Electroweak evolution equations and isospin conservation

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ABSTRACT: In processes taking place at energies much higher than the weak scale, electroweak corrections can be taken into account by using electroweak evolution equations, that are analogous to the DGLAP equations in QCD. We show that weak isospin conservation in these equations imposes to modify the expressions of the splitting functions commonly used in the literature. These modifications have a profound impact on the parton distribution functions.

Contents

1 Introduction

Electroweak evolution equations (EWEEs), which are analogous to the Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (DGLAP) equations in Quantum Chromodynamics (QCD) [1–4], are of primary importance when the energies of the processes considered are much larger than those of the weak scale. Indeed, at center-of-mass energies Q much higher than the electroweak (EW) symmetry breaking scale $M \sim 100\,\text{GeV}$, radiative EW corrections grow like $\log^2(Q/M)$ [5, 6]. One loop corrections reach the 30% level at the TeV scale, and, for this reason, keeping the perturbative series under control is challenging [7–13], and will be particularly important for next generation of very high energy colliders [14]. Moreover such EW corrections are present even for fully inclusive quantities [15] in contrast with QCD where large cancellations between real and virtual corrections take place, and are therefore ubiquitous whenever the initial state is charged under SU(2). With the purpose of considering these issues, EWEEs have been developed in [17, 18]. These new equations allow the resummation of all the terms $\mathcal{O}[\alpha \log^2(Q/M)]^n$ of infrared/collinear origin, and of the $\mathcal{O}[\alpha \log(Q/M)]^n$ terms of collinear origin (actually, terms of order $\alpha^n \log^k(Q/M)$ with $n \leq k \leq 2n$ are resummed). The EW evolution equations are integro-differential equations including kernels that are called splitting functions.

The main point of the present work is to advocate the use of splitting functions that differ from those commonly used in the literature, since they include a cutoff near $z = 0$, z being the momentum fraction. For instance, in the case of the kernel P_{gf}^R that describes the splitting of a fermion f into a gauge boson g and a final state fermion, we have that

$$\frac{1 + (1-z)^2}{z} \to \frac{1 + (1-z)^2}{z} \theta \left(z - \frac{\mu}{Q} \right), \tag{1.1}$$

where in the left hand side there is the standard expression of the splitting function and in the right hand side the one we propose here. The variable μ indicates the soft sliding scale with respect to which the functions are evolved (see section 2). As we explain in section 3, the need to modify the splitting functions arises on one hand from the probabilistic interpretation
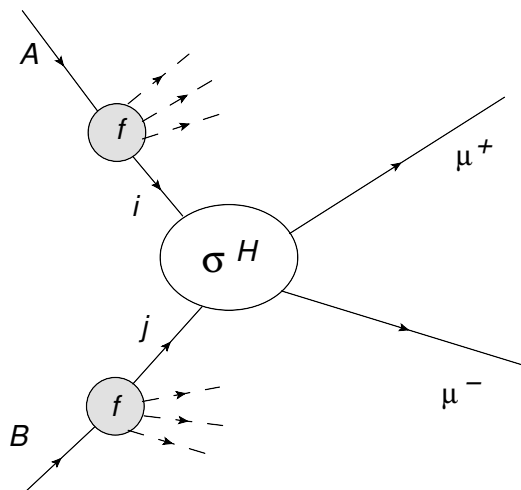
Figure 1. The cross section $\sigma(AB \to \mu^+\mu^- + X)$ can be written as a convolution of PDFs $f_{ij}s$ and a Hard partonic cross section (see text).

of the Parton Distribution Functions (PDFs), and on the other hand from quantum numbers conservation (symmetries of the theory).

The introduction of the modified splitting functions produces sizeable effects on the PDFs, as we show in section 4. Indeed, the distribution functions we obtain differ significantly from the standard ones, not only in the region close to $z = 0$ but also for z of order 1. In section 5 we discuss a point which has been overlooked in the literature so far: the equivalence between the ultraviolet (UV) evolution equations, that are evolved with respect to a hard scale q, and the infrared (IR) evolution equations, where the running scale is a soft one μ. We show that the two approaches are indeed equivalent, but they produce the same PDFs only with an appropriate choice of cutoffs.

2 The electroweak evolution equations

In this work we consider a cross section $\sigma(AB \to \mu^+\mu^- + X)$ involving initial states A, B provided by the collider. While A and B can be, in principle, leptons and/or hadrons, in this paper we limit ourselves to leptonic initial states. The final state features a tagged $\mu^+\mu^-$ pair and is completely inclusive over emitted radiation X. We can write:

$$\sigma(AB \to \mu^+\mu^- + X) = \sum_{i,j} \int dx_i dx_j \, f_{iA}(x_i; M, Q) \, \sigma_{ij}^H(Q^2 = x_j x_i s) \, f_{jB}(x_j; M, Q), \quad (2.1)$$

Here $\sqrt{s}$ is the collider c.m. energy and $Q^2 = (p_i + p_j)^2 \approx x_i x_j s$, is the squared mass of the colliding partons i and j with momenta p_i and p_j. We indicate with $x_i(x_j)$ the momentum fraction of parton $i(j)$ with respect to the initial particle $A(B)$. σ_{ij}^H is the hard cross section for the partonic subprocess $ij \to \mu^+\mu^-$, while i and j run over the Standard Model spectrum. Note that X includes, besides the customary QED and QCD radiation, also EW gauge bosons with their decay products. EWEEs describe the scale dependence of the PDFs f_{ij}, representing the distribution of parton i inside parton j. These functions depend on the momentum fraction x of the daughter particle and on a running scale μ having the physical

meaning of lower cutoff for the daughter's transverse momentum. The PDFs $f_{ij}(x; \mu, Q)$ are evaluated by solving the evolution equations starting from the initial condition at $\mu = Q$ and letting them evolve until μ reaches the weak scale value M. At this point, we obtain the final result $f_{ij}(x; M, Q)$. Since the PDFs are adimensional functions of μ and Q, they can only depend on the ratio μ/Q; therefore we switch to the notation:

$$f_{ij}(x, \varepsilon); \qquad \varepsilon = \frac{\mu}{Q} \tag{2.2}$$

In this work, we use a subset of the Standard Model (SM) equations, setting the U(1) coupling constant g' equal to zero. Furthermore, for illustrative purposes, we consider as hypothetical initial states only left leptons or transverse gauge bosons. As a consequence, we have to deal with only 7 different partons, that are gauge eigenstates: the left fermions electron e and neutrino ν, the corresponding antifermions $\bar{e}, \bar{\nu}$ and the transverse gauge bosons W_-, W_3, W_+. This limitation makes our results not suitable to be directly compared with experimental data. However, the purpose of our work consists in clearly showing that the splitting functions in the EWEEs need to be modified and that these modifications have a relevant effect on the PDFs.

We found convenient the use of a basis of definite isospin and CP quantum numbers. We label L_T^{CP} the left fermion eigenstate with isospin T and definite CP; in analogy, we indicate the transverse gauge boson states as G_T^{CP}. We adopt the classification of states defined in [18], with respect to which we use a slightly different normalization:

$$f_{*L_0^+} = \frac{f_{*\nu} + f_{*e} + f_{*\bar{\nu}} + f_{*\bar{e}}}{2}, \qquad f_{*L_0^-} = \frac{f_{*\nu} + f_{*e} - f_{*\bar{\nu}} - f_{*\bar{e}}}{2}, \tag{2.3}$$

$$f_{*L_1^+} = \frac{f_{*\nu} - f_{*e} + f_{*\bar{\nu}} - f_{*\bar{e}}}{2}, \qquad f_{*L_1^-} = \frac{f_{*\nu} - f_{*e} - f_{*\bar{\nu}} + f_{*\bar{e}}}{2}, \tag{2.4}$$

$$f_{*G_0^+} = \frac{f_{*W_+} + f_{*W_3} + f_{*W_-}}{\sqrt{3}}, \qquad f_{*G_1^-} = \frac{f_{*W_+} - f_{*W_-}}{\sqrt{2}},$$

$$f_{*G_2^+} = \frac{f_{*W_+} - 2f_{*W_3} + f_{*W_-}}{\sqrt{6}}, \tag{2.5}$$

where the asterisk stands for a generic index and similar expressions hold when we keep fixed the right index. We use capital letters for T, CP eigenstates: $A = (L_0^+, L_0^-, L_1^+, L_1^-, G_0^+, G_1^-, G_2^+)$ and small letters for gauge eigenstates $i = (\nu, e, \bar{\nu}, \bar{e}, W_+, W_3, W_-)$. The transition from one basis to the other is then given by a mixed indices unitary matrix U_{Ai} such that $f_{AB} = U_{Ai}U_{Bj}f_{ij}$ and

$$U = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix}. \tag{2.6}$$

The matrix f is a 7×7 square matrix, having, therefore 49 different matrix elements. However, working in the total isospin t-channel, due to isospin and CP conservation, the

majority of these elements vanish. In fact, isospin T and CP conservation implies

$$f_{A_{T_1}^{CP_1} B_{T_2}^{CP_2}}(x, \varepsilon) = \delta_{T_1 T_2} \, \delta_{CP_1 CP_2} \, f_{A_{T_1}^{CP_1} B_{T_1}^{CP_1}}(x, \varepsilon), \qquad \forall \, A, B = L, G \qquad (2.7)$$

for a total of 11 independent PDFs that can be grouped in the following combinations:

$$(T = 0, CP = +) : \begin{vmatrix} f_{L_0^+ L_0^+} & f_{L_0^+ G_0^+} \\ f_{G_0^+ L_0^+} & f_{G_0^+ G_0^+} \end{vmatrix}, \quad (T = 1, CP = -) : \begin{vmatrix} f_{L_1^- L_1^-} & f_{L_1^- G_1^-} \\ f_{G_1^- L_1^-} & f_{G_1^- G_1^-} \end{vmatrix}, \qquad (2.8)$$

$$(T = 0, CP = -) : f_{L_0^- L_0^-}, \quad (T = 1, CP = +) : f_{L_1^+ L_1^+}, \quad (T = 2, CP = +) : f_{G_2^+ G_2^+} .$$

By considering the various reaction channels, for the IR evolution equations we have the following leading-order kernels depending on the variable ε defined in (2.2):

$$P_{ff}^V = -\delta(1-z)\left(\log\frac{1}{\varepsilon^2} - \frac{3}{2}\right); \quad P_{ff}^R = \frac{1+z^2}{1-z}\theta(1-\varepsilon-z), \qquad (2.9)$$

$$P_{gg}^V = -\delta(1-z)\left(\log\frac{1}{\varepsilon^2} - \frac{5}{3}\right); \quad P_{gg}^R = 2\left(z(1-z) + \frac{z}{1-z} + \frac{1-z}{z}\right)\left[\theta(z-\varepsilon)\right]\theta(1-\varepsilon-z), \quad (2.10)$$

$$P_{gf}^R = \frac{1+(1-z)^2}{z}\left[\theta(z-\varepsilon)\right]; \qquad P_{fg}^R = z^2 + (1-z)^2, \qquad (2.11)$$

where with the upper index R (for real) or V (for virtual) we denote the origin of the corresponding contributions. The terms between square brackets in eqs. (2.10), (2.11) are not present in the usual expressions of the kernels, and produce the threshold effect proportional to $\theta(z - \varepsilon)$. In the next section we will discuss the origin of these terms. Throughout this paper we work with fixed value of the weak SU(2) coupling constant α.

Schematically, the equations in the two different basis can be written as:

$$-\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon} f_{ij}(x, \varepsilon) = [f_{ik} \otimes P_{kj}^G](x, \varepsilon) \ , \qquad -\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon} f_{AB}(x, \varepsilon) = [f_{AC} \otimes P_{CB}^I](x, \varepsilon) \, , \quad (2.12)$$

where we have defined the convolution:

$$[f \otimes P](x, \varepsilon) = \int_x^1 \frac{dz}{z} f(z, \varepsilon) P\left(\frac{x}{z}, \varepsilon\right) \, . \qquad (2.13)$$

In the above equations, P^G is a 7×7 matrix with all elements different from zero. Also P^I is a 7×7 matrix, but it is block-diagonal: as can be seen from eq. (2.8) it is a 2×2 submatrix in the $T = 0, CP = +1$ subspace and in the $T = 1, CP = -1$ subspace, while it is a 1×1 submatrix in the $T = 1, CP = +1; T = 0, CP = -1; T = 2, CP = +1$ subspaces. The evolution equations have therefore a much simpler form in the isospin basis; we now write such equations. We use the values of P^I taken from [18] and obtain:

$$-\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon} f_{L_0^- L_0^-}(x, \varepsilon) = \frac{3}{4}[f_{L_0^- L_0^-} \otimes (P_{ff}^R + P_{ff}^V)](x, \varepsilon) \, , \qquad (2.14)$$

$$-\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon} f_{L_1^+ L_1^+}(x, \varepsilon) = [f_{L_1^+ L_1^+} \otimes P_{ff}^V](x, \varepsilon) - \frac{1}{4} f_{L_1^+ L_1^+} \otimes (P_{ff}^R + P_{ff}^V) \, , \qquad (2.15)$$

$$-\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon} f_{G_2^+ G_2^+}(x, \varepsilon) = 3[f_{G_2^+ G_2^+} \otimes P_{gg}^V](x, \varepsilon) - [f_{G_2^+ G_2^+} \otimes (P_{gg}^R + P_{gg}^V)](x, \varepsilon) \, . \qquad (2.16)$$

In the 0^+ channel we have 2 sets of 2×2 systems, corresponding to the possible values $A = L_0^+$ and $A = G_0^+$:

$$\begin{cases} -\dfrac{\pi}{\alpha}\dfrac{\partial}{\partial \log \epsilon} f_{A\,L_0^+}(x,\varepsilon) = \dfrac{3}{4}[f_{A\,L_0^+} \otimes (P_{ff}^R + P_{ff}^V)](x,\varepsilon) + \dfrac{\sqrt{3}}{2}[f_{A\,G_0^+} \otimes P_{gf}^R](x,\varepsilon)\,, \\ -\dfrac{\pi}{\alpha}\dfrac{\partial}{\partial \log \epsilon} f_{A\,G_0^+}(x,\varepsilon) = \dfrac{\sqrt{3}}{2}[f_{A\,L_0^+} \otimes P_{fg}^R](x,\varepsilon) + 2[f_{A\,G_0^+} \otimes (P_{gg}^R + P_{gg}^V)](x,\varepsilon)\,. \end{cases} \tag{2.17}$$

The same happens in the 1^- channel, with $A = L_1^-$ and $A = G_1^-$:

$$\begin{cases} -\dfrac{\pi}{\alpha}\dfrac{\partial}{\partial \log \epsilon} f_{AL_1^-}(x,\varepsilon) = [f_{AL_1^-} \otimes P_{ff}^V](x,\varepsilon) - \dfrac{1}{4}[f_{AL_1^-} \otimes (P_{ff}^R + P_{ff}^V)](x,\varepsilon) + \dfrac{1}{\sqrt{2}}[f_{AG_1^-} \otimes P_{gf}^R](x,\varepsilon), \\ -\dfrac{\pi}{\alpha}\dfrac{\partial}{\partial \log \epsilon} f_{AG_1^-}(x,\varepsilon) = \dfrac{1}{\sqrt{2}}[f_{AL_1^-} \otimes P_{fg}^R](x,\varepsilon) + [f_{AG_1^-} \otimes P_{gg}^V](x,\varepsilon) + [f_{AG_1^-} \otimes (P_{gg}^R + P_{gg}^V)](x,\varepsilon). \end{cases} \tag{2.18}$$

Overall we have $2 \times 2 + 2 \times 2 + 1 + 1 + 1 = 11$ equations. In the EW framework, at difference with QCD, it is possible to express analytically the initial conditions, since we are in a perturbative regime. At the $\mu = Q$ scale, these initial conditions can be expressed as:

$$f_{AB}(x,\varepsilon = 1) = \delta_{AB}\delta(1 - x)\,. \tag{2.19}$$

3 Sum rules and cutoffs for the splitting functions

In this section we show that sum rules on the splitting functions enforce the presence of precisely determined cutoffs near $z = 0$. Sum rules are requirements on the integrals over z of $P_{ij}(z)$ and $zP_{ij}(z)$, obtained from the conservation of the total momentum and of the quantum numbers.

In the literature, in all the works where IR corrections are resummed, [17, 19–21], the cutoffs near $z = 1$ have been considered. In the present work, we introduce for the first time, to the best of our knowledge, additional cutoffs near $z = 0$. We present in this section the sources of these new cutoffs, while their quantitative relevance on the values of the distribution functions $f_{ij}(z,\varepsilon)$ will be discussed in section 4.

Conserved quantities involve integrals of the distribution functions. For instance, if the initial particle is a neutrino, then the probability to become an electron is $\int dz f_{e\nu}(z,\varepsilon)$. If we are interested in fermion number conservation, the initial fermion number, 1, must be conserved when we sum over all possible final states:

$$\begin{aligned} 1 &= \int_0^1 dz(f_{e\nu}(z,\varepsilon) + f_{\nu\nu}(z,\varepsilon) - f_{\bar{\nu}\nu}(z,\varepsilon) - f_{\bar{e}\nu}(z,\varepsilon)) \\ &= 2\int_0^1 dz f_{L_0^-\,\nu}(z,\varepsilon) = \int_0^1 dz f_{L_0^-\,L_0^-}(z,\varepsilon)\,, \end{aligned} \tag{3.1}$$

where we used eqs. (2.3)–(2.5) for both left and right indices, and considered the conservation of the weak isospin. This condition must be satisfied for every value of ε. This means that the evolution equations must have kernels which allows the conservation of the condition (3.1).

In order to explore the consequences of eq. (3.1), it is convenient to write the evolution in terms of the Mellin transform, i.e. by defining the N-th order moments:

$$\tilde{f}(N,\varepsilon) = \int_0^1 dz f(z,\varepsilon)z^{N-1}\,, \tag{3.2}$$

such that eq. (2.12) become factorized, with the ordinary product:

$$-\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon}\tilde{f}_{ij}(N,\varepsilon) = \tilde{f}_{ik}(N,\varepsilon)\,\tilde{P}^G_{kj}(N,\varepsilon)\,; \qquad -\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon}\tilde{f}_{AB}(N,\varepsilon) = \tilde{f}_{AC}(N,\varepsilon)\,\tilde{P}^I_{CB}(N,\varepsilon)\,.$$
(3.3)

Using these equations in the $N = 1$ case, and deriving eq. (3.1) with respect to ε we obtain:

$$0 = -\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon}\tilde{f}_{L_0^- \ L_0^-}(1,\varepsilon) = \frac{3}{4}\tilde{f}_{L_0^- \ L_0^-}(1,\varepsilon)\,(\tilde{P}^R_{ff}(1,\varepsilon) + \tilde{P}^V_{ff}(1,\varepsilon))\,.$$
(3.4)

Since the value of $\tilde{f}_{L_0^- \ L_0^-}(1,\varepsilon)$ depends on ε, therefore it is arbitrary, the above equation is satisfied for every value of ε if

$$\tilde{P}^R_{ff}(1,\varepsilon) + \tilde{P}^V_{ff}(1,\varepsilon) = 0\,.$$

We can now use eq. (2.9) to obtain:

$$\tilde{P}^R_{ff}(1,\varepsilon) + \tilde{P}^V_{ff}(1,\varepsilon) = \int_0^1 dz (P^R_{ff}(z,\varepsilon) + P^V_{ff}(z,\varepsilon)) = 2\varepsilon + \mathcal{O}(\varepsilon^2) \to 0\,.$$
(3.5)

The sum rule is not exact, but leading terms in $\log \varepsilon$ and subleading constant terms cancel with each other, while there is a remnant of (irrelevant) mass-suppressed terms.

We consider now the momentum conservation. Since z is the momentum fraction of the daughter (final) particle, considering a parent (initial) neutrino and operating as we have done in the case of the fermion number we have:

$$1 = \sum_j \int_0^1 dz\, z f_{j\nu}(z,\varepsilon) = 2\tilde{f}_{L_0^+ \ \nu}(2,\varepsilon) + \sqrt{3}\tilde{f}_{G_0^+ \ \nu}(2,\varepsilon) = \tilde{f}_{L_0^+ \ L_0^+}(2,\varepsilon) + \frac{\sqrt{3}}{2}\tilde{f}_{G_0^+ \ L_0^+}(2,\varepsilon)\,. \quad (3.6)$$

Analogously, for a neutral gauge boson initial state we obtain:

$$1 = \sum_j \int dz\, z f_{jW_3}(z,\varepsilon) = 2\tilde{f}_{L_0^+ \ W_3}(2,\varepsilon) + \sqrt{3}\tilde{f}_{G_0^+ \ W_3}(2,\varepsilon) = \frac{2}{\sqrt{3}}\tilde{f}_{L_0^+ \ G_0^+}(2,\varepsilon) + \tilde{f}_{G_0^+ \ G_0^+}(2,\varepsilon)\,.$$
(3.7)

We now use eq. (2.17) in the 0^+ channel:

$$0 = -\frac{\pi}{\alpha}\frac{\partial}{\partial \log \epsilon}\left(\tilde{f}_{L_0^+ L_0^+}(2,\varepsilon) + \frac{\sqrt{3}}{2}\tilde{f}_{G_0^+ L_0^+}(2,\varepsilon)\right)$$

$$= \frac{3}{4}\left[\left(\tilde{f}_{L_0^+ L_0^+}(2,\varepsilon) + \frac{\sqrt{3}}{2}\tilde{f}_{G_0^+ L_0^+}(2,\varepsilon)\right)(\tilde{P}^R_{ff} + \tilde{P}^V_{ff})(2,\varepsilon) + \left(\frac{2}{\sqrt{3}}\tilde{f}_{L_0^+ G_0^+}(2,\varepsilon) + \tilde{f}_{G_0^+ G_0^+}(2,\varepsilon)\right)\tilde{P}^R_{gf}(2,\varepsilon)\right],$$

and considering again eqs. (3.6), (3.7) we obtain the sum rule that can be seen to be obeyed by the splitting functions (2.9)–(2.11):

$$\tilde{P}^R_{ff}(2,\varepsilon) + \tilde{P}^V_{ff}(2,\varepsilon) + \tilde{P}^R_{gf}(2,\varepsilon) = \mathcal{O}(\varepsilon) \to 0\,.$$
(3.8)

Up to this point we have considered sum rules that the EWEEs have in common with those of the QCD. Indeed, in the case of strong interactions, due to confinement, initial states are color singlets, that correspond to the $T = 0$ evolution equations. In the case of EW interactions instead, initial states have isospin quantum numbers and, therefore, further

equations in the channels $T = 1$ and $T = 2$ are present. A consequence of this feature of the EW equations is that the double logs terms of IR origin do not cancel. This effect has been called "Bloch-Nordsieck (BN) violation" [15].

We now proceed to show that the EWEEs, together with quantum numbers conservation, lead to sum rules that are absent in QCD, and are responsible ultimately for the necessity of precise cutoffs in the splitting functions near $z = 0$.

The conservation of the third component of the weak isospin, T_3, is related to the $T = 1, CP = -$ structure function. Taking a neutrino and a neutral gauge boson as initial particles, we have:

$$t_\nu^3 = \frac{1}{2} = \sum_i t_i^3 \tilde{f}_{i\nu}(1, \varepsilon) = \frac{1}{2}(\tilde{f}_{L_1^- L_1^-}(1, \varepsilon) + \sqrt{2}\tilde{f}_{G_1^- L_1^-}(1, \varepsilon)), \tag{3.9}$$

$$t_{W_+}^3 = 1 = \sum_i t_i^3 \tilde{f}_{iW_+}(1, \varepsilon) = \frac{1}{\sqrt{2}}(\tilde{f}_{L_1^- G_1^-}(1, \varepsilon) + \sqrt{2}\tilde{f}_{G_1^- G_1^-}(1, \varepsilon)). \tag{3.10}$$

We use the evolution equation (2.18) and the relations (3.9), (3.10) and we operate as in the case of momentum conservation. We finally obtain:

$$3\tilde{P}_{ff}^V(1, \varepsilon) - \tilde{P}_{ff}^R(1, \varepsilon) + 4\tilde{P}_{gf}^R(1, \varepsilon) = 0 \Rightarrow \tilde{P}_{ff}^R(1, \varepsilon) = \tilde{P}_{gf}^R(1, \varepsilon), \tag{3.11}$$

$$\frac{1}{2}\tilde{P}_{fg}^R(1, \varepsilon) + \tilde{P}_{gg}^R(1, \varepsilon) + 2\tilde{P}_{gg}^V(1, \varepsilon) = 0. \tag{3.12}$$

where the last step in eq. (3.11) has been obtained by using eq. (3.5).

Let us consider first the sum rule (3.11). If we use the standard expression for P_{gf}^R, i.e. that without the cut close to $z = 0$, i.e.

$$P_{gf}^R(z) \to \frac{1 + (1 - z)^2}{z}, \tag{3.13}$$

eq. (3.11) cannot be satisfied since $\tilde{P}_{ff}^R(1, \varepsilon)$ is finite, while $\tilde{P}_{gf}^R(1, \varepsilon)$ diverges due to the singularity for $z \to 0$ proportional to $1/z$. If instead we impose a cutoff and use the expression for $\tilde{P}_{gf}^R(1, \varepsilon)$ of eq. (2.11), then the sum rule is satisfied.

The following points are worth to be emphasized.

- $\tilde{P}_{ff}^R(1, \varepsilon)$ is divergent for $z \to 1$ while $\tilde{P}_{gf}^R(1, \varepsilon)$ is divergent for $z \to 0$, therefore the sum rule connects the (known) cutoffs of IR origin close to 1 with the (new) cutoffs close to 0.

- The value of the cutoff $z > \varepsilon$ is precisely determined by the requirement of eq. (3.11). With a different value like $z > 2\varepsilon$ the sum rule would be violated.

- The presence of the cutoff is justified from a probabilistic point of view. We expect that the probability for a fermion to become a fermion of momentum fraction z has to be the same of the probability of becoming a gauge boson of momentum fraction $1 - z$: both are related to the same tree level diagram (see diagram (a) in figure 3).

Similar considerations hold for P_{gg}^R, whose integral in z must satisfy eq. (3.12). This is obtained by introducing the cutoff near $z = 0$ which has been highlighted by the square brackets in eq. (2.10).

Finally, we point out that the new splitting functions are P_{gf} and P_{gg} and they involve the splitting into a gauge boson. Indeed, the need for a cutoff close to 0 arises because of the $1/z$ singularity related to the IR dynamics of a soft gauge boson.

4 Quantitative results

In this section, we compare the PDFs obtained with the uncut splitting functions, i.e. the standard ones without cutoff, with those obtained with our splitting functions which include the cutoff.

In order to illustrate the different features characterizing the two types of PDFs, we consider first the analytical expressions of the first order terms of a perturbative expansion in powers of α:

$$f_{AB}(x,\varepsilon) = \sum_{i=0}^{\infty} \alpha^i f_{AB}^{(i)}(x,\varepsilon), \tag{4.1}$$

where the zeroth order term is dictated by the initial conditions

$$f_{AB}^{(0)}(x,\varepsilon) = \delta_{AB}\delta(1-x), \tag{4.2}$$

which indicate that, in the no-emission case, the probability of finding a particle inside itself is equal to the unity.

Using the perturbative expansion in eqs. (2.14)–(2.18), we obtain analytical results for all first order terms of the PDFs. For instance, in the case of the (L_0^+, G_0^+) channel we have

$$f_{L_0^+,G_0^+}^{(1)}(x,\varepsilon) = \frac{1}{\pi}\frac{\sqrt{3}}{2}\int_\varepsilon^1 \frac{d\varepsilon'}{\varepsilon'} P_{fg}^R(x,\varepsilon'), \tag{4.3}$$

and we obtain two different results for our, cut (c), and standard, uncut (u), cases:

$$f_{G_0^+ L_0^+}^{(1)c}(x,\varepsilon) = \frac{1}{\pi}\frac{\sqrt{3}}{2}\frac{1+(1-x)^2}{x}\theta(x-\varepsilon)\log\frac{x}{\varepsilon}; \qquad f_{G_0^+ L_0^+}^{(1)u}(x,\varepsilon) = \frac{1}{\pi}\frac{\sqrt{3}}{2}\frac{1+(1-x)^2}{x}\log\frac{1}{\varepsilon}. \tag{4.4}$$

Already from these first order expressions we see that the differences between the PDFs obtained with our splitting functions and the standard ones are relevant. Indeed, in the standard case the behaviour close to 0 is $\propto 1/x$ and therefore divergent, while in our case the PDF is $\propto (1/x)\log(x/\varepsilon)$ therefore it is continuous and, because of the step function, is 0 for $0 \leq x \leq \varepsilon$. Physically, the latter situation is more plausible, as has already been noticed in [16], since the fraction of energy Q of a massive gauge boson with mass M cannot be smaller than M/Q.

In addition to the first order, we obtained the complete solutions by numerically solving eqs. (2.14)–(2.18). We consider their evolution in the variable $t \equiv \log\varepsilon = \log(\mu/Q)$, which ranges in the interval $[t_{\min}, 0]$ with $t_{\min} \equiv \log(M/Q) < 0$. We used a discrete two-dimensional grid $[t_{\min}, 0] \times [0, 1]$ in the (t, x)-plane to transform the differential equation (2.12) into a finite-difference one. The z-integration occurring in eq. (2.13) has been carried out by using an adaptive method which evaluates the exact values of the splitting functions and an interpolation of the evolving PDFs. This procedure has the advantage of a better treatment
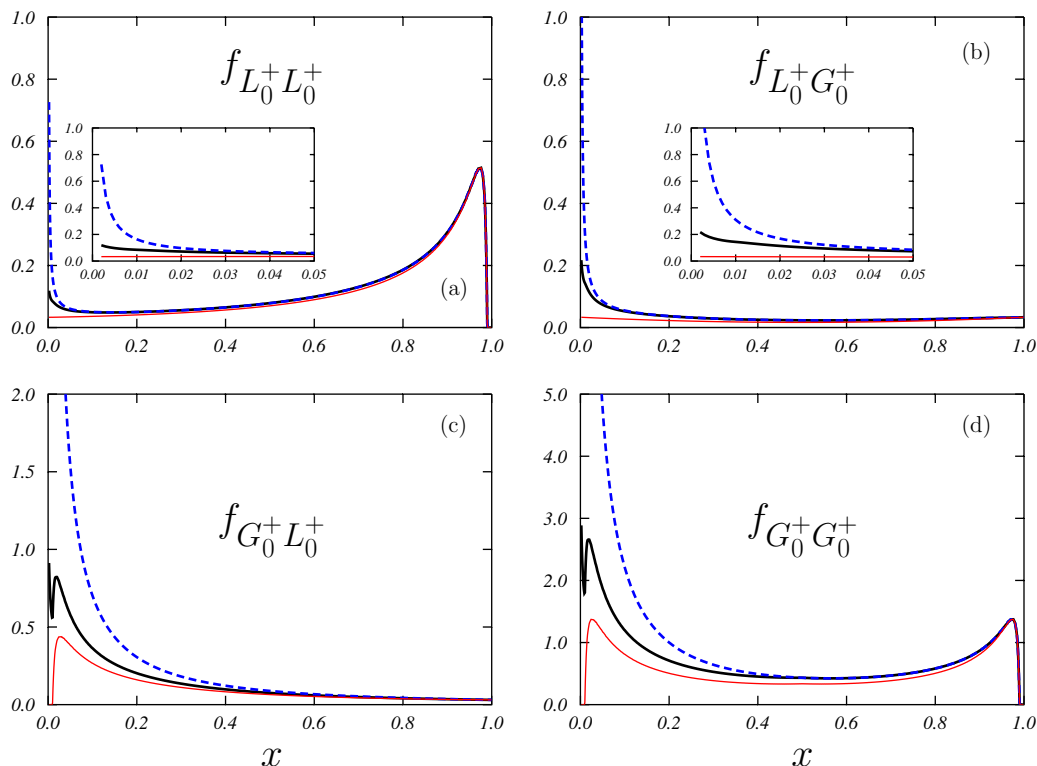
Figure 2. PDFs of the $T = 0$ isospin channels. The full black lines show the results of our full calculations where the new constraints on the P_{gg}^R and P_{gf}^R splitting functions have been considered. The thin red lines show the results obtained by considering only the first order term in the α power expansion of eq. (4.1). The results of the complete, numerical, calculation carried out without considering the constrains in the splitting functions are shown by the blue dashed lines.

of the t-dependent cuts on the z-integration, since the boundaries of the z-integration are almost never found on the discretized x_i points. We implemented the t-evolution via a 4-point Runge-Kutta algorithm and increase the number of points in the grid until we reach the required precision.

We studied the impact of the new constraints on the splitting functions by solving eqs. (2.14)–(2.18) with and without the $\theta(x - \varepsilon)$ terms in the expressions (2.10) and (2.11) of P_{gg}^R and P_{gf}^R. We carried out calculations for three different values of ε, specifically 0.01, 0.001, 0.0001. We present here only some selected results for $\varepsilon = 0.01$, since the physics contents of the other cases is analogous.

We show in figure 2 the PDFs as a function of the momentum fraction x for the four $T = 0$ isospin channels. The PDFs obtained with our splitting functions, f^c, are indicated by the black lines, while those obtained with the standard splitting functions, f^u, by the dashed blue lines. The thin red lines show the first order solution $f^{(1)c}$ in an expansion in powers of α (see eq. (4.1)).

As expected, the differences between f^c and f^u are more significant at small values of x. We emphasize these difference in the insets of the panels (a) and (b) where we show the PDFs for $x \ll 1$.
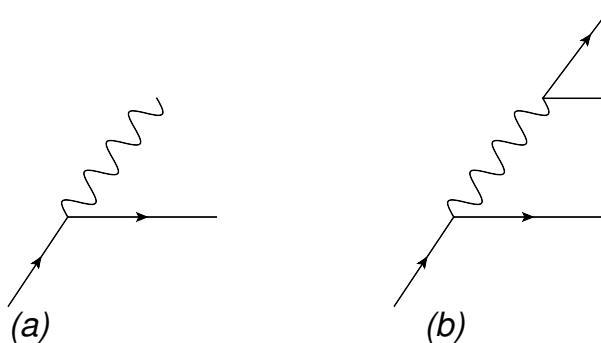
Figure 3. First order contribution to f_{GL} (a) and second order contribution to f_{LL} (b).

These results show that the difference between f^c and f^u is significant in the isospin channels describing gauge bosons distributions like $f_{G_0^+ G_0^+}$ and $f_{G_0^+ L_0^+}$, while in the channels describing lepton distributions like $f_{L_0^+ G_0^+}$ and $f_{L_0^+ L_0^+}$ these differences are less pronounced. The reason for this is that the new constraint are present in P_{gf} and P_{gg} and they involve the splitting into a gauge boson. These splitting functions contribute at first order for a gauge boson distribution but only at second order for a lepton distribution (see figure 3). For this reason the effect of the new constraints is more pronounced for gauge bosons distributions.

For small x values, and precisely when $|\log x| \gg 1$, the DGLAP picture breaks down, and one should resort to the BFKL picture instead [47–49]. Moreover, in the case of EW interactions, the situation at small x is further complicated by the possible presence of additional terms [50]. Nevertheless, we point out that the effect of the new constraints is not limited to very small values of x: for example, for the complete solution, in the case of $f_{G_0^+ L_0^+}$, at $x = 0.4$, f^u is larger than f^c by 25%; in this case $|\log x|$ is of order one and no small x issues are present. This could surprise because our splitting functions differs significantly from the standard ones only in the region $x \simeq \varepsilon$, and we choose very small values of ε. However, one has to consider that in the EWEEs, the splitting functions are integrated on all the possible values of ε', i.e. from $\varepsilon' = \varepsilon$ to $\varepsilon' = 1$, and this means that the new cutoffs generate effects for every value of x.

We evaluated the PDFs in the physical channels by inverting the U matrix of eq. (2.6). We show in figure 4 the results for four selected channels representative of the 49 ones. The meaning of the lines is the same as in figure 2.

In the isospin representation we have pointed out that the gauge boson channels are most sensitive to the new cutoffs, and this is reflected also in the physical channels.

The results of the panel (a) of figure 4 show that the new constraints modify the PDFs only at very small values of x, as it is shown in the inset. This is a purely leptonic channel with an electron producing a neutrino. In this case, the three curves almost overlap, except when $x \simeq 0$.

The situation is very different when, at least, one gauge boson channel appears, as it is shown in the other panels of the figure. The relative difference between f^c and f^u at $x = 0.4$ is about 22% in the $W_- e$ and $W_3 \nu$ channels, panels (b) and (c), and about 10% in the $W_+ W_+$ channels, panel (d).

In these channels the solution for the first order expansion term fails in describing the correct behaviour of the complete solution. The difference in the W_+, W_+ channel is

$$\epsilon = 0.01$$
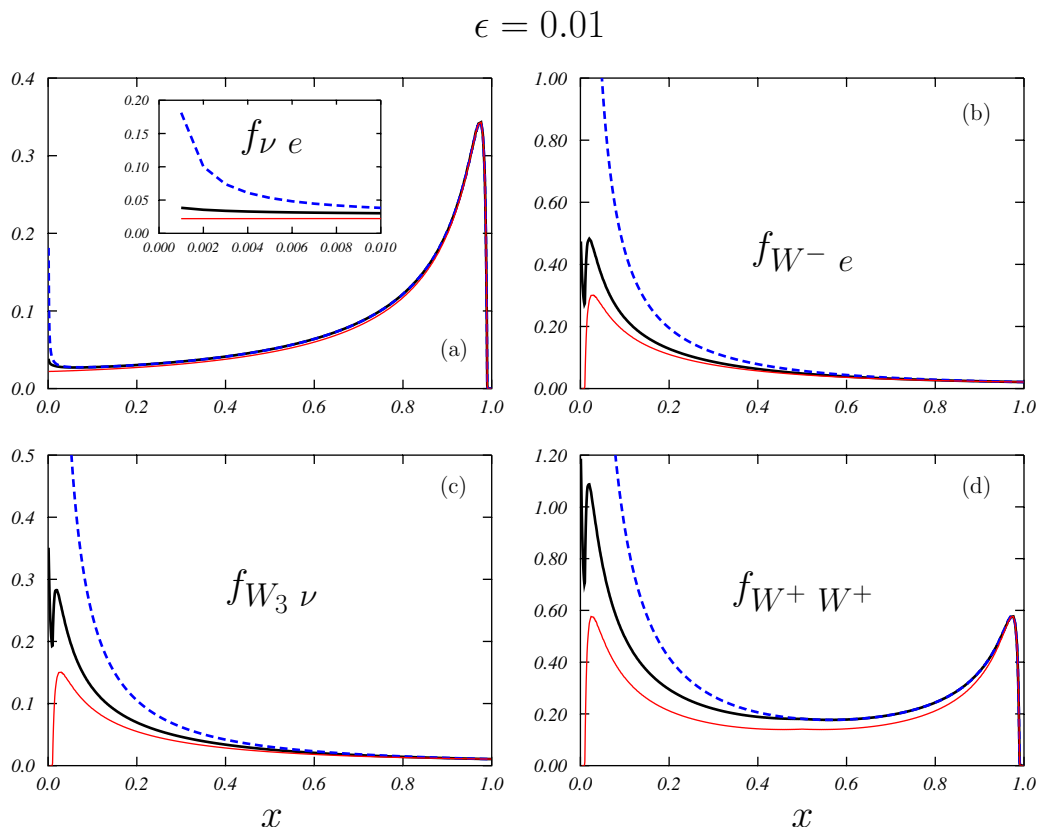
Figure 4. PDFs for four selected physical channels. The meaning of the lines is the same as in figure 2.

remarkable even from the qualitative point of view. The first order solution is symmetric around $x = 0.5$, while the full solution is clearly asymmetric with values at small x remarkably larger than those around $x = 1$.

5 IR and UV evolution equations

The EWEEs were originally derived as IR equations [17, 18], i.e. equations where the varying scale is an infrared parameter μ having the meaning of *lower bound* on transverse momentum of the emitted particles. This approach was historically used to calculate double-logarithmic (DL) contributions of IR origin, first calculated by Sudakov in QED [22–26], generalized as a factorization theorem of photon bremsstrahlung for small $k_\perp$ in high energy cross sections [28–35] and then extended in QCD in [36] and EW [19–21, 37]. From an alternative point of view, different works [39–46, 51] used UV evolution equations, where the varying scale is a parameter q having the meaning of *upper bound* on the transverse momentum of the emitted particles. It is not clear under which conditions the two approaches produce the same results, i.e. the same PDFs. In the first part of this section we show that the solutions of the UV and IR evolution equations coincide when the same splitting functions, including the cutoffs, are consistently used in both approaches.
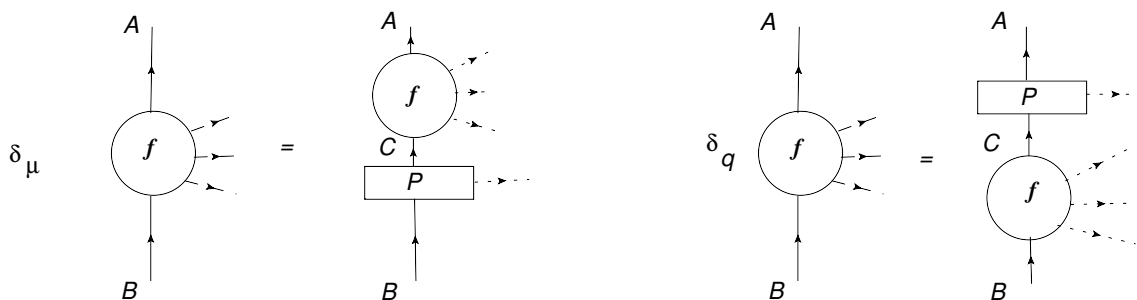
Figure 5. IR (left) and UV (right) evolution equations. The dashed lines represent particles belonging to the final state.

In the EW case we have to consider splitting functions depending not only on the momentum fraction z but also on the transverse momentum $k_\perp$: $P_{AB}(z, k_\perp)$. In the IR equation the contribution of the emitted particle with lower momentum is factorized (figure 5, left side); calling μ the lower cutoff on this momentum we have:

$$\mu \frac{\partial}{\partial \mu} f_{AB}^{IR}(x; \mu, Q) = -\frac{\alpha}{\pi} f_{AC}^{IR} \otimes P_{CB}(\mu); \qquad f^{IR}(x, \mu = Q) = f_0(x). \tag{5.1}$$

In the case of UV equations instead, the contribution of the hardest emitted particle is factorized (figure 5, right side). We call q the upper bound on the transverse momentum and obtain:

$$q \frac{\partial}{\partial q} f_{AB}^{UV}(x; M, q) = \frac{\alpha}{\pi} P_{AC}(q) \otimes f_{CB}^{UV}; \qquad f^{UV}(x, q = M) = f_0(x). \tag{5.2}$$

Note that, apart from a trivial sign difference due to the fact that the f's are ultimately functions of μ/q, eqs. (5.1) and (5.2) are truly different because of the different running of the indices. In other words, by considering f and P as matrices, in the IR case the matrix product $f \cdot P$ appears in the right-hand side of the equations, while in the UV case $P \cdot f$ appears; therefore there is no guarantee that the equations will produce the same solutions. In this section we prove that, provided that certain conditions are fulfilled, the solutions of (5.1) and (5.2) coincide.

The formal solution, in the IR case, is given by:

$$f^{IR}(\mu, Q) = f_0 \otimes K\left\{ \exp\left[\frac{\alpha}{\pi} P\right] \right\}(\mu, Q), \tag{5.3}$$

having defined:

$$K\left\{ \exp\left[\frac{\alpha}{\pi} P\right] \right\}_{ij}(\mu, q) = \delta(1 - z)\delta_{ij} + \frac{\alpha}{\pi} \int_\mu^q P_{ij}(k) \frac{dk}{k} \tag{5.4}$$

$$+ \sum_{n \geq 2} \left(\frac{\alpha}{\pi}\right)^n \int\limits_{\mu < k_n < \cdots < k_1 < q} \left(\Pi_i \frac{dk_i}{k_i}\right) P_{ii_1}(k_1) \otimes \cdots \otimes P_{i_n j}(k_n)$$

Eq. (5.3) is indeed solution of eq. (5.1), since the only variable depending on μ in the ordered product of eq. (5.4) is k_n. When we derive with respect to μ we obtain (minus) the value of

the integrand calculated for $k_n = \mu$ and this gives the solution of (5.1). It is then sufficient to implement the initial condition f_0.

A similar reasoning applies to the UV case. In this case, only k_1 depends on q and we have:

$$f^{UV}(M, q) = \mathrm{K}\left\{ \exp\left[\frac{\alpha}{\pi}P\right] \right\}(M, q) \otimes f_0 \,. \tag{5.5}$$

Since we eventually set $\mu = M$ in (5.3) and $q = Q$ in (5.5), the IR case and UV case differ only because of a different placement of the f_0 function. However, since we have $f_0 = \delta(1-z)\delta_{AB}$ and since $\delta(1-z)$ is the identity for convolutions and δ_{AB} is the identity in the P matrices space, the two solutions are equal:

$$f^{IR}(M, Q) = f^{UV}(M, Q) = \mathrm{K}\left\{ \exp\left[\frac{\alpha}{\pi}P\right] \right\}(M, Q) \,. \tag{5.6}$$

This establishes the equivalence between the IR and the UV approaches.

We remark that eqs. (5.1) and (5.2) are equivalent, i.e. they produce the same solutions for the PDFs, only when the same splitting functions are used, albeit with different arguments: μ in the IR case and q in the UV one. If the kernels P_{AB} do not depend on $\mu(q)$ at all, the equivalence between IR and UV approach is straightforward on the basis of the proof given above. This is the case in many applications of QCD (DGLAP) equations, where only the "+" distribution, defined as

$$P^+(x) = \lim_{\varepsilon \to 0}(P^V(x, \varepsilon) + P^R(x, \varepsilon)) \tag{5.7}$$

appears.[1]

However, in the EW case treated here, there is a cutoff for $x < 1 - \mu/Q$ in $P_{AB}(z, \mu)$ for the IR equations. This cutoff is well justified in terms of kinematics and because it reproduces the correct IR properties of the theory, i.e. the eikonal approximation [17]. Then, in the ultraviolet equations, a cutoff at $x < 1 - q/Q$ must appear in $P_{AB}(z, q)$. This might sound trivial, but the point is that in the literature different cutoffs appear. For instance in ref [39] the cutoff is rather $z < 1 - M/q$, leading, in general, to PDFs that may differ from those we obtain. This possible difference, due to the upper cutoff on z, comes on top of the more important difference due to the lower cutoff on z that we have analysed in previous sections. Even though a detailed comparison with ref. [39] of the impact of the upper cutoff is beyond the scope of this work, we now show that, at least in the case when radiation is forbidden and only virtual corrections are present, our results (PDFs) coincide.

From eqs. (2.14)–(2.18) we see that in the limit $P_{ij}^R \to 0$ the four diagonal leptonic PDFs $f_{L_0^+ L_0^+}, f_{L_0^- L_0^-}, f_{L_1^+ L_1^+}, f_{L_1^- L_1^-}$ share the same equation and the same initial condition, producing the same solution that we call $\Delta_{ff}^{(IR)}(x, \frac{\mu}{Q})$. This function is called the (virtual) Sudakov form factor and is related to the infrared properties of the Standard Model. Analogous situation holds for the gauge bosons case, where $f_{G_0^+ G_0^+}, f_{G_1^- G_1^-}, f_{G_2^+ G_2^+}$ all become equal to the Sudakov form factor $\Delta_{gg}^{(IR)}(x, \frac{\mu}{Q})$ in the purely virtual case. First of all let us show explicitly that the

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[1]In chapter 1.3.2 of [27] it was shown that, in the case where the kernels do not depend on the evolution variable, the UV and the IR approaches in QCD produce the same PDFs.

same purely virtual Sudakov Form Factors are generated in our IR evolution equations and in the UV evolution equations with the cutoff defined in [39]. We obtain:

$$-\frac{\pi}{\alpha}\mu\,\frac{\partial}{\partial\mu}\Delta_{ff}^{(IR)}\left(x,\frac{\mu}{Q}\right) = C_f\,P_{ff}^V(\mu)\,\Delta_{ff}^{(IR)}\left(x,\frac{\mu}{Q}\right),\quad C_f=\frac{3}{4},\ P_{ff}^V(\mu)=\frac{3}{2}+2\log\frac{Q}{\mu} \quad (5.8)$$

$$-\frac{\pi}{\alpha}\mu\,\frac{\partial}{\partial\mu}\Delta_{gg}^{(IR)}\left(x,\frac{\mu}{Q}\right) = C_g\,P_{gg}^V(\mu)\,\Delta_{gg}^{(IR)}\left(x,\frac{\mu}{Q}\right),\quad C_g=2,\ P_{gg}^V(\mu)=\frac{5}{3}+2\log\frac{Q}{\mu} \quad (5.9)$$

From [39] the Sudakov form factors $\Delta^{(UV)}$ can be read off directly; they satisfy:

$$\frac{\pi}{\alpha}q\,\frac{\partial}{\partial q}\Delta_{ff}^{(UV)}\left(x,\frac{M}{q}\right) = C_f\,P_{ff}^V(q)\,\Delta_{ff}^{(UV)}\left(x,\frac{M}{q}\right),\quad C_f=\frac{3}{4},\ P_{ff}^V(q)=\frac{3}{2}-2\log\frac{q}{M} \quad (5.10)$$

$$\frac{\pi}{\alpha}q\,\frac{\partial}{\partial q}\Delta_{gg}^{(UV)}\left(x,\frac{M}{q}\right) = C_g\,P_{gg}^V(q)\,\Delta_{gg}^{(UV)}\left(x,\frac{M}{q}\right),\quad C_g=2,\ P_{gg}^V(q)=\frac{5}{3}-2\log\frac{q}{M} \quad (5.11)$$

Once we integrate in the range $M \le \mu, q \le Q$ the final solutions are

$$\Delta_{ff}^{(IR)}\left(x,\frac{M}{Q}\right) = \Delta_{ff}^{(UV)}\left(x,\frac{M}{Q}\right) = \exp\left[\frac{\alpha\,C_f}{\pi}\left(\log^2\frac{Q}{M}-\frac{3}{2}\log\frac{Q}{M}\right)\right], \quad (5.12)$$

$$\Delta_{gg}^{(IR)}\left(x,\frac{M}{Q}\right) = \Delta_{gg}^{(UV)}\left(x,\frac{M}{Q}\right) = \exp\left[-\frac{\alpha\,C_g}{\pi}\left(\log^2\frac{Q}{M}-\frac{5}{3}\log\frac{Q}{M}\right)\right] \quad (5.13)$$

We have shown that at the leading Sudakov form factor level our results and those of ref. [39] coincide; these results also coincide with the analogous ones in ref. [40]. Understanding more in detail the (possible) differences between the two approaches is an important point; however it goes beyond the scope of the present work. For the moment let us stress an important point. Regardless of the type of approach (IR or UV) and of the upper cut on z, the arguments that lead to the presence of a *lower* cutoff, which has an important impact on the PDFs as we have seen, hold just the same. This is because the sum rules (3.11), (3.12) only depend on isospin conservation and not on the type of approach.

6 Summary, conclusions and perspectives

At c.m. energies much greater than the weak scale, energy growing Electroweak Radiative corrections can be taken into account by defining Parton Distribution Functions (PDFs) that obey Electroweak Evolution Equations (EWEEs), in analogy with DGLAP in QCD. In this work we propose to modify EWEEs with respect to what has been done until now in the literature and we analyse the impact on PDFs of these modifications. In particular, electroweak interactions are characterized by isospin 1 evolution equations that are absent in the corresponding isospin 0 QCD (DGLAP) and QED equations. Isospin conservation, related to these isospin 1 equations, requires to modify the splitting functions (that are the kernels of EWEEs) by adding suitable cutoffs. The solutions (PDFs) obtained with these new kernels differ significantly from those obtained with the standard kernels used in the literature until now (see figure 4). We think that all future works on the subject will have to introduce the new kernels we propose.

In this work we have also addressed the issue of comparing the results obtained by using a IR approach (ours) with previous results obtained using a UV approach, as is customary

in the literature. We have shown that UV equations indeed produce the same PDFs as IR equations, but only if a careful choice of the cutoffs in the splitting functions is made. Finally, let us note that work has still to be done in order to provide theoretical results that can be compared with the experimental measurements. First, QCD and QED interactions have to be added and then the full particle spectrum of the Standard Model has to be considered, while we chose to consider only a subset. Then, the case of hadronic colliders have to be included, while we consider only leptonic initial states. Finally, for small momentum fraction x, additional terms proportional to $\log x$ should be added to the equations [50].

The modifications described here will be particularly relevant if a 100 TeV hadronic collider [52] and/or a TeV scale muon collider [53] will see the light.

Acknowledgments

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