E C O N O M I C S B U L L E T I N

# Quasi−option value under ambiguity

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## *Abstract*

Real investments involving irreversibility and ambiguity embed a positive quasi−option value under ambiguity (q.o.v.a.), which modifies the evaluation of an investment decision involving depletion of natural resources by increasing the value of delaying. Q.o.v.a. depends on the specific decision−maker attitude towards ambiguity, expressed by a capacity on the state space. An empirical measure of q.o.v.a. is pointed out. Exploiting the properties of a capacity and its conjugate, the relationship has been established between the upper and lower Choquet integral with respect to a subadditive capacity and the bid and ask price of the underlying asset (output) of the investment decision. The empirical measure of q.o.v.a. is defined as the upper bound of the opportunity value. As an example, q.o.v.a. is applied to evaluate an off−shore petroleum lease under ambiguity.

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# 1 Introduction.

Irreversibility and uncertainty characterize development decisions involving non-renewable assets, such as energy-generating resources like coal, oil and natural gas. Irreversibility breaks the temporal symmetry between the past and the future in the consumption decision of natural resources. Uncertainty occurs when consequences of development decisions cannot be fully determined *ex-ante* and all the uncontrolled variables of the decision process are random variables, which only depend on the possible state of nature that will occur in the future.

In seminal articles, Arrow and Fisher (1974) and Henry (1974a, 1974b) independently point out that under uncertainty, when a given decision could (at least partially) have irreversible effects and learning is possible before future decisions have to be made, it is generally valuable to keep open an option, even if the decision-maker is risk neutral and her marginal utility is constant. They call quasi-option value the extra value attached to the preservation of an option in order to stress the crucial role played by irreversibility and learning and show its independence from risk attitude.

Some different methods of measuring quasi-option value have been suggested in the context of empirical decision problems. The most notable of these is real-option analysis.[1] Real-option pricing theory considers an irreversible investment as a financial call option, which provides the possibility of waiting for new information to arrive that might affect the desirability or timing of the expenditure. Applications involving financial instruments and project-investment valuations have a common element for using option pricing.

It has been argued (Basili, 1998) that the tool of quasi-option value (real-option value) should be augmented to encompass ambiguity. In this paper an empirical measure of quasi-option value under ambiguity ($q.o.v.a.$) is proposed and evaluated. Both the decision-maker's priors and conjugate beliefs are considered and the upper and lower value of the contingent claim are derived. For each investment project, two quasi-option values are obtained: the greater of them is taken as the measure of $q.o.v.a.$. As an example, an application of $q.o.v.a.$ is considered for the case of an offshore petroleum lease. It is shown that the $q.o.v.a.$ modifies the value of the exclusive rights to the

[1] Myers and Majd, 1983; McDonald and Siegel, 1986; Majd and Pindyck, 1987; Paddock 'et. al.',1988.

project.

The plan of the paper is as follows. Section 2 describes quasi-option value and quasi-option value under ambiguity. In Section 3 an empirical measure of *q.o.v.a.* is proposed. Section 4 contains the example of a possible application to an offshore oil tract. Section 5 concludes. The proof of our main claim is in the appendix.

## 2 Quasi-option value and quasi-option value under ambiguity.

Arrow and Fisher (1974) introduce the notion of quasi-option value and argue that whenever uncertainty is assumed, "even where it is not appropriate to postulate risk aversion in evaluating an activity, something of the feel of risk aversion is produced by a restriction on reversibility of decision" (Arrow and Fisher, 1974, p.318). Henry (1974b) shows that replacing the initial random problem by an associated riskless problem, i.e. an equivalent certainty case, the decision-maker could obtain a non-optimal solution, even if she is risk neutral and the payoff function is quadratic. Quasi-option value ($q.o.v.$) is equal to the maximum difference between $R^*$, the expected revenue of the random problem, and $R$, the expected revenue of the riskless problem, that is $q.o.v. = max[R^* - R, 0]$. The quasi-option value represents the conditional value of information, conditional to the reversible action.

It is worth noting that an irreversible investment opportunity is equivalent to a financial perpetual call option on common stock,[2] where the investment expenditure is the exercise price and the project value, which is the expected payoff from investing, is a share of the underlying asset. Dixit and Pindyck derive "the value of the extra freedom, namely the option to postpone the decision" (Dixit and Pindyck, 1994, p.97), as the difference between the expected net present values of random problem and associated riskless one. Pindyck (1991) observes that dynamic programming and contingent claims analysis yield the identical solution (rule that maximizes the market value of the investment opportunity), if the decision-maker is risk neutral[3] and

[2] A disinvestment opportunity (partial reversibility) is equivalent to a put option and the act to disinvest is equivalent to exercise such an option.

[3] Risk neutrality means that the discount rate equals the risk-free rate (e.g., Cox and Ross, 1976).

the risk-free interest rate replaces the discount rate. Given markets complete or at least sufficiently complete (spanning assumption), the value of a project and the value of the option to invest is determined by constructing a replicating portfolio or finding some perfectly correlated assets and using option-pricing theory.[4] That is, the value of the option to invest is based on the construction of a risk-free portfolio in which the asset is traded (long or short position) or by finding another asset or a combination of some assets, whose prices are perfectly correlated with the price of the output of the investment project, if that asset is not traded.

Models explaining quasi-option value (real option value) assume that states of nature have an additive probability of occurring, that is, the decision-maker's description of states of nature is exhaustive. The decision-maker has (explicitly or implicitly) a unique probability distribution over events and points out an expected utility function linear in probabilities. It is therefore impossible to deal with ambiguous decision problems. On the contrary, if decision-maker faces ambiguity, it becomes necessary to generalize the *q.o.v.* to assess the value of her uncertain and irreversible decision. Indeed, it can be done introducing the notion of quasi-option value under ambiguity, *q.o.v.a.* This differs from Arrow and Fisher and Henry, who assume events have an additive probability to occur and can thus derive the expected value associated to each possible decision. Events under ambiguity are measured by capacities, that express both events' uncertainty and decision maker's attitude towards it. As a consequence, the quasi-option value measure is generalized by means of the proper Choquet Integral of the decision maker on the underlying assets.

# 3 An empirical measure of quasi-option value under ambiguity.

Real option-pricing theory considers that the decision-maker faces various forms of risk, such as uncertainty over future product prices, operating costs, future interest rates, cost and timing of the investment itself. Uncertainty is represented by a set of states of nature, one of which will be revealed as true

[4] If the spanning assumption does not hold, it is possible to value the investment project and the decision to invest by dynamic programming with an exogenous discount rate (Pindyck, 1991, p.1116).

and option-pricing will determine the optimal exercise rule of an investment. Given competitive markets, no transaction costs and asset prices that follow diffusion processes, it is assumed a unique probability distribution $p \to [0,1]$ on the measurable space $(\Omega, S)$, such that market value of any asset is the expectation of its discounted payments. The asset, which spans the stochastic changes in the project worth, may be considered a random variable $\beta : \Omega \to \mathbb{R}$ of its expected discounted payments and its unique market value equals $\int_\Omega \beta dp$. As a result, there is only one opportunity value or quasi-option value for each irreversible investment project.

Consider an optimistic decision-maker: she has a concave[5] capacity $v : S \to [0,1]$ on the measurable space $(\Omega, S)$ and the valuation of the asset will not be the Lebesgue integral of its expected discounted payments (linear pricing rule), but it will be obtained by the Choquet integral of the expected discounted payments of the assets (non-linear pricing rule) with respect to $v$, i.e. $\int_\Omega \beta dv$.

The optimist decision-maker might be considered as a financial dealer[6] who has both long and short positions on the asset $\beta$, respectively $\int_\Omega -\beta dv$ and $\int_\Omega \beta dv$. By the subadditivity of the Choquet integral, $\int_\Omega \beta dv + \int_\Omega -\beta dv \geq \int_\Omega (\beta - \beta) dv$, which implies that $\int_\Omega \beta dv \geq -\int_\Omega -\beta dv$ : there is a bid-ask spread of the asset $\beta$, and the dealer makes a positive profit.

The bid and ask prices of the underlying asset can be seen as respectively the worst and the best expected payoff of an irreversible investment project. The optimistic decision-maker considers the ask price of the asset $\beta$ as the lowest price (upper bound) at which she will wish to sell the asset, consistent with her priors. She considers the bid price of the asset $\beta$ as the highest price (lower bound) up to which she will wish to buy the asset $\beta$, compatible with her beliefs (Basili and Fontini, 2002).

Summing up, the decision-maker assumes that the true probability distribution of the asset $\beta$ payments is located in the set $P$ of probability

[5]We suppose that optimists (pessimists) hold concave (convex) capacities. This is coherent with Ghirardato and Marinacci (2002), where it is shown that concavity (convexity) is a sufficient conditions for preferences to exhibit uncertainty favour (aversion). For a different approach, see Epstein (1999).

[6]There is some empirical evidence that professional delaers hold subadditive (concave) capacities (Fox 'et. al.', 1996)

distributions, even if she has complete ignorance about its location. As a consequence, the ask price of the asset $\beta$ is supposed to be the supremum of the family of mathematical expectation with respect to every probability distribution in $P$, whereas the bid price is supposed to be the infimum of the same family of mathematical expectation. These two asset values crucially depend on ambiguity: the higher the degree of uncertainty (Marinacci, 2000) the longer the interval. Given the relationship between the lower and upper Choquet integral with respect to the subadditive capacity $v$ and the bid and ask price of the asset $\beta$, it is possible to compute the lower and upper bound of the investment opportunity value, by considering the diffusion processes of the bid and ask prices. Therefore, the following holds true:

**Proposition 1** *The empirical measure of quasi-option value under ambiguity (q.o.v.a.) included in an investment project is the maximum value of a development opportunity.*

**Proof.** In Appendix ■

# 4 An example: the q.o.v.a. of an offshore petroleum lease

In the case of a natural resource the value of the underlying asset largely depends on the price and the available quantity of the resource. Consider the case of an offshore petroleum lease.[7] The valuation and exploitation of an off-shore oil tract can be considered as a compound option (exploration, development and extraction), whose value depends on production cost, current oil price, expected changing of oil price, volume of the undeveloped reserve. For simplicity's sake, assume that the company concluded the exploration phase with an estimated oil reserve of 50 million barrels and obtained a production license or concession. At this stage of the investment process, the company has to decide whether to exploit the reserve and start development or to abandon it. The company faces the development stage that involves consequences, i.e. a monetary revenue, to each possible state of nature and cost of installing productive capacity (i.e. constructing rig and drilling production wells). Assume that costs of developing the reserve are fixed and

[7] See Paddock 'et al.' 1988.

occur in one instantaneous lump sum. Relinquishment requirement lasts 20 years and oil is available for sale after one year.

The development decision is represented as a sequential process in which all available choices are either to invest in development at once or to wait for additional information. At the beginning, the company faces ambiguity in oil price. Ambiguity is resolved over time and developing the reserve is not a "now or never"' opportunity but a "now or next period" one. Yet, if development is exercised, the option to invest will be killed and lost forever. In this investment process involving ambiguity and learning, the notion of *q.o.v.a.* emerges and can be determined by considering the bid and ask prices of the oil.

Consider the following *q.o.v.* formula:[8]

$$q.o.v. = S \cdot e^{-yt} N(d_1) - K \cdot e^{-rt} N(d_2) \tag{1}$$

where:

- $S$ = net current value of the underlying asset[9]
- $K$ = strike price of the option (cost of developing the reserve)
- $t$ = time to expiration on the option (relinquishment requirement)
- $r$ = riskless interest rate corresponding to the life of the option
- $\sigma^2$ = variance in the $ln(price)$ of the underlying asset
- $y$ = dividend yield or cost to delay
- $N(d_1), N(d_2)$ = normal standard cumulative distributions[10]
- $d_1 = [\ln(\frac{S}{K}) + (r - y + \frac{\sigma^2}{2})t]/\sigma\sqrt{t}$
- $d_2 = d_1 - \sigma\sqrt{t}$

[8] See, for instance, Damodaran, 2000.

[9] In our example the net current value of the underlying asset is equal to the price minus the average cost of each barrel times the overall amount of the reserve (50 million barrels) discounted by the annual rate derived from the life of the option (20 years).

[10] $N(d_1)$ and $N(d_2)$ are probabilities and represent the likelihood that the option will have a positive cash-flow at the exercise. They are components of the replicating portfolio.

In this example, it is assumed that the spot price of offshore oil[11] is \$43.34, marginal cost per barrel is \$11.50,the the strike price equals \$1600 millions and the riskless rate is 6%. Since cash flows are evenly distributed over 20 years (life of the option) the dividend yield equals 5%.

Taking the ask price of oil[12] we obtain: $\sigma^2 = 0.002012$; $\sigma = 0.04485447$; $d_1 = 0.829115$; $d_2 = 0.62852$; which yield: $N(d_1) = 0.7965$; $N(d_2) = 0.7352$.

Given equation 1 the $q.o.v.$ (in million of dollars) is:

$$q.o.v._1 = \$38.9017 \tag{2}$$

Let us take now the bid price of oil. We have: $\sigma^2 = 0.001880$; $\sigma = 0.043356703$; $d_1 = 0.850943$; $d_2 = 0.657046$, and obtain $N(d_1) = 0.8026$; $N(d_2) = 0.7444$. Equation 1 yields the following:

$$q.o.v._2 = \$38.5486 \tag{3}$$

The $q.o.v.a.$ is the highest $q.o.v.$ of the uncertain decision problem. It corresponds in this example to $q.o.v._1 = \$38.9017$. In other words, even though the present value of developing the reserve is lower than its cost ($1516.19M\$ < 1600M\$$), its development has a positive quasi-option value under uncertainty. This is due to the variance (risk) of future prices and the uncertainty about it, which induces the optimistic dealer to take the highest value of $q.o.v.$, i.e., the $q.o.v.$ evaluated using the ask prices.

# 5 Concluding remarks.

Ambiguity is the prevalent condition in projects of development that involve energy-generating natural resources and this paper suggests an empirical measure of the quasi-option value under ambiguity. Generalizing quasi-option pricing theory under ambiguity and evaluating the underlying asset by the Choquet integral of its expected payments it is possible to obtain two quasi-option values. They are derived by considering the relationship between the decision-maker's priors and conjugate beliefs and the bid and ask prices of the underlying asset. The empirical measure of quasi-option value under ambiguity equals the maximum expected value of the investment opportunity, compatible with the decision-maker's priors. As a consequence,

[11] Reference price: Crude Oil (NYM), 11/30/2004.

[12] Source: MRCI's delayed quotes and bid-asks, available on-line (*http://www.mrci.com*).

the value of an investment project increases when ambiguity increases. Indeed, the lesser the trust in the likelihood of events, the larger the difference between the two investment opportunity values.

The *q.o.v.a* can induce a more conservative policy whenever investment decisions about the exploitation of energy-leading natural resources involve their depletion. It generalizes real option applications and could be taken into account when valuing R&D programs, patents, internet firms and investment decisions characterized by vague information.

# 6 Appendix.

Given normalization and monotonicity with respect to set inclusion, suppose that the capacity $v$ is monotonically sequentially continuous and compatible with a probability $p$, that is, for all $s_1, s_2 \in S$, $p(s_1) \leq p(s_2)$ implies $v(s_1) \leq v(s_2)$,and for all $s \in S$, $s_n \uparrow s$ implies $v(s_n) \uparrow v(s)$ and $s_n \downarrow s$ implies $v(s_n) \downarrow v(s)$. The proof of the Proposition derives from the following two Lemmas:

**Lemma 1** *Under the hypotheses of monotonicity with respect to set inclusion, monotone sequential continuity and compatibility, there exists a unique concave capacity $v$ on $(\Omega, S)$, such that:* $\int_{\Omega} \beta dv = \max\{\int_{\Omega} \beta dp \mid p \in P = core(v)\}$.

**Proof.** See Chateauneuf, 1991, Theorem 3' ■

**Remark 1** *The Choquet integral of $\beta$ with respect to $v$ is equal to the maximum of a family of Lebesgue integrals with respect to the family of probability distributions $P$ on $(\Omega, S)$, such that for all $s_1 \in S$, $v(s_1) \leq p(s_1)$. It corresponds to the ask price of the replicating asset $\beta$.*

**Lemma 2** *Define $v^*(s_i) = v(\Omega) - v(s_i^c)$, where $s_i^c$ is the complement of $s_i$. Call $v^*$ the dual capacity of $v$. It provides a measure of the extent to which the decision-maker believes the negation of $s_i$ is true. If no monotonicity with respect to set inclusion, monotone sequential continuity and compatibility holds, there exists a unique dual capacity $v^*$ on $(\Omega, S)$, such that:* $\int_{\Omega} \beta dv^* = \min\{\int_{\Omega} \beta dp^* \mid p^* \in P = core(v)\}$.

**Proof.** See Schmeidler, 1989, Proposition and Remark 6. ■

**Remark 2** *The Choquet integral of $\beta$ with respect to $v^*$ equals the minimum Lebesgue integral with respect to the family of probability distributions $P$; it reveals the worst expectation of the optimistic decision-maker; it corresponds to the bid, because of the asymmetry of the Choquet Integral: $-\int_\Omega -\beta dv = \int_\Omega \beta dv^*$ (Denneberg, Proposition 5.1 (iii), pag 64).*

**Proof of the Proposition.** Because of Lemma 1, the $q.o.v.$ evaluated with respect to the capacity $v$ is not smaller than the $q.o.v.$ evaluated with respect to an additive probability $p \in core(v)$, where the latter is the $q.o.v.$ without ambiguity (since it is the value of the investment opportunity derived by using option-pricing theory, taking into account stochastic changes in the underlying asset $\beta$ only). On the other hand, Lemma 2 shows that the $q.o.v.$ evaluated taking the bid price is not bigger than the ambiguity-free $q.o.v.$ (since it is the minimum expectations of additive probabilities that are in the core of $v$). Therefore, for optimistic decision makers whenever there is a bid-ask spread, the $q.o.v.$ using the ask price is higher that the $q.o.v.$ using the bid, and it is the highest $q.o.v.$ compatible with $P$. This proves the Proposition: $q.o.v.a.$ is computed with respect to the maximum probability distribution in $P$. ■

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