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Original Article

A novel fractional memristor-based Grassi-Miller map: Hyperchaotic behavior and coexistence of attractors

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ABSTRACT

Referring to fractional memristor-based discrete systems, this paper contributes to the field by presenting a new fourth-dimensional (4D) hyperchaotic memristor-based fractional map. The conceived system, obtained by combining a non-integer order discrete memristor with the Grassi-Miller map, is characterized by some special features, which include the absence of equilibrium point and the coexistence of various chaotic and hypechaotic attractors. Numerical techniques including phase plots, Lyapunov exponents and bifurcation diagrams are used to highlight the complex dynamic behavior of the suggested 4D fractional memristor-based Grassi-Miller map.

1. Introduction

Fractional calculus deals with fractional derivatives (for continuous-time systems) as well as fractional difference operators (for discrete-time systems) [1]. Starting from 1974, interest has been directed towards exploring the potentials of discrete fractional calculus, so that several non-integer order difference operators have been proposed [2,3]. During the last decade, great efforts have been made to explore the chaotic behaviors of fractional discrete models, which are nonlinear maps distinguished by their heightened sensitivity to the initial conditions. For example, in the pioneering references [4,5] chaotic behaviors in fractional logistic maps and their delayed versions were discovered. In [6] the chaotic fractional Hénon map has been presented, while in [7] an example of fractional logistic map has been explored via Julia sets. In [8] the chaotic dynamic of the fractional standard map and the fractional sine map have been studied, meanwhile, in [9], the existence of chaos in a discrete fractional Hénon-Lozi map has been investigated. In [10] the permutation entropy has been exploited to study a non-integer order multi-cavity map, whereas in [11] control laws to stabilize at zero the chaotic dynamic of some fractional maps have been developed. In [12] bifurcations, phase portraits, largest Lyapunov exponent and 0-1 test have been used to investigate the chaotic behavior of a variable-order fractional Tinkerbell map.

Recently, interest has been directed towards fractional maps with some special features. These maps are distinguished by particular dynamic behaviors (including hyperchaos) and/or specific properties related to the location of the fixed points. For instance, in [13] the dynamics of a simple symmetrical map, which includes only five nonlinear terms in its fractional model, has been studied. In [14] a chaotic discrete fractional map with hidden attractor has been analyzed, whereas in [15] the properties of some hidden attractor in a novel noninteger order map have been investigated. In [16] an unprecedented chaotic fractional map, which is distinguished by the coexistence of several types of different attractors, has been presented. In [17] the properties of some non-integer order maps with special features have been discussed, whereas in [18] the chaotic behaviors of fractional maps distinguished by special symmetries in the fixed points have been investigated. Referring to complex dynamic behaviors, in [19] the hyperchaotic dynamics of the fractional double-scroll map has been explored, while in [20] the presence of hyperchaos in the fractional generalized Hénon map has been discovered. In [21] the complex dynamics of the fractional discrete Grassi-Miller model described by the ν-Caputo-like operators have been studied, whereas in [22] particular dynamic behaviors in the

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Grassi-Miller discrete system described by the Caputo-like h-difference operator have been illustrated. In [23] the hyperchaotic fractional discrete Grassi–Miller system is presented. In particular, the authors of reference [23] have investigated the dynamics and implementation of an innovative version of the Grassi-Miller discrete system employing the Grunwald–Letnikov difference operator.

Recently, there has been limited exploration of discrete memristor maps, and their chaotic and hyperchaotic behavior and characteristics have not received as much attention [24–26]. Very recently, some attempts have been made to investigate the existence of chaos in fractional memristor-based discrete systems [27–29]. These non-integer order maps originate from the concept of memristor, which represents a non-linear circuit element relating electric charge and magnetic flux linkage [30]. For example, in [27] a new fractional non-equilibrium point memristor-based map has been presented. In particular, in [27] hidden hyperchaotic attractors have been studied via Lyapunov exponents, phase attractors, bifurcation diagrams and approximation entropy. In [28] the implementation of noval fractional discrete memristor-based chaotic model with hidden attractors has been illustrated. In particular, reference [28] shows that the conceived memristor-based discrete model generates rich dynamical behaviors, such as coexisting hidden dynamic and initial offset boosting. Finally, in [29] some mathematical models of fractional discrete memristors have been illustrated. Moreover, different chaotic behaviors have been found, showing that the complexity of the maps is controlled within the memristor parameters [29]. The research underscores the complex and diverse dynamics of the system, underscoring the importance of fractional components in contributing to the intricate nature and adaptability of memristor-based maps. However, there is a noticeable gap in the existing literature regarding the exploration of hidden chaotic attractors in such maps. This points to an unexplored area in the discrete memristor domain, especially concerning fractional memristors. Exploring the behavior and characteristics of fractional memristors holds valuable insights and potential applications across various domains. Hence, it is imperative to delve deeper into this area through additional investigation and research, aiming to unveil the distinctive characteristics and potential advantages of fractional discrete memristors.

Based on the previous considerations regarding fractional memristor-based discrete systems, this paper aims to provide a contribution to the field by presenting an innovative fourth-dimensional (4D) hyperchaotic memristor-based fractional map. By combining a non-integer order discrete memristor with the Grassi-Miller map, the conceived system is characterized by some special features, which include the coexistence of various chaotic and hypechaotic hidden attractors. Numerical techniques including phase plots, Lyapunov exponents and bifurcation diagrams are used to highlight the complex dynamic behavior of the suggested 4D fractional memristor-based Grassi-Miller map. The paper is divided as follows. In Section 2, besides giving necessary fundamental concepts of fractional calculus, a non-integer order discrete memristor is presented. In Section 3 the equations of the proposed 4D fractional memristor-based Grassi-Miller map are presented, showing that the system has no equilibrium point. In Section 4 coexistences of hyerchaotic attractors as well as coexistences of chaos/hyperchaos phenomena are illustrated. Finally, phase diagrams, bifurcation diagrams and Lyapunov exponents are reported to highlight the complex dynamic behavior of the conceived system.

2. Non-integer order discrete memristor

The memristor is recognized as the fourth fundamental circuit element, bridging the relationship between magnetic flux and charge. The generalized continuous memristor is defined by

$$\begin{cases} i(t) = G(x)\upsilon(t), \\ \frac{dx}{dt} = g(x, \upsilon), \end{cases} \tag{1}$$

where υ is the voltage, i is the current, $G(x)$ denotes a function representing the memristor value, and $g(x,\upsilon)$ is a continuous function. Employing the forward Ruler difference method [31], the continuous memristor in equations (1) can be converted into a discrete memristor. Let $\upsilon(n)$, $i(n)$ and $x(n)$ be the sampling values of $i(t), \upsilon(t)$ and $x(t)$ at the n-th iteration, respectively, and $x(n+1)$ be the sampling value of $x(t)$ at the $(n+1)$-th iteration. Therefore, a discrete memristor can be modeled by

$$\begin{cases} i(n) = G\left(x(n)\right)\upsilon(n), \\ x(n+1) = x(n) + hg\left(x(n), \upsilon(n)\right), \end{cases} \tag{2}$$

where $h = 1$ is the iteration step size. The generalized fractional discrete memristor is defined by incorporating the Caputo-like difference operator into the discrete memristor (2), as follows:

$$\begin{cases} i(n) = G\left(x(n)\right)\upsilon(n), \\ {}^{C}\Delta_a^{\nu}x(t) = g\left(x(t+1-\nu), \upsilon(t+1-\nu)\right), \end{cases} \tag{3}$$

where $t \in \mathbb{N}_{a-\nu+1}$, $\mu \in (0,1]$ is the fractional order and $\mathbb{N}_a = \{a, a+1, a+2, \cdots\}$ $a \in \mathbb{R}$. This paper introduces a new type of fractional-order discrete memristor (FODM), which is presented as follows:

$$\begin{cases} i(n) = \left(a_1 + a_2 tanh\left(x_n\right)\right)\upsilon(n), \\ {}^{C}\Delta_a^{\nu}x(t) = \upsilon(t+1-\nu). \end{cases} \tag{4}$$

Using the ν-fractional sum, the equivalent discrete integral form of the FODM is expressed as:

$$\begin{cases} i(n) = \left(a_1 + a_2 tanh\left(x_n\right)\right)\upsilon(n), \\ x(n) = x(0) + \frac{1}{\Gamma(\nu)}\sum_{j=0}^{n-1}\frac{\Gamma(n-j+\nu)}{\Gamma(n-j+1)}\upsilon(j). \end{cases} \tag{5}$$

The distinctive features of the introduced mathematical model for memristors, such as amplitude-dependent and frequency-dependent pinched hysteresis loop are verified using numerical simulations. Pinched hysteresis loop is considered the distinctive signature for recognizing memristors [32]. In order to verify this notable feature of the proposed discrete memristors, we take a discrete sinusoidal voltage $\upsilon(n) = Asin(fn)$ with amplitude A and frequency f and system parameters $a_1 = 2, a_2 = 2$, and order $\nu = 0.1$. When the amplitude $A = 1$ is held and f is set to $f = 0.5$, 1, 10. The $\upsilon - i$ plots are depicted in Fig. 1(a). This figure vividly depicts 8-shaped pinched hysteresis loops with different input frequencies. As the excited frequencies increase from 0.5 to 10 the pinches hysteresis loop gradually decreases and shrinks into a single valued function. Furthermore, as the frequencies of the applied voltage approach infinity, the pinched hysteresis loops of the memristor will contract into a singlevalued function. While when the frequency is fixed to $f = 0.5$ and A is set to 0.5, 1 and 1.5, the corresponding $\upsilon - i$ plots are depicted in Fig. 1(b). This explains that the pinched hysteresis is achieved irrespective of the stimulus amplitude. These numerical results presented in Figs. 1(a) and 1(b) perfectly indicate that the discrete memristors can emulate the behavior of memristors in the continuous time domain.

To investigate the effect of fractional order on the features of the discrete memristor, we can set parameters $a_2 = 0.08$, $a_1 = 0.3$, with amplitude $A = 0.5$ and frequency $f = 0.5$ with different fractional order values $\nu = 0.1, 0.4, 1$. It is observed that the discrete memristors showed pinched hysteresis loop under various values of fractional order, as depicted from Fig. 2. Altering the fractional order induces variations in the local active properties, leading to complex dynamical behaviors when the discrete memristors are integrated into a discrete map [33].

3. 4D fractional memristor-based Grassi-Miller map

Grassi and Miller presented a 3-dimensional map, displaying hyperchaotic attractors. The state equation can be presented as

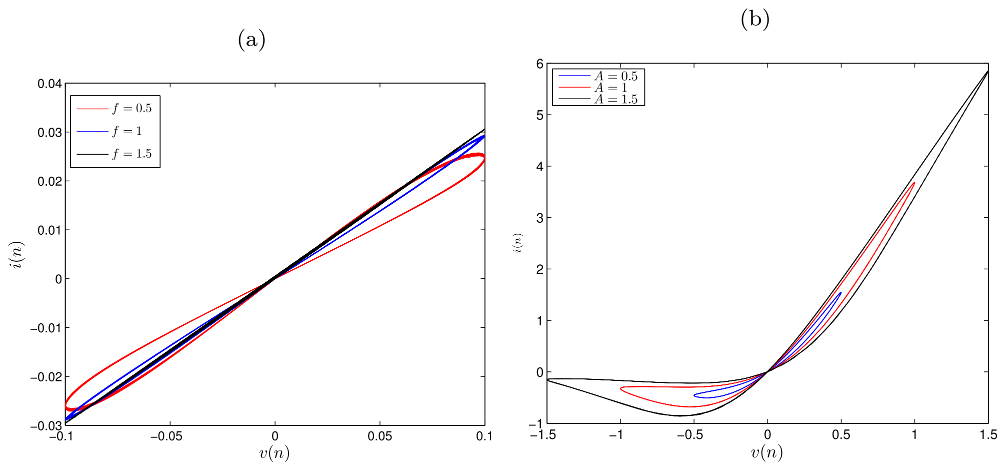

Fig. 1. (*a*) Frequency-dependent pinched hysteresis loops of the discrete-memristor with $A = 1$. (*b*) Amplitude-dependent pinched hysteresis loops of the discrete-memristor with $f = 0.5$.

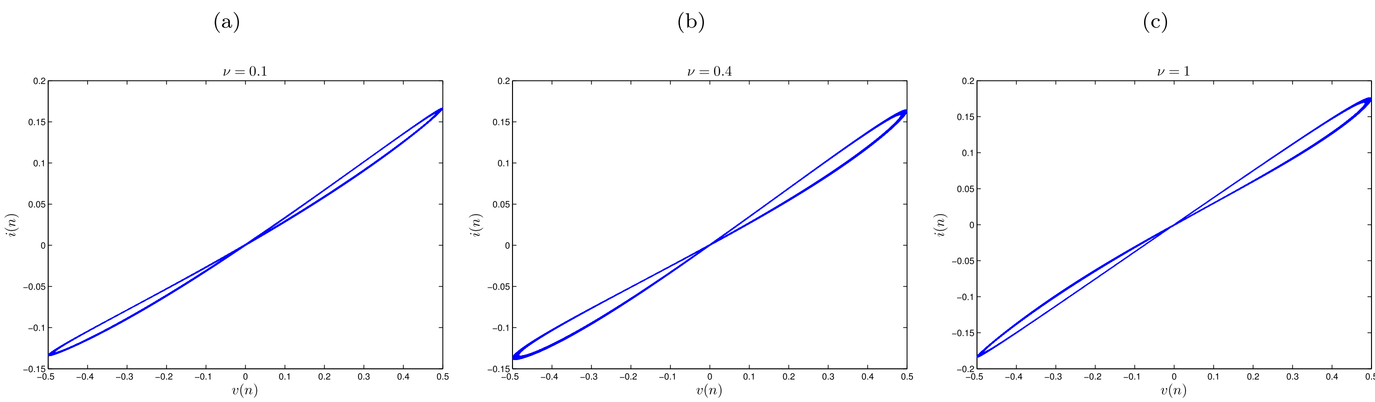

Fig. 2. Frequency-dependent pinched hysteresis loops of the discrete-memristor with $A = 0.5$ and $f = 0.5$ for three order values.

$$\begin{cases} x(i) = \alpha - y(i-1)^2 - \beta z(i-1), \\ y(i) = x(i-1), \\ z(i) = y(i-1), \end{cases} \tag{6}$$

where $\alpha \neq 0$ and β are bifurcation parameters, and x, y, z are three state variables of map (6).

In this part, employing the relationship mentioned in equation (5) a 4D fractional discrete memristor based on the Grassi-Miller is established as follows:

$$\begin{cases} x(i) = \alpha - y^2(i-1) - \beta z(i-1), \\ y(i) = x(i-1), \\ z(i) = y(i-1) + (a_1 + a_2 tanh\,(w(i-1))\, z(i-1), \\ w(n) = w(0) + \frac{L}{\Gamma(\nu)} \sum_{j=0}^{n-1} \frac{\Gamma(n-j+\nu)}{\Gamma(n-j+1)} z(j). \end{cases} \tag{7}$$

As it is known, the stability of a map can be measured by its fixed points. The fixed points of system (7) can be obtained from the following equation:

$$\begin{cases} x = \alpha - y^2 - \beta z, \\ y = x, \\ z = y + \left(a_1 + a_2 tanh\,(w)\right) z, \\ 0 = z. \end{cases} \tag{8}$$

Through equation (8) we get $x = y = z = 0$, whereas when we substitute it into the first equation $x = \alpha - y^2 - \beta z$, this equation does not have a solution, indicating that the fractional-order memristor-based Grassi–miller map has no fixed point. Because of the no fixed point, any chaotic attractors in the map are certainly hidden. Consider the parameters system $\alpha = 1.1$, $\beta = 0.1$ and $a_1 = 0.3$, $a_2 = -0.45$, fractional order $\nu = 0.2$ and we assign the initial conditions as $(1, 0.1, 0.2, 0)$. We used MATLAB to perform numerical simulations of the system and various useful results were obtained, which are displayed in Fig. 3. The results unveil chaotic behavior characterized by a hidden attractor. The finite-time Lyapunov exponent, calculated using Wolf algorithm, are determined as $\lambda_1 = 0.1995$, $\lambda_2 = 0.1560$, $\lambda_3 = -1.5662$, and $\lambda_4 = -2.4141$, respectively.

4. Coexistences of chaotic and hyperchaotic hidden attractors

The coexisting bifurcation is a bifurcation phenomena that depends on initial conditions. The given map can generate diverse types of attractors depending on the initial condition of the memristor w_0. By configuring the control parameters as $\alpha = 1.7$, $\beta = 0.1$, $a_2 = -0.45$, $a_1 \in [0.05, 1.3]$ and $\nu = 0.1$ with the initial states are assigned as $x_0 = 1, y_0 = 0.1, z_0 = 0.2$, the coexistence of attractor of the model is analyzed. As shown in Fig. 4, the coexisting bifurcations of the state $z(n)$ and the Lyapunov exponents under $w_0 = 40$ (red diagram) and $w_0 = -40$ (blue diagram). Complex dynamical phenomena, including period window, chaos and hyperchaos can be observed in Fig. 4. For $w_0 = 40$, when the bifurcation parameter a_1 is increased in $[0.05, 1.3]$, observing the bifurcation diagrams depicted in Fig. 4 (*a*), it is evident that the discrete model (7) starts from hyperchaos and goes into chaos via tangent bifurcation route with some narrow period windows. We notice that the system exhibits hyperchaotic dynamics for values of a_1 less than or equal to 1.225, and beyond that value $a_1 \geq 1.225$, it transitions into chaotic behavior with some periodic windows. For the initial

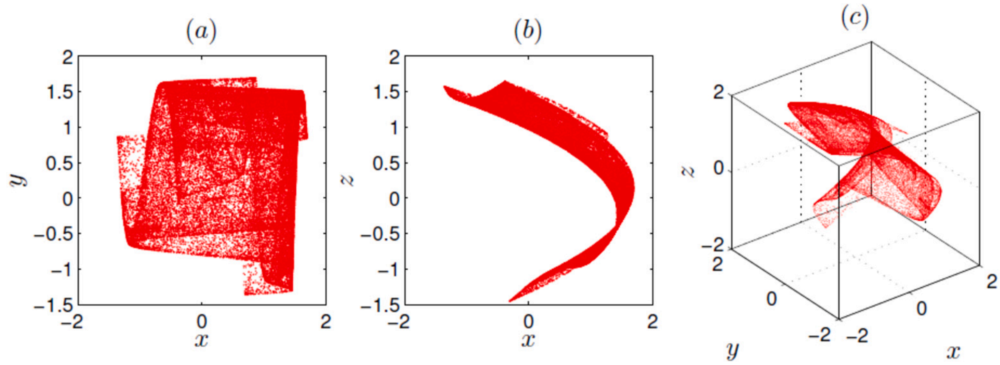

Fig. 3. Phase diagrams in different projections of hyperchaotic hidden attractors in system (7) for parameters $\alpha = 1.7$, $\beta = 0.1$, $a_1 = 0.3$, $a_2 = -0.45$, and fractional order $\nu = 0.2$: (a) in the x-y plane, (b) in the x-z, (c) in the 3D plane.

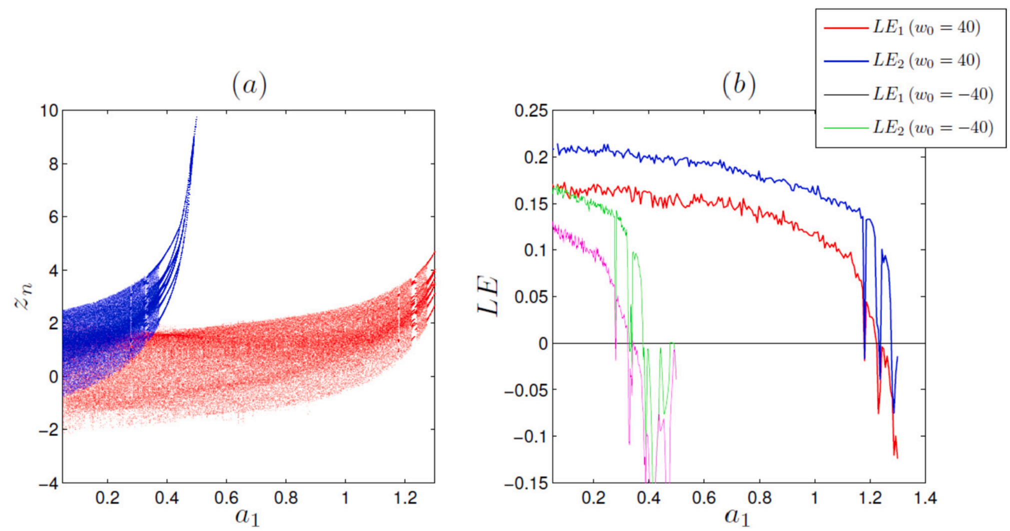

Fig. 4. Bifurcation diagrams and LEs of the FODMS (7) with infinite fixed point for fractional order value $\nu = 0.1$: (a) for $a1 \in [0.05, 1.3]$, $\alpha = 1.7$, $\beta = 0.1$, $a_1 = 0.3$, $a_2 = -0.45$, and initial condition $(1, 0.1, 0.2, 40)$ (red diagram) and $(1, 0.1, 0.2, -40)$ (blue diagram).

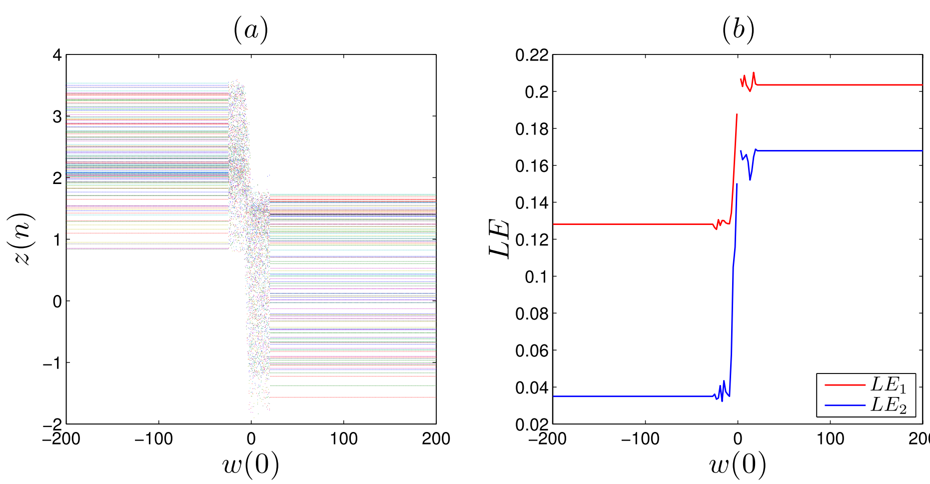

Fig. 5. Bifurcations and LEs of system (7) with infinite fixed point versus memristor initial condition w_0 for fractional order value $\nu = 0.1$ and system parameter $\alpha = 1.7$, $\beta = 0.1$, $a_1 = 0.3$, $a_2 = -0.45$.

condition $w_0 = -40$ and variation of the system parameter a_1 within the interval $[0.05, 0.5]$, the bifurcation diagrams of the state z_n are numerically simulated in blue diagram. The Lyapunov exponents reveals that the model (7) is in hyperchaotic state at the beginning with periodic window at 0.2807 and $[0.326, 0.3417[$, and then goes into chaotic state via chaos crisis. The period orbits evolves from chaos in the interval $]0.3779, 0.5]$.

Moreover, we plot the bifurcation diagrams of the fractional discrete memristive-based Grassi-Miller model varying with the intial value of memristor, respectively as shown in Fig. 5. During this period, a preliminary assessment indicates the presence of coexisting attractors in the

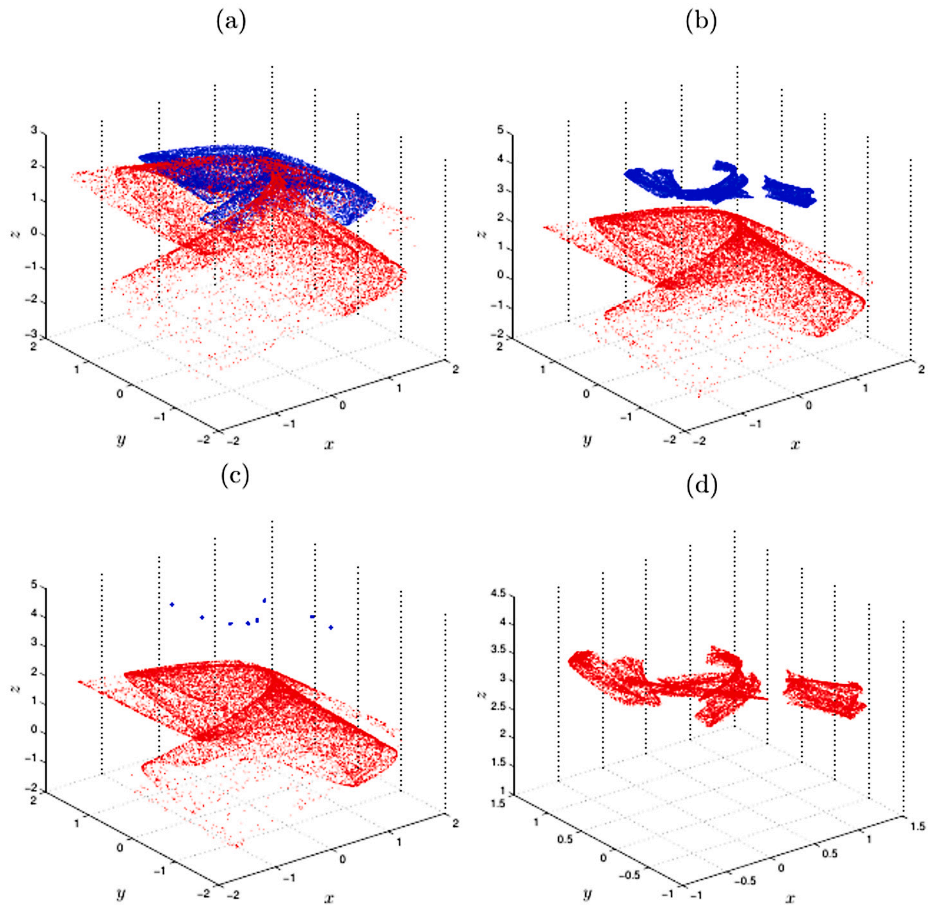

Fig. 6. Several attractors of system (7) with infinite fixed points for the symmetrical IV $(x_0, y_0, z_0, w_0) = (1, 0.1, 0.2, \pm 40)$, fractional value $\nu = 0.1$, positive parameters system in red color, and negative parameters system in blue color: (a) Coexistence of hyperchaotic attractors for $a_1 = 0.1$, (b) Coexistence of chaotic and hyperchaotic attractors for $a_1 = 0.35$, (c) Coexistence of periodic and chaotic attractor $a_1 = 0.4$. (d) Chaotic attractor for $a_1 = 1.25$.

model. To further evaluate the state of coexisting attractor, several coexisting attractors was analyzed with the varying of parameter a_1, as shown in Fig. 6. The attractors from initial condition (IC) $w_0 = 40$ are drawn in red and the attractors drawn in blue are from the IC $w_0 = -40$. The fractional discrete memristive-based Grassi-Miller map displays coexisting hyperchaotic attractors for system parameter $a_1 = 0.1$ as shown in Fig. 6(a). At the same time when $a_1 = 0.35$, it can be seen that there is coexistence hyperchaotic (for IC $w_0 = 40$) and chaotic attractor for (IC $w_0 = -40$). Furthermore, when $a_1 = 0.4$ system (7) coexist chaotic and periodic attractor; however when $a_1 = 1.25$ the fractional discrete memristive-based Grassi-Miller map chaotic attractor as shown in Fig. 6(d).

Fig. 7 shows the phase portraits of the coexisting hyperchaotic attractors for different values of w_0. The coexisting hyperchaotic attractors were calculated by using the initial conditions $w_0 = 38$, $w_0 = 39$, $w_0 = 40$, $w_0 = 41$, $w_0 = 42$, respectively. From Fig. 7, these hyperchaotic attractor have a similar structure in the $z - w$ plane.

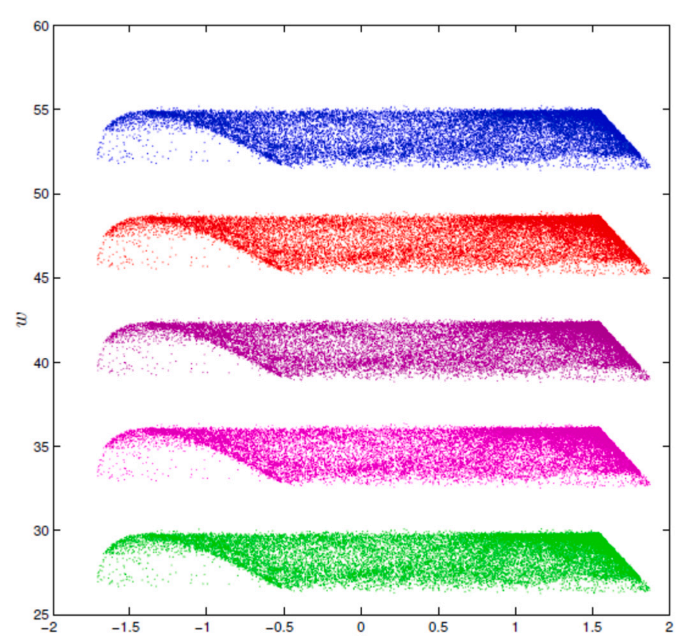

Fig. 7. Coexisting multiple hyperchaotic attractor for different memristor initial value $w_0 = 38$ (blue), $w_0 = 39$ (red), $w_0 = 40$ (Purple), $w_0 = 41$ (Magenta), $w_0 = 42$ (green).

5. Conclusion

By including Grassi as a coauthor, the manuscript has introduced a novel 4D fractional memristor-based Grassi-Miller map. The authors have shown that the conceived system (derived by combining a non-integer order discrete memristor with the Grassi–Miller map) is characterized by some special features. Namely, the proposed memristor-based map features both no equilibria and the coexistence of various chaotic/hypechaotic attractors. Numerical techniques including phase plots, Lyapunov exponents and bifurcation diagrams have been exploited to highlight the complex dynamic behavior of the suggested 4D fractional memristor-based Grassi-Miller map. Finally, as a future work, the plan is to implement in hardware the proposed memristive map, based on the expertise that some of the co-authors of the present manuscript have already achieved in implementing a previous version of the fractional discrete Grassi–Miller model [23].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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