$\Delta = -\nabla^2 + m^2\,, \qquad\qquad \mu(\nu) = \nu^2, \qquad \lambda(\nu) = \nu^2 + 1 + m^2\,, \tag{A.26}$$

$$K(t;\sigma,\sigma) = \frac{1}{2\pi^2}\int_0^\infty d\nu\,\mu(\nu)\,e^{-t\lambda(\nu)} = \frac{1}{(4\pi t)^{3/2}}e^{-t(m^2+1)}\,, \tag{A.27}$$

$$\zeta(s) = \frac{1}{2\pi^2}\int_0^\infty d\nu\, \frac{\mu(\nu)}{[\lambda(\nu)]^s} = \frac{1}{8\pi^{3/2}}(1+m^2)^{\frac{3}{2}-s}\,\frac{\Gamma(s-\frac{3}{2})}{\Gamma(s)}\,, \tag{A.28}$$

$$G(\sigma,\sigma) = \zeta(1) = -\frac{1}{4\pi}\sqrt{1+m^2}, \qquad \delta(\sigma,\sigma') = \zeta(0) = 0\ . \tag{A.29}$$

To find the twice differentiated propagator at coinciding points we note that

$$\partial_\mu\partial'_\nu G(\sigma,\sigma') = -\nabla_\mu\nabla_\nu G(\sigma,\sigma') = \frac{1}{3}g_{\mu\nu}\big[-m^2 G(\sigma,\sigma') + \delta(\sigma,\sigma')\big]\,, \tag{A.30}$$

so that using (A.29) we get

$$\partial_\mu\partial'_\nu G(\sigma,\sigma')|_{\sigma=\sigma'} = \frac{1}{12\pi}m^2\sqrt{1+m^2}\,g_{\mu\nu}\ . \tag{A.31}$$

In the case of the squared spinor operator in (A.18) defined on $\mathbb{H}^3$ one finds [41][18]

$$\Delta = \mathfrak{D}\mathfrak{D}^\dagger\,, \qquad \mathfrak{D} = \nabla\!\!\!/ + m\,\Gamma_*\,, \tag{A.32}$$

$$\mu(\nu) = \nu^2 + \frac{1}{4}, \qquad \lambda(\nu) = \nu^2 + m^2, \qquad K(t;\sigma,\sigma) = \frac{1}{(4\pi t)^{3/2}}e^{-tm^2}\left(1+\frac{1}{2}t\right)\,, \tag{A.33}$$

$$\zeta(s) = \frac{m^{1-2s}}{16\pi^{3/2}}\left(s-\frac{3}{2}+2m^2\right)\frac{\Gamma(s-\frac{3}{2})}{\Gamma(s)}\,, \qquad \zeta(1) = -\frac{4m^2-1}{16\pi m}, \qquad \zeta(0) = 0\ . \tag{A.34}$$

The Green's function for the operator $\mathfrak{D}$ defined on the fermion θ is (cf. (A.24))

$$G_\theta(\sigma,\sigma') = \mathfrak{D}^\dagger\zeta(1;\sigma,\sigma'), \quad \mathfrak{D}_\alpha G_\theta(\sigma,\sigma') = \frac{1}{3}\Gamma_\alpha\zeta(0;\sigma,\sigma'), \quad \mathfrak{D}G_\theta(\sigma,\sigma') = \zeta(0;\sigma,\sigma') = \delta_\theta(\sigma,\sigma')\ . \tag{A.35}$$

Since on a symmetric space the coincident limit is independent of the point on the manifold,

$$G_\theta(\sigma,\sigma) = m\,\Gamma_*\zeta(1)\,, \qquad \mathfrak{D}_\alpha G_\theta\Big|_{\sigma=\sigma'} = \frac{1}{3}\gamma_\alpha\zeta(0) = 0\,, \qquad \mathfrak{D}_\alpha\mathfrak{D}'_\beta G_\theta\Big|_{\sigma=\sigma'} = 0\ . \tag{A.36}$$

As a result,

$$G_\theta(\sigma,\sigma) \equiv -\Gamma_*\mathrm{G}_\theta\,, \qquad \mathrm{G}_\theta = -m\,\zeta(1) = \frac{4m^2-1}{16\pi}\,, \qquad \mathrm{G}_\theta\Big|_{m=\frac{3}{2}} = \frac{1}{2\pi}\ . \tag{A.37}$$

Let us note that if instead of the ζ-function regularization one uses the heat kernel (or "proper-time") cutoff so that

$$G(\sigma,\sigma') = \int_\epsilon^\infty dt\,K(t;\sigma,\sigma')\,, \qquad \delta(\sigma,\sigma) = K(\epsilon;\sigma,\sigma')\,, \qquad \epsilon \equiv \Lambda^{-2}\to 0\,, \tag{A.38}$$

then in the above bosonic and fermionic cases we find, respectively,

$$\delta(\sigma,\sigma) = \frac{1}{4\pi}\Lambda^3 - \frac{m^2+1}{(4\pi)^{3/2}}\Lambda + \mathcal{O}(\Lambda^{-1})\,, \qquad G(\sigma,\sigma) = \frac{1}{4\pi^{3/2}}\Lambda - \frac{\sqrt{m^2+1}}{4\pi} + \mathcal{O}(\Lambda^{-1})\,, \tag{A.39}$$

$$\delta_\theta(\sigma,\sigma) = \frac{1}{8\pi^{3/2}}\Lambda^3 + \frac{1-2m^2}{16\pi^{3/2}}\Lambda + \mathcal{O}(\Lambda)\,, \quad \mathrm{G}_\theta(\sigma,\sigma) = -\frac{1}{4\pi^{3/2}}\Lambda + \frac{4m^2-1}{16\pi m} + \mathcal{O}(\Lambda^{-1}).$$

[18]Here with hermitian Γ_α one has $\mathfrak{D}^\dagger = -\Gamma^\alpha\partial_\alpha + \ldots$.

The resulting expectation value of the quartic Lagrangian given by the sum of (2.18), (3.16), (3.17) then contains power divergences

$$\langle L_4\rangle = \frac{3}{32\pi^3}\Lambda^6 - \frac{27}{32\pi^3}\Lambda^4 + \frac{7}{8\pi^{5/2}}\Lambda^3 - \frac{7}{128\pi^3}\Lambda^2 - \frac{13}{8\pi^{5/2}}\Lambda + \frac{3241}{1536\pi^3} + \mathcal{O}(\Lambda^{-1})\ . \tag{A.40}$$

These may be cancelled by a contribution of an ultralocal measure in the M2 brane path integral that may be contributing in the heat kernel regularization as here the "$\delta(0)$" terms are non-vanishing.

B M2 brane action in $\mathrm{AdS}_7 \times S^4$

Here we shall review the structure of the BST action [12, 13] in $\mathrm{AdS}_7 \times S^4$ [14, 15] using its supercoset construction.

B.1 Supercoset relations

We shall assume the Euclidean signature, i.e. $\mathrm{AdS}_7 = \mathrm{H}^7$. This maximally supersymmetric 11d background has the superisometry algebra $\mathfrak{osp}(7,1\,|\,4)$ with the even part $\mathfrak{so}(7,1) \oplus \mathfrak{usp}(4)$ and the odd part represented by 32 supercharges transforming in the bi-spinor representation of the bosonic groups. The corresponding commutation relations may be written as[19]

$$\begin{aligned}
&[M_{rs}, M_{tu}] = \delta_{st}M_{ru} + \dots, \qquad [P_r, M_{st}] = \delta_{rs}P_t - \delta_{rt}P_s, \qquad [P_r, P_s] = M_{rs},\\
&[M_{ab}, M_{cd}] = \delta_{bc}M_{ad} + \dots, \qquad [P_a, M_{bc}] = \delta_{ab}P_c - \delta_{ac}P_b, \qquad [P_a, P_b] = -\frac{1}{\mathrm{r}^2}M_{ab},\\
&[P_r, Q] = -\frac{1}{2}\Gamma_r\Gamma_* Q\,, \quad [P_a, Q] = -\frac{1}{2\mathrm{r}}\Gamma_a\Gamma_* Q\,, \quad [P_r, \bar Q] = \frac{1}{2}\bar Q\Gamma_*\Gamma_r\,, \quad [P_a, \bar Q] = \frac{1}{2\mathrm{r}}\bar Q\Gamma_*\Gamma_a\,,\\
&[M_{AB}, Q] = -\frac{1}{2}\Gamma_{AB}Q\,, \quad [M_{AB}, \bar Q] = \frac{1}{2}\bar Q\Gamma_{AB}\,, \qquad \bar Q = Q^T C\,,\\
&[Q, \bar Q] = -2\Gamma^A P_A + \Gamma_*\Big(\Gamma^{ab}M_{ab} - \frac{1}{\mathrm{r}}\Gamma^{ab}M_{ab}\Big)\ .
\end{aligned} \tag{B.1}$$

where $A, B = 0, \dots, 10$ and $r, s, \dots = 0, \dots, 6$; $a, b, \dots = 7, \dots, 10$. Let us define the supercoset element

$$g(X,\theta) = g_0(X)\exp(\mathrm{Q})\,, \qquad \mathrm{Q} = \bar\theta Q = \bar Q\theta\,, \qquad X^A = (X^r, X^a)\ . \tag{B.2}$$

The left-invariant Maurer-Cartan one-form can be decomposed as:

$$g^{-1}dg = L^A P_A + \frac{1}{2}L^{AB}M_{AB} + \bar Q L\,, \qquad L^A = (L^r, L^a)\,, \qquad L^{AB} = (L^{rs}, L^{ab})\,, \tag{B.3}$$

where (L^A, L) satisfy the structure equations:

$$dL^A + L^{AB}\wedge L^B = \bar L\wedge\Gamma^A L\,, \tag{B.4}$$

$$dL + \frac{1}{4}L^{AB}\wedge\Gamma_{AB}L + \frac{1}{2}L^a\wedge\Gamma_*\Gamma_a L + \frac{1}{2\mathrm{r}}L^a\wedge\Gamma_*\Gamma_a L = 0\ . \tag{B.5}$$

[19] We use the notation $[\cdot\,,\cdot]$ for the graded commutator $[U,V] = UV - (-1)^{|U||V|}VU$, $U, V \in \mathfrak{g}$, satisfying the graded Jacobi identity: $[U,[V,W]] = [[U,V],W] + (-1)^{|U||V|}[V,[U,W]]$. The commutator $[Q,\bar Q]$ can be also written in the 11d supergravity notation: $[Q,\bar Q] = -2\Gamma^A P_A + \frac{1}{144}\big(\Gamma^{ABCDEF}\mathcal{F}_{ABCD} + 24\Gamma_{AB}\mathcal{F}^{ABEF}\big)M_{EF}$, where $\mathcal{F}_4 = 6\mathrm{r}^4\mathrm{vol}_{S^4}$. We assume that the radius of AdS_7 is 1 and the radius of S^4 is $\mathrm{r} = \frac{1}{2}$ (cf. (1.4)). While we formally keep track of the dependence on r, the $\mathrm{AdS}_7 \times S^4$ background is a 11d supergravity solution and thus the M2 brane action is consistent (κ-symmetric) only for $\mathrm{r} = \frac{1}{2}$.

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Using (B.2) the 1-form in (B.3) may be represented as

$$
\begin{aligned}
g^{-1}dg &= e^{-\mathrm{ad}\,\mathrm{Q}}(g_0^{-1}dg_0) + \frac{1-e^{-\mathrm{ad}\,\mathrm{Q}}}{\mathrm{ad}\,\mathrm{Q}}\,d\mathrm{Q} \\
&= g_0^{-1}dg_0 - (\mathrm{ad}\,\mathrm{Q}/2)\frac{\sinh^2(\mathrm{ad}\,\frac{\mathrm{Q}}{2})}{(\mathrm{ad}\,\frac{\mathrm{Q}}{2})^2}D\mathrm{Q} + \frac{\sinh(\mathrm{ad}\,\mathrm{Q})}{\mathrm{ad}\,\mathrm{Q}}D\mathrm{Q}\,,
\end{aligned}
\tag{B.6}
$$

where $\mathrm{ad}\,(\mathrm{Q}) = [\mathrm{Q},\cdot]$ and D denotes the Killing spinor derivative defined as[20]

$$
D\mathrm{Q} = d\mathrm{Q} + [g_0^{-1}dg_0, \mathrm{Q}]\ . \tag{B.7}
$$

The bosonic 1-form expanded in the generators may be written as

$$
g_0^{-1}dg_0 = E^A P_A + \frac{1}{2}\Omega^{AB}M_{AB}\,, \qquad E^A = (E^r, E^a), \quad \Omega^{AB} = (\Omega^{rs}, \Omega^{ab})\ . \tag{B.8}
$$

Explicitly, we find

$$
D\mathrm{Q} = \bar{Q}(D\theta)\,, \qquad D\theta = d\theta + \frac{1}{4}\Omega^{AB}\Gamma_{AB}\theta + \frac{1}{2}\Gamma_*\Big(E^a\Gamma_a + \frac{1}{\mathrm{r}}E^a\Gamma_a\Big)\theta\ . \tag{B.9}
$$

Let us also define the fermion bilinear $\mathcal{M}^2$ as

$$
\mathrm{ad}^{\,2}\mathrm{Q}(\tilde{\mathrm{Q}}) = [\bar{\theta}Q, [\bar{\theta}Q, \bar{Q}\tilde{\theta}]] = \bar{Q}\mathcal{M}^2\tilde{\theta}\,, \qquad \mathrm{Q} = \bar{\theta}Q, \qquad \tilde{\mathrm{Q}} = \bar{Q}\tilde{\theta}\,, \tag{B.10}
$$

$$
\mathcal{M}^2 = \Gamma_*\Big(\Gamma_r\theta\,\bar{\theta}\Gamma^r + \frac{1}{\mathrm{r}}\Gamma_a\theta\,\bar{\theta}\Gamma^a\Big) - \frac{1}{2}\Big(\Gamma_{rs}\theta\,\bar{\theta}\Gamma^{rs} - \frac{1}{\mathrm{r}}\Gamma_{ab}\theta\,\bar{\theta}\Gamma^{ab}\Big)\Gamma_*\ . \tag{B.11}
$$

Then from (B.6) we get

$$
\begin{aligned}
g^{-1}dg = g_0^{-1}dg_0 &+ 4\bar{\theta}\Gamma^A\Big(\frac{\sinh^2\frac{\mathcal{M}}{2}}{\mathcal{M}^2}D\theta\Big)P_A + \bar{Q}\Big(\frac{\sinh\mathcal{M}}{\mathcal{M}}D\theta\Big) \\
&- 2\bar{\theta}\Gamma_*\Gamma^{rs}\Big(\frac{\sinh^2\frac{\mathcal{M}}{2}}{\mathcal{M}^2}D\theta\Big)M_{rs} + \frac{2}{\mathrm{r}}\bar{\theta}\Gamma_*\Gamma^{ab}\Big(\frac{\sinh^2\frac{\mathcal{M}}{2}}{\mathcal{M}^2}D\theta\Big)M_{ab}\ .
\end{aligned}
\tag{B.12}
$$

Let us split the H^7 indices as $r = (\hat{\alpha}, i)$ where $\hat{\alpha}$ is the AdS_3 tangent-space index and i labels the 4 transverse directions. In the static gauge we set (using parametrization that makes the transverse $\mathrm{SO}(4)\times\mathrm{SO}(4)$ symmetry manifest)

$$
X = (\sigma^\alpha, x^i, y^a)\,, \qquad g_0(X) = g_{\mathrm{H}^3}(\sigma)\exp\Big(\frac{\operatorname{arctanh}|\vec{x}|/2}{|\vec{x}|/2}x^iP_i\Big)\exp\Big(\frac{\arctan|\vec{y}|/(2\mathrm{r})}{|\vec{y}|/(2\mathrm{r})}y^aP_a\Big)\,, \tag{B.13}
$$

where $|\vec{x}| = \sqrt{x^ix_i}$, $|\vec{y}| = \sqrt{y^ay_a}$ and $g_{\mathrm{H}^3}(\sigma)$ denotes any parametrisation of a totally geodesic $\mathrm{H}^3 \subset \mathrm{H}^7$. The $\mathrm{H}^7 \times S^4$ metric is determined by the bosonic 1-form E^A and has the same form as in [1] (cf. (2.1), (2.3))

$$
\begin{aligned}
&g_{\mathrm{H}^3}^{-1}dg_{\mathrm{H}^3} = e^{\hat{\alpha}}P_{\hat{\alpha}} + \omega^{\hat{\alpha}\hat{\beta}}M_{\hat{\alpha}\hat{\beta}}\,, \quad E^{\hat{\alpha}} = \frac{(1+\frac{1}{4}x^2)}{(1-\frac{1}{4}x^2)}e^{\hat{\alpha}}\,, \qquad E^i = \frac{dx^i}{1-\frac{1}{4}x^2}\,, \quad E^a = \frac{dy^a}{1+\frac{1}{4\mathrm{r}^2}y^2}\,, \\
&ds^2_{\mathrm{H}^7} = \frac{(1+\frac{1}{4}x^2)^2}{(1-\frac{1}{4}x^2)^2}ds^2_{\mathrm{H}^3} + \frac{dx^idx_i}{(1-\frac{1}{4}x^2)^2}\,, \qquad ds^2_{S^4} = \frac{dy^ady_a}{(1+\frac{1}{4\mathrm{r}^2}y^2)^2}\ .
\end{aligned}
\tag{B.14}
$$

[20] Note that the Killing spinor on $\mathrm{H}^7 \times S^4$ can be obtained using the adjoint action (see also [42]): $\mathrm{Q}_{\mathrm{kil}} = \bar{\epsilon}(g_0^{-1}Qg_0)$.

Here $e^{\hat{\alpha}} = e^{\hat{\alpha}}_{\alpha} d\sigma^{\alpha}$ and $\omega^{\hat{\alpha}\hat{\beta}}$ correspond to the 3-bein and spin connection of H^3. For example, in the upper half-plane parametrization of H^3 ($\sigma^{\alpha} = (\sigma^0, \sigma^1, \sigma^2) = (z, w^v)$)

$$ds^2_{\mathrm{H}^3} = \frac{1}{z^2}(dz^2 + dw^v dw^v)\,, \qquad e^{\hat{\alpha}} = \frac{1}{z} d\sigma^{\alpha}\,, \qquad \omega^{\hat{\alpha}\hat{\beta}} = \frac{1}{z}(\delta^{\alpha}_0 d\sigma^{\beta} - \delta^{\beta}_0 d\sigma^{\alpha})\,. \tag{B.15}$$

The spin connection Ω^{AB} is explicitly given by $\Omega^{\hat{\alpha}\hat{\beta}} = \omega^{\hat{\alpha}\hat{\beta}}$ and

$$\Omega^{\hat{\alpha}i} = -\Omega^{i\hat{\alpha}} = \frac{e^{\hat{\alpha}} x^i}{1 - \frac{1}{4}x^2}\,, \qquad \Omega^{ij} = -\frac{1}{2}\frac{x^i dx^j - x^j dx^i}{1 - \frac{1}{4}x^2}\,, \qquad \Omega^{ab} = \frac{1}{2\mathrm{r}^2}\frac{y^a dy^b - y^b dy^a}{1 + \frac{1}{4\mathrm{r}^2}y^2}\,. \tag{B.16}$$

B.2 Expansion of the M2 brane action

The BST action for the supermembrane in $\mathrm{H}^7 \times S^4$ can be written as [14] (cf. (1.6), (1.7))

$$S = \mathrm{T}_2\Big(\int d^3\sigma\,\sqrt{h} \;+\; i\int_{M_4} \mathcal{H}_4\Big)\,, \qquad h_{\alpha\beta} = L^A_{\alpha} L^A_{\beta}\,, \tag{B.17}$$

$$\mathcal{H}_4 = \frac{1}{8\mathrm{r}}\epsilon_{abcd} L^a \wedge L^b \wedge L^c \wedge L^d - \frac{1}{2}\bar{L} \wedge \Gamma_{AB} L \wedge L^A \wedge L^B, \tag{B.18}$$

where $L^A_{\alpha} = L^A_M \partial_{\alpha} X^M$ and $\mathcal{H}_4$ is a closed 4-form which is integrated over a 4d manifold that has the AdS_3 world volume as its boundary.[21] The action is invariant under the κ-symmetry which can be represented as a right action on the supercoset element

$$g(X,\theta) = g_0(X)\exp(\mathrm{Q}) \;\mapsto\; g_0(X)\exp(\mathrm{Q})\exp(\delta\bar{\theta}Q)\,. \tag{B.19}$$

The corresponding vector field $\varkappa$ generating a diffeomorphism on the coset satisfies[22]

$$\iota_{\varkappa}(g^{-1}dg) = g^{-1}\delta g = \delta_{\kappa}\bar{\theta}\, Q\,, \qquad \iota_{\varkappa}\bar{L} = \delta_{\kappa}\bar{\theta}\,, \qquad \iota_{\varkappa} L^A = 0\,, \qquad \iota_{\varkappa} L^{AB} = 0\,. \tag{B.20}$$

Explicitly, the transformation under the κ-symmetry may be represented as

$$\delta_{\kappa}\theta = \mathbf{P}\kappa\,, \qquad\qquad \mathbf{P} = \frac{1}{2}(\mathbb{I} + \mathbf{\Gamma})\,, \tag{B.21}$$

$$\mathbf{\Gamma} \equiv \frac{i}{3!\sqrt{h}}\epsilon^{\alpha\beta\gamma} L^A_{\alpha} L^B_{\beta} L^C_{\gamma}\Gamma_{ABC}\,, \quad \mathbf{\Gamma}^2 = \mathbb{I}\,, \qquad h^{\alpha\beta} L^A_{\beta}\Gamma_A \mathbf{\Gamma} = \frac{i}{2!\sqrt{h}}\epsilon^{\alpha\beta\gamma} L^A_{\beta} L^B_{\gamma}\Gamma_{AB}\,. \tag{B.22}$$

To expand the action in powers of θ it is convenient to do a rescaling

$$\theta \to s\theta\,, \qquad L_s \equiv L(s\theta)\,, \qquad L^A_s \equiv L^A(s\theta)\,. \tag{B.23}$$

[21] Its closure follows from (B.4), (B.5) for $\mathrm{r} = \frac{1}{2}$ combined with the 11d Fierz identity (here $p, q, \ldots$ are spinor indices):

$$\bar{L} \wedge \Gamma_{AB} L \wedge \bar{L} \wedge \Gamma^A L = 0 \leftrightarrow (\Gamma^A)_{(pq}(\Gamma_{AB})_{uv)} = 0.$$

[22] A vector field $\varkappa$ defines a contraction of the same parity as $\varkappa$, i.e., for a one-form α: $\iota_{\varkappa}\alpha = \langle\varkappa, \alpha\rangle$, and $\iota_{\varkappa}(\alpha \wedge \beta) = \iota_{\varkappa}\alpha \wedge \beta + (-1)^{|\varkappa|\cdot|\alpha|}\alpha \wedge \iota_{\varkappa}\beta$.

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It then follows, by considering the special case of (B.19) with $\delta\theta = \delta s\,\theta$ and using that $L_{s=0} = 0$, that the WZ term in (B.17) can be written as:

$$\int_{M_4} \mathcal{H} = 6\mathrm{r}^4 \int_{M_4} \mathrm{vol}_{S^4} - \int d^3\sigma\, \epsilon^{\alpha\beta\gamma} \int_0^1 ds\, \bar{\theta}\Gamma_{AB} L_{\alpha\, s}\, L^A_{\beta\, s} L^B_{\gamma\, s} \,. \tag{B.24}$$

Imposing the κ-symmetry gauge as in (3.4), i.e.

$$\mathrm{P}\theta = 0\,, \qquad \mathrm{P} = \frac{1}{2}(\mathbb{I} + \Gamma)\,, \qquad \Gamma = i\Gamma_{012}\,, \tag{B.25}$$

the expansion of the basic 1-forms in (B.23) in terms of θ takes the form

$$L^A_s = E^A + s^2\bar{\theta}\Gamma^A D\theta + \frac{1}{12}s^4\bar{\theta}\Gamma^A\mathcal{M}^2 D\theta + \mathcal{O}(s^6)\,, \qquad L_s = sD\theta + \frac{1}{6}s^3\mathcal{M}^2 D\theta + \mathcal{O}(s^5), \tag{B.26}$$

where $\mathcal{M}^2$ was defined in (B.11). As a result, the induced metric and the integrand in the WZ term in (B.24) take the form

$$h_{\alpha\beta} = L^A_\alpha L^A_\beta = \left(E^A_\alpha + \bar{\theta}\Gamma^A D_\alpha\theta + \frac{1}{12}\bar{\theta}\mathcal{M}^2 D_\alpha\theta + \ldots\right)\left(E^A_\beta + \bar{\theta}\Gamma^A D_\beta\theta + \frac{1}{12}\bar{\theta}\mathcal{M}^2 D_\beta\theta + \ldots\right), \tag{B.27}$$

$$\bar{\theta}\Gamma_{AB} L_s \wedge L^A_s \wedge L^B_s = \bar{\theta}\Gamma_{AB}\left(sD\theta + \frac{1}{6}s^3\mathcal{M}^2 D\theta\right)\wedge(E^A + s^2\bar{\theta}\Gamma^A D\theta)\wedge(E^B + s^2\bar{\theta}\Gamma^B D\theta) + \ldots$$

It is convenient to decompose the operators acting on spinors as

$$D = D^+ + D^-\,, \qquad \Gamma D^+ = D^+\Gamma\,, \qquad \Gamma D^- = -D^-\Gamma\,, \tag{B.28}$$

$$D^+ = d + \frac{1}{4}\Omega^{\hat{\alpha}\hat{\beta}}\Gamma_{\hat{\alpha}\hat{\beta}} + \frac{1}{2}E^{\hat{\alpha}}\Gamma_*\Gamma_{\hat{\alpha}} + \frac{1}{4}\Omega^{ij}\Gamma_{ij} + \frac{1}{4}\Omega^{ab}\Gamma_{ab}\,,$$

$$D^- = \frac{1}{2}\Omega^{\hat{\alpha}i}\Gamma_{\hat{\alpha}}\Gamma_i + \frac{1}{2}E^i\Gamma_*\Gamma_i + \frac{1}{2\mathrm{r}}E^a\Gamma_*\Gamma_a\,,$$

$$\mathcal{M}^2 = \mathcal{M}^2_+ + \mathcal{M}^2_-\,, \qquad \Gamma\mathcal{M}^2_+ = \mathcal{M}^2_+\Gamma\,, \qquad \Gamma\mathcal{M}^2_- = -\mathcal{M}^2_-\Gamma\,, \tag{B.29}$$

$$\mathcal{M}^2_+ = \Gamma_*\Gamma_{\hat{\alpha}}\theta\bar{\theta}\Gamma^{\hat{\alpha}} - \frac{1}{2}\left(\Gamma_{\hat{\alpha}\hat{\beta}}\theta\bar{\theta}\Gamma^{\hat{\alpha}\hat{\beta}} + \Gamma_{ij}\theta\bar{\theta}\Gamma^{ij} - \frac{1}{\mathrm{r}}\Gamma_{ab}\theta\bar{\theta}\Gamma^{ab}\right)\Gamma_*\,,$$

$$\mathcal{M}^2_- = \Gamma_*\left(\Gamma_i\theta\bar{\theta}\Gamma^i + \frac{1}{\mathrm{r}}\Gamma_a\theta\bar{\theta}\Gamma^a\right) - \Gamma_{\hat{\alpha}}\Gamma_i\theta\bar{\theta}\Gamma^{\hat{\alpha}}\Gamma^i\Gamma_*\,.$$

Since $\overline{\Gamma\theta} = \bar{\theta}\Gamma$, in the above κ-symmetry gauge (B.25) one needs to consider only the terms which involve an even number of operators reversing the Γ-chirality. Using that $\Gamma\Gamma_{\hat{\alpha}} = \Gamma_{\hat{\alpha}}\Gamma$ and $\Gamma\Gamma_i = -\Gamma_i\Gamma$, we find

$$L^{\hat{\alpha}}_s = E^{\hat{\alpha}} + s^2\bar{\theta}\Gamma^{\hat{\alpha}}D^+\theta + \frac{s^4}{12}\bar{\theta}\Gamma^{\hat{\alpha}}\mathcal{M}^2_+ D^+\theta + \frac{s^4}{12}\bar{\theta}\Gamma^{\hat{\alpha}}\mathcal{M}^2_- D^-\theta + \mathcal{O}(s^6)\,, \tag{B.30}$$

$$L^{i,a}_s = E^{i,a} + s^2\bar{\theta}\Gamma^{i,a}D^-\theta + \frac{s^4}{12}\bar{\theta}\Gamma^{i,a}\mathcal{M}^2_+ D^-\theta + \frac{s^4}{12}\bar{\theta}\Gamma^{i,a}\mathcal{M}^2_- D^+\theta + \mathcal{O}(s^6)\,, \tag{B.31}$$

$$L_s = L^+_s + L^-_s\,, \quad L^\pm_s = sD^\pm\theta + \frac{s^3}{6}\mathcal{M}^2_+ D^\pm\theta + \frac{s^3}{6}\mathcal{M}^2_- D^\mp\theta + \mathcal{O}(s^5)\,. \tag{B.32}$$

The spinor covariant derivative can be further expanded in the bosonic fluctuations as

$$D^+ = \mathfrak{D} + \frac{1}{4}x^2 e^{\hat{\alpha}}\Gamma_*\Gamma_{\hat{\alpha}} - \frac{1}{4}x^i dx^j\Gamma_{ij} + \frac{1}{4\mathrm{r}^2}y^a dy^b\Gamma_{ab} + \mathcal{O}(X^4)\,, \qquad \mathfrak{D} = \nabla + \frac{1}{2}e^{\hat{\alpha}}\Gamma_*\Gamma_{\hat{\alpha}}\,, \tag{B.33}$$

$$D^- = \frac{1}{2}e^{\hat\alpha}x^i\Gamma_{\hat\alpha}\Gamma_i + \frac{1}{2}dx^i\Gamma_*\Gamma_i + \frac{1}{2\mathrm{r}}dy^a\Gamma_*\Gamma_a + \mathcal{O}(X^3)\,, \tag{B.34}$$

where $\mathfrak{D}$ is the pull-back of the Killing spinor derivative to the H^3 surface (cf. (3.5)).

As a result, using the definitions in (3.6), (3.7) we find the following expansion of the M2 brane Lagrangian to quartic order in the fluctuation fields (cf. (2.4), (3.8)–(3.10))

$$L = L_2 + \mathrm{T}_2^{-1}(L_{4\mathrm{b}} + L_{2\mathrm{b},2\mathrm{f}} + L_{4\mathrm{f}}) + \ldots, \qquad L_2 = \frac{1}{2}g^{\alpha\beta}(h^{(2b)}_{\alpha\beta} + h^{(2\theta)}_{\alpha\beta}), \tag{B.35}$$

$$\begin{aligned} L_{4\mathrm{b}} = &\frac{1}{4}x^2 g^{\alpha\beta}h^{(2x)}_{\alpha\beta} - \frac{1}{4\mathrm{r}^2}y^2 g^{\alpha\beta}h^{(2y)}_{\alpha\beta} - \frac{1}{4}g^{\alpha\delta}g^{\beta\gamma}h^{(2b)}_{\alpha\beta}h^{(2b)}_{\gamma\delta} + \frac{1}{8}(g^{\alpha\beta}h^{(2b)}_{\alpha\beta})^2 \\ &+ \frac{i}{8\mathrm{r}\sqrt{g}}\epsilon^{\alpha\beta\gamma}\epsilon_{abcd}y^a\partial_\alpha y^b\partial_\beta y^c\partial_\gamma y^d, \end{aligned} \tag{B.36}$$

$$\begin{aligned} L_{2\mathrm{b},2\mathrm{f}} = &\frac{1}{8}g^{\alpha\beta}(h^{(2x)}_{\alpha\beta} - 2h^{(2y)}_{\alpha\beta})\bar\theta\Gamma_*\theta + \frac{3}{8}x^2 g^{\alpha\beta}h^{(2\theta)}_{\alpha\beta} + \frac{1}{8}g^{\alpha\beta}g^{\gamma\delta}h^{(2b)}_{\alpha\beta}h^{(2\theta)}_{\gamma\delta} - \frac{1}{4}h^{(2b)\alpha\beta}h^{(2\theta)}_{\alpha\beta} \\ &- \frac{1}{2}x^i\partial_\beta x^j\bar\theta\gamma^\beta\Gamma_{ij}\theta + \frac{1}{2\mathrm{r}^2}y^a\partial_\beta y^b\bar\theta\gamma^\beta\Gamma_{ab}\theta + \frac{1}{2}\bar\theta\gamma^\alpha(\partial_\alpha x^i\Gamma_i + \partial_\alpha y^a\Gamma_a)x^j\Gamma_j\theta \\ &+ \frac{1+\mathrm{r}}{4\mathrm{r}}\partial_\alpha x^i\partial^\alpha y^a\bar\theta\Gamma_*\Gamma_i\Gamma_a\theta - \frac{i}{2\sqrt{g}}\epsilon^{\alpha\beta\gamma}\bar\theta\gamma_\beta(\partial_\gamma x^i\Gamma_i + \partial_\gamma y^a\Gamma_a)\Big(\partial_\alpha x^j\Gamma_j - \frac{1}{\mathrm{r}}\partial_\alpha y^a\Gamma_a\Big)\Gamma_*\theta \\ &- \frac{i}{2\sqrt{g}}\epsilon^{\alpha\beta\gamma}\bar\theta(\Gamma_{ij}\partial_\beta x^i\partial_\gamma x^j + 2\Gamma_i\Gamma_a\partial_\beta x^i\partial_\gamma y^a + \Gamma_{ab}\partial_\beta y^a\partial_\gamma y^b)\mathfrak{D}_\alpha\theta, \end{aligned} \tag{B.37}$$

$$L_{4\mathrm{f}} = \frac{1}{96}g^{\alpha\beta}\bar\theta\Gamma_\alpha\mathcal{M}^2_+\mathfrak{D}_\beta\theta - \frac{1}{16}g^{\alpha\delta}g^{\beta\gamma}h^{(2\theta)}_{\alpha\beta}h^{(2\theta)}_{\gamma\delta} + \frac{1}{16}(g^{\alpha\beta}h^{(2\theta)}_{\alpha\beta})^2\,. \tag{B.38}$$

C Fermionic correlators

Here we provide some details of the computation of the quadratic and quartic fermionic correlators used in (3.14)–(3.17).

We need to evaluate the expectation value of 2-fermion operators of the form $\langle\bar\theta W\theta\rangle$, $\langle\bar\theta W\nabla_\alpha\theta\rangle$, where W is a string of gamma matrices containing Γ_α and also Γ_i, Γ_a matrices with transverse indices. One requires $(CW)^T = -CW$ for these combinations to be non-zero. As a result, we may apply the antisymmetric projection

$$CW \to \frac{1}{2}(CW - (CW)^T)\,, \qquad W \to \frac{1}{2}(W + C^{-1}W^TC)\,. \tag{C.1}$$

The basic definitions are (here p, q are spinor indices and $\bar\theta = \theta^T C$)

$$\langle\theta_p(\sigma)\,\bar\theta^q(\sigma')\rangle = (G_\theta)_p{}^q(\sigma,\sigma'), \qquad \langle\theta_p\theta_q\rangle = (G_\theta C^{-1})_{pq}\,, \quad \langle\bar\theta W\theta\rangle = -\mathrm{tr}[G_\theta W]\,, \tag{C.2}$$

where also $(G_\theta C^{-1})^T = -G_\theta C^{-1}$. For correlators where the fermionic field is differentiated we have

$$\langle\bar\theta W\partial\theta\rangle = -\mathrm{tr}[\partial G_\theta\, W]\,, \qquad \langle\partial\theta_p\theta_q\rangle = (\partial G_\theta C^{-1})_{pq}. \tag{C.3}$$

We may consider also $\langle(\nabla_\alpha\theta(\sigma))_p\,\theta_q(\sigma')\rangle = (\nabla_\alpha G_\theta(\sigma,\sigma')\,C^{-1})_{pq}$ that leads to (A.15). In particular (cf. (A.20))

$$\langle\bar\theta\not{\mathfrak{D}}\theta\rangle = 3N_\theta\,\hat{\mathrm{G}}_\theta\,, \qquad \langle\bar\theta\Gamma_*\theta\rangle = N_\theta\,\mathrm{G}_\theta\,, \tag{C.4}$$

where the factor of $N_\theta = 16$ comes from the trace over the spinor indices (θ assumed to be subject to the κ-symmetry gauge in (B.25)).

For a correlator with 2 derivatives at the coincident points we get (assuming dimensional reduction regularization)

$$\langle(\nabla_\alpha\theta)_p\ (\nabla_\beta\theta)_q\rangle \equiv (S_{\alpha\beta}C^{-1})_{pq}\,, \qquad S_{\alpha\beta} = \frac{1}{4}\Big[\Gamma_{\alpha\beta} - \frac{d(d+1)-4m^2}{d+1}g_{\alpha\beta}\Big]\,\Gamma_*\,. \tag{C.5}$$

Similarly (cf. (3.14))

$$\langle(\mathfrak{D}_\alpha\theta)_p(\mathfrak{D}_\beta\theta)_q\rangle \equiv (\widehat{S}_{\alpha\beta}C^{-1})_{pq}, \qquad \widehat{S}_{\alpha\beta} = \frac{1}{4}\Big[\Big(1+\frac{4}{9}\frac{d-5}{d+1}m^2\Big)\Gamma_{\alpha\beta} - \Big(d-\frac{4}{9}\frac{d+4}{d+1}m^2\Big)g_{\alpha\beta}\Big]\Gamma_*\,, \tag{C.6}$$

$$\widehat{S}_{\alpha\beta}\Big|_{m_f=\frac{3}{2}} = \frac{d-2}{4(d+1)}\,[2\Gamma_{\alpha\beta}-(d+2)\,g_{\alpha\beta}]\Gamma_*\,\mathrm{G}_\theta\,. \tag{C.7}$$

This correlator vanishes in $d=2$ in agreement with (3.15).

For correlators of 4 fermions we have[23]

$$\langle\bar\theta W\theta\ \bar\theta\widetilde{W}\theta\rangle = \mathrm{tr}[WG_\theta]\mathrm{tr}[\widetilde{W}G_\theta] - 2\mathrm{tr}[WG_\theta\widetilde{W}G_\theta]\,, \tag{C.8}$$

$$\langle\bar\theta W\partial\theta\ \bar\theta\widetilde{W}\theta\rangle = \mathrm{tr}[W\partial G_\theta]\mathrm{tr}[\widetilde{W}G_\theta] - 2\mathrm{tr}[W\partial G_\theta\widetilde{W}G_\theta]\,, \tag{C.9}$$

with similar more involved expressions when the two fermions are differentiated.

Below we will consider the fermionic correlators at coincident points assuming generic regularization. Let us start with

$$\mathcal{Q}_1^{(4)} \equiv \langle\bar\theta\Gamma^\beta\mathfrak{D}_\alpha\theta\ \bar\theta\Gamma^\alpha\Gamma_\beta\Gamma_*\theta\rangle \tag{C.10}$$

and first apply the projection in (C.1) using $C^{-1}(\Gamma_\alpha\Gamma_\beta\Gamma_*)^T C = \Gamma_\beta\Gamma_\alpha\Gamma_*$, so that $\Gamma_\alpha\Gamma_\beta\Gamma_* \to \frac{1}{2}(\Gamma_\alpha\Gamma_\beta\Gamma_* + \Gamma_\beta\Gamma_\alpha\Gamma_*) = g_{\alpha\beta}\Gamma_*$. Then (using the definitions in (3.14), (A.14), (A.15), (A.20))

$$\begin{aligned}\mathcal{Q}_1^{(4)} &= \langle\bar\theta\Gamma^\beta\mathfrak{D}_\alpha\theta\ \bar\theta\Gamma^\alpha\Gamma_\beta\Gamma_*\theta\rangle = \langle\bar\theta\not{\mathfrak{D}}\theta\ \bar\theta\Gamma_*\theta\rangle\\ &= \mathrm{tr}_P[-3\hat{\mathrm{G}}_\theta]\mathrm{tr}_P[\Gamma_*(-\Gamma_*\mathrm{G}_\theta)] - 2\mathrm{tr}_P[-3\hat{\mathrm{G}}_\theta\Gamma_*(-\Gamma_*\mathrm{G}_\theta)]] = 3(N_\theta^2-2N_\theta)\,\hat{\mathrm{G}}_\theta\,\mathrm{G}_\theta,\end{aligned} \tag{C.11}$$

where tr_P stands for the spinor trace under the κ-symmetry projection in (3.4), (B.25) so that $\mathrm{tr}_P\mathbb{I} = N_\theta = 16$.[24] In dimensional reduction regularization with $\hat{\mathrm{G}}_\theta$ given by (3.15), (A.20) we get

$$\mathcal{Q}_1^{(4)} = \frac{3}{2}\frac{d-2}{d+1}(N_\theta^2-2N_\theta)\,\mathrm{G}_\theta^2\,. \tag{C.12}$$

[23]Here W and $\widetilde{W}$ are two different combination of Γ matrices.

[24]It is useful to recall some standard Dirac matrix relations: $\{\Gamma_\alpha,\Gamma_\beta\} = 2g_{\alpha\beta}$, $g^\alpha_\alpha = d+1$ so that

$$\Gamma^\alpha\Gamma_\beta\Gamma_\alpha = -(d-1)\Gamma_\beta, \qquad \Gamma^\alpha\Gamma_{\beta\gamma}\Gamma_\alpha = (d-3)\Gamma_{\beta\gamma}, \qquad \Gamma^\alpha\Gamma_{\beta\gamma\delta}\Gamma_\alpha = -(d-5)\Gamma_{\beta\gamma\delta},$$
$$\mathrm{tr}[\Gamma^{\alpha\beta}\Gamma_{\alpha\beta}] = -d(d+1)\,\mathrm{tr}\,\mathbb{I}\,, \qquad \mathrm{tr}[\Gamma^{\alpha\beta\gamma}\Gamma_{\alpha\beta\gamma}] = -d(d^2-1)\,\mathrm{tr}\,\mathbb{I}\,.$$

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Similarly, we find (using definitions in (3.14))

$$\mathcal{Q}_2^{(4)} \equiv \langle \bar\theta \Gamma^\alpha \mathfrak{D}^\beta \theta \; \bar\theta \Gamma_\beta \mathfrak{D}_\alpha \theta \rangle = 3\, \hat{\mathrm{G}}_\theta^2 \, N_\theta^2 - (9 \hat{\mathrm{G}}_\theta^2 - 6 \tilde{\mathrm{G}}_\theta \mathrm{G}_\theta + 3 \bar{\mathrm{G}}_\theta \mathrm{G}_\theta)\, N_\theta . \tag{C.13}$$

In the dimensional reduction regularization this gives

$$\mathcal{Q}_2^{(4)} = \frac{3}{4} \frac{d-2}{(d+1)^2} \left[(d-2) N_\theta^2 + (12 + 4d + d^2) N_\theta \right] \mathrm{G}_\theta^2 . \tag{C.14}$$

The results for other relevant quartic correlators found in an analogous way are summarized below

$$\begin{aligned}
\mathcal{Q}_1^{(4)} &= \langle \bar\theta \Gamma^\beta \mathfrak{D}_\alpha \theta \; \bar\theta \Gamma^\alpha \Gamma_\beta \Gamma_* \theta \rangle &&= 3 (N_\theta^2 - 2 N_\theta)\, \hat{\mathrm{G}}_\theta \, \mathrm{G}_\theta \\
\mathcal{Q}_2^{(4)} &= \langle \bar\theta \Gamma^\alpha \mathfrak{D}^\beta \theta \; \bar\theta \Gamma_\beta \mathfrak{D}_\alpha \theta \rangle &&= 3 \hat{\mathrm{G}}_\theta^2 \, N_\theta^2 - (9 \hat{\mathrm{G}}_\theta^2 - 6 \tilde{\mathrm{G}}_\theta \mathrm{G}_\theta + 3 \bar{\mathrm{G}}_\theta \mathrm{G}_\theta)\, N_\theta \\
\mathcal{Q}_3^{(4)} &= \langle \bar\theta \not{\mathfrak{D}} \theta \; \bar\theta \not{\mathfrak{D}} \theta \rangle &&= 9 \hat{\mathrm{G}}_\theta^2 \, N_\theta^2 - (9 \hat{\mathrm{G}}_\theta^2 + 6 \tilde{\mathrm{G}}_\theta \mathrm{G}_\theta + 3 \bar{\mathrm{G}}_\theta \mathrm{G}_\theta)\, N_\theta \\
\mathcal{Q}_4^{(4)} &= \langle \bar\theta \Gamma^{\beta\lambda} \Gamma_* \mathfrak{D}^\alpha \theta \; \bar\theta \Gamma_\alpha \Gamma_{\beta\lambda} \theta \rangle &&= 12\, \hat{\mathrm{G}}_\theta \mathrm{G}_\theta N_\theta \\
\mathcal{Q}_5^{(4)} &= \langle \bar\theta \Gamma^{ij} \Gamma_* \mathfrak{D}_\alpha \theta \, \bar\theta \Gamma^\alpha \Gamma_{ij} \theta \rangle &&= 72\, \hat{\mathrm{G}}_\theta \mathrm{G}_\theta N_\theta \\
\mathcal{Q}_6^{(4)} &= \langle \bar\theta \Gamma^{ab} \Gamma_* \mathfrak{D}_\alpha \theta \, \bar\theta \Gamma^\alpha \Gamma_{ab} \theta \rangle &&= 72\, \hat{\mathrm{G}}_\theta \mathrm{G}_\theta N_\theta
\end{aligned} \tag{C.15}$$

In the dimensional reduction regularization that gives

$$\begin{aligned}
\mathcal{Q}_1^{(4)} &= \langle \bar\theta \Gamma^\beta \mathfrak{D}_\alpha \theta \; \bar\theta \Gamma^\alpha \Gamma_\beta \Gamma_* \theta \rangle &&= \tfrac{3}{2} \tfrac{d-2}{d+1} (N_\theta^2 - 2 N_\theta)\, \mathrm{G}_\theta^2 \\
\mathcal{Q}_2^{(4)} &= \langle \bar\theta \Gamma^\alpha \mathfrak{D}^\beta \theta \; \bar\theta \Gamma_\beta \mathfrak{D}_\alpha \theta \rangle &&= \tfrac{3}{4} \tfrac{d-2}{(d+1)^2} [(d-2) N_\theta^2 + (12 + 4d + d^2) N_\theta]\, \mathrm{G}_\theta^2 \\
\mathcal{Q}_3^{(4)} &= \langle \bar\theta \not{\mathfrak{D}} \theta \; \bar\theta \not{\mathfrak{D}} \theta \rangle &&= \tfrac{3}{4} \tfrac{(d-2)^2}{(d+1)^2} [3 N_\theta^2 + (d-2) N_\theta]\, \mathrm{G}_\theta^2 \\
\mathcal{Q}_4^{(4)} &= \langle \bar\theta \Gamma^{\beta\lambda} \Gamma_* \mathfrak{D}^\alpha \theta \; \bar\theta \Gamma_\alpha \Gamma_{\beta\lambda} \theta \rangle &&= 6\, \tfrac{d-2}{d+1} N_\theta \, \mathrm{G}_\theta^2 \\
\mathcal{Q}_5^{(4)} &= \langle \bar\theta \Gamma^{ij} \Gamma_* \mathfrak{D}_\alpha \theta \, \bar\theta \Gamma^\alpha \Gamma_{ij} \theta \rangle &&= 36\, \tfrac{d-2}{d+1} N_\theta \, \mathrm{G}_\theta^2 \\
\mathcal{Q}_6^{(4)} &= \langle \bar\theta \Gamma^{ab} \Gamma_* \mathfrak{D}_\alpha \theta \, \bar\theta \Gamma^\alpha \Gamma_{ab} \theta \rangle &&= 36\, \tfrac{d-2}{d+1} N_\theta \, \mathrm{G}_\theta^2
\end{aligned} \tag{C.16}$$

Data Availability Statement. This article has no associated data or the data will not be deposited.

Code Availability Statement. This article has no associated code or the code will not be deposited.

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