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ORIGINAL RESEARCH

Multilevel modeling for investigating the probability of digital innovation in museums

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Abstract
Museums represent a fundamental asset for the Italian cultural and social background, and the use of digital technologies can be considered as a keystone for their attractiveness. Thus, assessing the specific determinants which stimulate to invest in new digital solutions and to provide a competitive museum offer is of crucial interest. For this reason, a performing multilevel approach for modeling the probability of including digital innovations in museums will be discussed and different modeling options will be compared. In particular, the implementation of a multilevel binary logit model will be useful to detect the factors of adopting at least basic digital tools. Then, the development of an innovative and flexible multilevel multinomial ordered model will be suitable to further investigate on the probability for the museums to move towards medium/low or high levels of digitalization, on the basis of an increasing sorting criterion. This will be realized by considering the variation of such probability both at regional and provincial levels for some key specific museums features, as well as by including some regional/provincial contextual factors.

Keywords Multilevel ordered logit model · Partial proportional odds ratios · Museum digitalization determinants

1 Introduction

In the last years, the digital revolution has led to a radical change that has also involved museums, starting from the introduction of websites to the use of advanced digital tech-

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nologies to enhance the visitors experience (Hooper-Greenhill, 1999; Romanelli, 2018). As recommended by UNESCO in 2015 (UNESCO, 2015), information and communication technologies play a key role for museums in terms of preservation, study, creation and transmission of heritage and related knowledge. Moreover, UNESCO Member States are encouraged to support museums in sharing and disseminating knowledge, using such technologies whenever necessary. In Italy, since 2006 the Italian Ministry of Culture has promoted the first web channel where information about a large part of the museum heritage is available. A digital Italian platform for Cultural Heritage (named *CulturaItalia*) was created in 2008, with the aim of gathering and organizing in a unique portal all the information on the cultural resources (museums, photographs, libraries, archives, galleries, exhibitions, monuments, films, records, etc.).

After the experience of *CulturaItalia*, the Italian Ministry of Culture has launched many other initiatives with a strong acceleration in the last few years, thanks both to public contributions and to the interest of private institutions, devoting their energies in the developing of an avant-garde Italian cultural system, compatible with the international dynamics. Finally, the latest action is the Three-Year Plan for the Digitalization and Innovation of Museums, approved by the Italian Ministry of Culture in July 2019 with the goal to provide a useful tool to support the digitalization process of museums.

Another active institution in the field of digital research is the Observatory for Digital Innovation in Heritage and Culture, which studies the impact of digital innovation on cultural heritage.

Today, the acceleration towards museums' digitalization is also caused by the impact of the pandemic crisis, which has highlighted the need to improve online access to cultural heritage. In any case, the digitalization in museums and its determinants are still little explored in the literature. Among the few works on this subject it is interesting to mention a contribution (Raimo et al., 2021) which applied a mixed methodological approach based on the descriptive comparison of three different museum organizations operating in the Apulian region. A few years earlier, another work (Lazzeretti & Sartori, 2016) investigated the adoption of information technologies and innovation processes in the Uffizi Gallery in Florence, a prominent art museum in the region of Tuscany, and their interaction dynamics between curators and technology developers, according to the Virtual Value Chain Model.

On the other hand, to our knowledge, no work in this field of application has ever provided a thorough study at a national level on the factors that incentivize the introduction of digital technologies. For this reason, an advanced multilevel approach has been proposed and different modeling options have been discussed. In particular, a multilevel binary logit model aimed at modeling the probability for the museums of using at least basic digital technologies has been fitted and the factors which influence the digital innovation process have been analysed. Then, a multilevel multinomial ordered logit model has been defined for measuring the probability for the museums of exploiting different degrees of digitalization (ranging from no, medium/low to high digitalization). Therefore, after a first step where the key dynamics which affect the digital innovation process in museums have been explored by fitting a multilevel binary logit model, in the second step selected covariates have been employed in the multinomial multilevel ordered model for estimating their effects on the probability of registering ever-increasing levels of digitalization for museums. The transition from the multilevel binary logit model to the multinomial ordered model allows to evaluate the propensity to invest in different levels of digital innovations, classified according to an increasing sorting criterion. For this reason, a polychotomous dependent variable associated to three ordered categories (no, medium/low, high level of digitalization), has been used instead of a binary logit variable. In particular, a multinomial ordered model for this polychotomous variable has been defined by means of two sub-equations based on cumulative probabilities (i.e. cumulative logits or log odds), conditioned by a set of covariates, added in the model as separate or common effects (Rasbash et al., 2009).

After a short theoretical discussion on the multilevel modelling (Sect. 2), a description of the ISTAT microdata concerning the Italian survey on the museums and cultural institutions during 2018 has been proposed (Sect. 3). The probability for the museums to include digital innovations has been estimated through a multilevel binary logistic regression model (Sect. 4) able to reveal the positive or negative impact generated by some internal or contextual variables on the choice of digitalization. Then, the interest in the evaluation of different levels of digitalization has led towards the implementation of a multilevel multinomial ordered logit model (Sect. 5). The introduction of this new multinomial model has been justified by the opportunity of classifying the level of digitalization for museums, by resorting to an ordinal scale (criterion), such as "no, medium/low, high" grade of digitalization. More specifically, the expected probabilities of being without digital innovations, as well as having a medium/low or high grade of digitalization, have been measured, with respect to the regions and provinces where the museums are placed. The empirical evidence highlights the pattern of variables which encourage to invest in increasingly performing digital innovations.

2 Theoretical framework on multilevel models

The multilevel approach is often recalled in Statistics for the study of hierarchical data structure characterized by complex patterns of variability (Goldstein, 2011; Scott et al., 2013; Snijders & Bosker, 2012). This structure organizes the cases into known clusters and a set of explanatory variables (covariates) associated with each group level. The multilevel models can be considered as a natural extension of classical linear models or generalized linear models. Nevertheless, unlike traditional regression models, covariates in multilevel models can be picked for each cluster with the assessment of variability at the selected levels of aggregation, in order to measure the cluster effects on the outcome variable. In the literature, multilevel regression models are also called as *Variance components models*, *Hierarchical linear models* and *Random coefficient models* (De Leeuw & Kreft Ita, 1986; Longford, 1993; Grilli & Rampichini, 2015). Indeed, over the past years multilevel regression models have been the object of many papers and books (Snijders & Bosker, 2012; Goldstein, 2011; Raudenbush & Bryk, 2002; Reise & Duan, 2003; De Iaco & Maggio, 2021; De Iaco et al., 2019).

In the following, the multilevel binary and multinomial ordered logistic regression models are presented.

2.1 Multilevel binary logit model

Let $Y_{ijk} \sim Ber(\pi_{ijk})$ be the binary response variable which takes values 0/1 (response categories), with the index i ($i = 1, \ldots, n_{jk}$) representing the level 1 unit, the index j ($j = 1, \ldots, N_k$) corresponding to the level 2 unit and the index k ($k = 1, \ldots, K$) indicating the level 3 unit.

Given the set of covariates $\{X_{1\cdot}, X_{2\cdot}, \ldots, X_{H\cdot}\}$, which influence the dependent response variable $Y_{ijk} \sim Ber(\pi_{ijk})$, the three-level logit model is defined as follows:

$$\eta_{ijk} = \beta_{0jk} + \sum_{h=1}^{H} \beta_{hjk}\, x_{hijk}, \tag{1}$$

where link between the mean π_{ijk} and the linear predictor η_{ijk} is given by the $logit$ function (well-known as link function),

$$\eta_{ijk} = \log it(\pi_{ijk}) = \ln \frac{\pi_{ijk}}{1 - \pi_{ijk}} \quad \text{and} \quad \pi_{ijk} = \frac{\exp\{\eta_{ijk}\}}{1 + \exp\{\eta_{ijk}\}}, \tag{2}$$

with

- $\beta_{hjk} = \beta_h + \nu_{hk} + u_{hjk}, \qquad h = 0, 1, \ldots, H,$
- $\begin{bmatrix} \nu_{0k} \\ \nu_{1k} \\ \vdots \\ \nu_{hk} \\ \vdots \\ \nu_{Hk} \end{bmatrix} \sim N\Big(\mathbf{0}, \boldsymbol{\Omega}_\nu\Big), \quad \boldsymbol{\Omega}_\nu = \begin{bmatrix} \sigma^2_{\nu 0} & & & & & \\ \sigma_{\nu 01} & \sigma^2_{\nu 1} & & & & \\ \vdots & \vdots & \ddots & & & \\ \sigma_{\nu 0h} & \sigma_{\nu 1h} & \ldots & \sigma^2_{\nu h} & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ \sigma_{\nu 0H} & \sigma_{\nu 1H} & \ldots & \sigma_{\nu hH} & \ldots & \sigma^2_{\nu H} \end{bmatrix},$
- $\begin{bmatrix} u_{0jk} \\ u_{1jk} \\ \vdots \\ u_{hjk} \\ \vdots \\ u_{Hjk} \end{bmatrix} \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_u\right), \quad \boldsymbol{\Omega}_u = \begin{bmatrix} \sigma^2_{u0} & & & & & \\ \sigma_{u01} & \sigma^2_{u1} & & & & \\ \vdots & \vdots & \ddots & & & \\ \sigma_{u0h} & \sigma_{u1h} & \ldots & \sigma^2_{uh} & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ \sigma_{u0H} & \sigma_{u1H} & \ldots & \sigma_{uhH} & \ldots & \sigma^2_{uH} \end{bmatrix}.$

Note that the vectors and matrices associated to the parameters ν_{hk} and u_{hjk}, specify the variation across the 3rd level and the 2nd level.

Thus, the model in (1) allows the intercept and the slopes to vary across both the 2nd level and the 3rd level.

The parameters of such a model can be estimated through the marginal maximum likelihood estimation, where the marginal likelihood of the observed data, obtained by integrating out the distribution of the random effects, is maximized. It is worth pointing out that the estimated regression coefficients stand for changes in the log-odds (logits) and their exponentials correspond to the odds ratios (ORs) which can take any value from 0 to infinity. The ORs point out multiplicative effects rather than additive effects. Since their interpretation is less friendly than probabilities, many authors explain and deepen the models' results in terms of estimated probabilities (Skrondal & Rabe-Hesketh, 2009), which can be computed by replacing the parameters with the estimates obtained from the fitted model and from the estimated group effects of the model.

The model (1), structured in three levels, can be easily extended to many more levels.

2.2 Multinomial ordered logit model

The multinomial ordered logistic model is a cumulative regression model that links an ordinal variable of multiple categories (i.e. polychotomous outcome variable consisting of ordered categories) to a set of covariates (Grilli & Rampichini, 2012). In the literature, the early works regarding multilevel regression models for ordinal data are introduced by McKelvey and Zavoina (1975); McCullagh and Nelder (1989) and Winship and Mare (1984). Furthermore, reviews about ordered logistic regression models are provided by Agresti and Natarajan (2001), Hedeker (2008), Agresti (2010), as well as by Fullerton (2009) among others.

The most common multilevel ordinal logistic regression model is based on the assumption of proportional odds, according to which the effects of the covariates included in the model are the same across all the categories of the ordinal dependent variable and only the intercept changes for each level. As underlined by Peterson and Harrell (1990), an extension of this model is represented by the partial proportional odds model, where this proportionality hypothesis can be relaxed by allowing non-proportional odds for a set of regressors (predictors). In other terms, in case of violation of this proportional odds assumption, covariates can produce "differential effects" on the cumulative logits. A Wald test can be applied in order to test the proportional odds assumption (Hedeker, 2008; Fullerton & Xu, 2012; Peterson & Harrell, 1990).

In the case of rejection of the proportional odds hypothesis, the partial proportion odds model is applied, allowing separate effects for some variables without no potential loss in accuracy of prediction.

Let $Y_{ijk}^{(s)}$ be a multinomial ordered response variable which takes values $s = 1, 2, ..., t$ (t response categories), where t is chosen as the reference category and with the index i ($i = 1, \ldots, n_{jk}$) representing the level 1 unit, the index j ($j = 1, \ldots, N_k$) corresponding to the level 2 unit and the index k ($k = 1, \ldots, K$) indicating the level 3 unit.

Given the probability $\pi_{ijk}^{(s)}$ that the i-th first level unit presents a response variable value equal to s (i.e. the probability of being in category s, with $s = 1, \ldots, t$), let $\{X_{1\cdot}, X_{2\cdot}, \ldots, X_{H\cdot}\}$ be a set of covariates which influences the dependent response variable.

In order to take into account the ordering, the selected model is based on the cumulative response probabilities defined as follows:

$$\psi_{ijk}^{(s)} = \sum_{h=1}^{s} \pi_{ijk}^{(h)}, \qquad s = 1, 2, \ldots, t-1.$$

Thus, the category probabilities can be expressed in terms of the cumulative probabilities, as specified below:

$$\pi_{ijk}^{(h)} = \psi_{ijk}^{(h)} - \psi_{ijk}^{(h-1)}, \qquad 1 < h < t,$$
$$\pi_{ijk}^{(1)} = \psi_{ijk}^{(1)}, \qquad \psi_{ijk}^{(t)} = 1.$$

Then, a multinomial ordered model is constructed as follows:

$$\psi_{ijk}^{(s)} = \left\{1 + \exp\left[-\left(\beta_{0jk}^{(s)} + \sum_{h=1}^{H} \beta_{hjk}^{(s)} x_{hijk}\right)\right]\right\}^{-1}, \qquad s = 1, 2, \ldots, t-1,$$

or analogously:

$$\eta_{ijk}^{(s)} = \log it(\psi_{ijk}^{(s)}) = \beta_{0jk}^{(s)} + \sum_{h=1}^{H} \beta_{hjk}^{(s)} x_{hijk}, \qquad s = 1, 2, \ldots, t-1,$$

with

- $\beta_{hjk}^{(s)} = \beta_h^{(s)} + \nu_{hk}^{(s)} + u_{hjk}^{(s)}, \qquad h = 0, 1, \ldots, H, \qquad s = 1, 2, \ldots, t-1,$
- $\begin{bmatrix} \nu_{0k}^{(1)} \\ \nu_{0k}^{(2)} \\ \vdots \\ \nu_{hk}^{(s)} \\ \vdots \\ \nu_{Hk}^{(t-1)} \end{bmatrix} \sim N(\mathbf{0}, \mathbf{\Omega}_\nu),$ • $\begin{bmatrix} u_{0jk}^{(1)} \\ u_{0jk}^{(2)} \\ \vdots \\ u_{hjk}^{(s)} \\ \vdots \\ u_{Hjk}^{(t-1)} \end{bmatrix} \sim N(\mathbf{0}, \mathbf{\Omega}_u).$

This implies that increasing values of the linear components are associated with increasing probabilities as s increases (Rasbash et al., 2009). Note that all of the covariate effects $\beta_{hjk}^{(s)}$ vary across categories ($s = 1, 2, \ldots, t-1$); similarly for the random-effect variance term (Hedeker, 2008).

Remark

The multilevel multinomial ordered models give a flexible method of analysing ordinal response variables (Peterson & Harrell, 1990), since the explanatory variables may have either proportional odds (with one parameter for each covariate), unconstrained non-proportional odds (with $t-1$ parameters for each covariate) or constrained non-proportional odds (with a trend in log-odds ratios, where the odds parameter can increase or decrease in a monotonic way through the cut-points of the ordinal values). In particular, as previously mentioned, the multilevel multinomial logit proportional odds model is characterized by the restrictive proportionality assumption which can be often released. Indeed, researchers may choose either a *partial* model by releasing the proportionality hypothesis only for a subset of variables or the *generalized* model where the assumption is violated for every independent variable (Fullerton & Xu, 2012).

In this paper, a partial proportional odds model will be fitted, in which the hypothesis of the proportional odds is partially violated only for some explanatory variables.

3 Data and modeling setup for museum digitalization analysis

The microdata used in this paper were collected during an Italian National Institute of Statistics (ISTAT) census survey conducted in 2018 on the public and private museums located in Italy. In particular, this census survey is carried out yearly by ISTAT on the basis of a specific protocol signed in 2017 with the Ministry for Cultural Heritage and Tourism. It offers an updated and detailed description of all the museums and other similar institutions located in Italy, i.e. all those non-profit permanent structures (public or private, state or non-state) open to the public. In the ISTAT questionnaire, the characteristics of each museum, classified with respect to different categories, such as management, access and visits, type and stock of items, staff, financial resources, structures, support of fruition, activities and services, relationship with the territory, were investigated. The population size of italian museums was equal to 4,908 units (museums and similar institutions), of which 3,882 museums, galleries or collections (on which the study is concerned), 327 archaeological areas and parks and 630 monuments and monumental complexes as well as 69 ecomuseums. After excluding missing values, the data set includes 3,217 museums, galleries or collection (2,043 public and 1,174 private). Museum supply is widespread all over the country with one municipality out of three having at least one museum (ISTAT, 2018). In particular, 48% of the total is located in the Northern regions, 28% in the Central regions and the remaining 24% in the Southern regions including islands.

Despite the progressive diffusion of digital technologies in the museum system, some specific innovations appear to be limited, such as the use of a complete digital catalogue. Indeed, in Italy the 15% of the structures, concerning museums, galleries or collection, have a complete digital scientific catalogue of its assets. However, the use of interactive technologies and digital tools that allow to enrich the visit experience and public engagement is spreading: almost half of the surveyed structures (44.7%) makes available at least one device including smartphone, tablet, touch screen, visiting aids such as video and/or multimedia rooms, QR Code technology and augmented reality paths. Online communication involves an ever-increasing number of structures: in fact, half of the institutions have a dedicated website and 53.4% an account on the most important social media (such as Facebook, Twitter, Instagram, etc.). In the period 2015–2018 both the number of structures which offer the possibility to buy tickets online (from 6.6 in 2015 to 14% in 2018) and the number of museums offering free Wi-Fi access (from 18.6 in 2015 to 25.1% in 2018) have increased. The 38.4% of museums publishes links to digital maps and/or to the GPS coordinates useful for the geographical location of the structure and the 9.9% of museums offers the possibility to virtually visit the exhibitions.

In this context, a two-step procedure is proposed by applying, first of all, a multilevel binary logit model for measuring the probability of including digital innovations in museums management and fruition (first step). Then, a multilevel multinomial ordered model suitable to estimate the probability for the museum of being without significant digital innovations, or adopting medium/low or high level of digitalization, is also implemented (second step).

In particular, three hierarchical levels have been considered:

- *the first level*, that is the museums (in total 3,217 museums);
- *the second level*, corresponding to the Italian provinces where the museums are located (108 provinces),
- *the third level*, referred to the Italian regions where the museums are placed (20 regions).

The three levels of aggregation are justified by the inherent hierarchical structure of data, where the regions represent the highest level in which the cultural heritage can offer different opportunities; on the other hand, the museums are considered as the lowest level of nesting. A thorough descriptive analysis on ISTAT microdata has been performed on the characteristics of museums, in terms of management, access, visits, staff, financial resources, structures, support of fruition, activities and services, relationship with the territory. The multinomial structure of the model enables the discernment of the different effects that regional and provincial characteristics might cause on the level of digitalization, that is a dependent variable, which can be naturally ordered into three categories (no, medium/low, high digitalization). More specifically, the level of digitalization has been recoded and then derived by taking into account the presence of at most 15 significant digital innovations, such as a digital inventory, a digital catalogue, video and audio-guides, applications dedicated for smartphones and tablets, interactive installations and/or virtual reconstructions (touch screen tables, videos), QR Code and/or proximity devices (Bluetooth, Wi-Fi, etc.), tablets available for the visitors, immersion video/multimedia rooms, free Wi-Fi connection, dedicated website, online ticketing service, digital catalogue accessible online, virtual tours online, social media accounts (Facebook, Twitter, Instagram, Pinterest, Foursquare, etc.), link to digital maps and/or to GPS coordinates for the location of the museum.

Table 1 Covariates selected for the study

Questionnaire variables	Questionnaire modality
Network of museums	“0”=no network “1”= presence of network
Access	“0”=with admission fee “1”=absolutely free
Research activities	“0”=no research activities “1”= presence of research activities
Partnership	“0”=no partnership “1”= presence of partnership
Guided tours	“0”=no guided tours “1”= presence of guided tours
Exhibition space	“0”=not greater than 201 square meters “1”=between 201 and 500 square meters “2”= more than 500 square meters
Type of institution	“0”=public “1”=private
Total number of visitors	“0”=lower than 700 “1”=between 700 and 3,000 “2”=between 3,002 and 10,000 “3”= more than 10,000
Presence of systematic or occasional satisfaction campaigns in the last five years	“0”=no satisfaction campaigns “1”=systematic satisfaction campaigns “2”=occasional satisfaction campaigns “3”=systematic and occasional satisfaction campaigns
Percentage of Italians vs Foreigners	“0”=lower than 50% “1”=greater than or equal to 50%
Provincial-level covariates	
Number of tourist accommodation establishments	“0”=lower than 1,000 “1”=greater than or equal to 1,000
Number of bedrooms in the tourist accommodation establishments	“0”=lower than 7,000 “1”=greater than or equal to 7,000
Regional-level covariates	
Expenditure for recreation, culture and religion	“0”=lower than 470 euro “1”=greater than or equal to 470 euro
Gross domestic product per capita, current prices	“0”=lower than 0.028 “1”=greater than or equal to 0.028
Motorway density (km per 100 km^2)	“0”=up to 0.9 “1”=between 1 and 2 “2”=greater than 2
Number of airports	“0”=no airports “1”=between 1 and 3 “2”=greater than 3
Number of ports	“0”=no ports “1”=greater than or equal to 1

From the exploratory data analysis, the covariates shown in Table 1 have been selected and recoded for computational purposes.
Starting from a full model which includes, as covariates, the variables in Table 1, the backward deletion procedure has been used in order to select the right pattern of covariates. At the end of this process, some covariates (such as the total number of visitors, the presence of systematic or occasional satisfaction campaigns in the last five years, the number of bedrooms in the tourist accommodation establishments, the motorway density, the number of airports, the number of ports) as well as the interactions terms have been neglected since they were not statistically significant.

4 Modeling the probability of digitalization

Let $Y_{ijk} \sim Ber(\pi_{ijk})$ be the binary response variable which takes values 0 for "Italian museums without digital innovations" with probability $(1-\pi_{ijk})$ and 1 for " Italian museums with digital innovations" with probability π_{ijk}. Moreover, let $\{X_{1\cdot}, X_{2\cdot}, \ldots X_{12\cdot}\}$ be a set of covariates selected for modeling purposes (Table 2). Thus, the following binary logistic regression random slope model has been fitted:

$$\begin{aligned}\eta_{ijk} = \beta_0 + \beta_1\, x_{1ijk} + \beta_{2k}\, x_{2ijk} + \beta_3\, x_{3ijk} + \beta_{4k}\, x_{4ijk} + \beta_5\, x_{5ijk} + \beta_6\, x_{6ijk} + \\ + \beta_7\, x_{7ijk} + \beta_{8j}\, x_{8ijk} + \beta_{9j}\, x_{9ijk} + \beta_{10}\, x_{10k} + \beta_{11}\, x_{11k} + \beta_{12}\, x_{12jk},\end{aligned} \tag{3}$$

where

- $\eta_{ijk} = \log it(\pi_{ijk}) = \ln \dfrac{\pi_{ijk}}{1-\pi_{ijk}}$ and $\pi_{ijk} = \dfrac{\exp\{\eta_{ijk}\}}{1+\exp\{\eta_{ijk}\}}$,
- $\beta_{2k} = \beta_2 + \nu_{2k}$,
- $\beta_{4k} = \beta_4 + \nu_{4k}$,
- $\beta_{8j} = \beta_8 + u_{8jk}$,
- $\beta_{9j} = \beta_9 + u_{9jk}$,
- $\begin{bmatrix} \nu_{2k} \\ \nu_{4k} \end{bmatrix} \sim N(\mathbf{0}, \mathbf{\Omega}_\nu)$, $\quad \mathbf{\Omega}_\nu = \begin{bmatrix} \sigma^2_{\nu 2} & \\ \sigma_{\nu 24} & \sigma^2_{\nu 4} \end{bmatrix}$,
- $\begin{bmatrix} u_{8jk} \\ u_{9jk} \end{bmatrix} \sim N(\mathbf{0}, \mathbf{\Omega}_u)$, $\quad \mathbf{\Omega}_u = \begin{bmatrix} \sigma^2_{u8} & \\ \sigma_{u89} & \sigma^2_{u9} \end{bmatrix}$,
- $i = 1, \ldots n_{jk},\ j = 1, \ldots, N_k,\ k = 1, \ldots, K(K = 20)$,
- $\sum_{k=1}^{K} N_k = 108,\ \sum_{k=1}^{K} \sum_{j=1}^{N_k} n_{jk} = 3{,}217.$

This model allows the slopes of the covariates:

- "Research activities" and "Partnership" to vary across the 3rd level,
- "Network of museums" and "Type of institution" to vary across the 2nd level.

On the other hand, both the coefficients of the contextual factors ("Expenditure for recreation", "Gross domestic product per capita, current prices", "Number of tourist accommodation establishments") and the remaining covariates implemented into the model have been assumed to be constant with respect to the hierachical levels.

The reliability of the assumption of this model lies in the hypothesis of normality residuals, as confirmed by the residual diagnostics performed both for the third and second level of aggregation. Computational aspects associated to the fitting process have been faced by using a specific statistical software for multilevel analysis, called *MLwiN* (Rasbash et al., 2009).

Table 2 Estimates of fixed and random parameters, together with the standard errors (SE), the Wald statistic, the *p-value* and the ORs for the binary logit model

Fixed parameters (Covariate's category)	$\hat{\beta}$	$SE(\hat{\beta})$	Wald statistic	*p-value*	$OR = \exp(\hat{\beta})$
Individual-level covariates					
constant	1.107	0.302	3.666	0.000***	3.025
Access ($x_{1\,ijk}$)	− 1.305	0.168	−7.768	0.000***	0.271
Research activities ($x_{2\,ijk}$)	1.208	0.208	5.808	0.000***	3.347
Percentage of Italians vs Foreigners ($x_{3\,ijk}$)	0.171	0.085	2.012	0.044**	1.186
Partnership ($x_{4\,ijk}$)	0.781	0.165	4.733	0.000***	2.184
Guided tours ($x_{5\,ijk}$)	0.703	0.153	4.595	0.000***	2.020
Exhibition space between 201 e 500 square meters ($x_{6\,ijk}$)	0.567	0.151	3.755	0.000***	1.763
Exhibition space of more than 500 square meters ($x_{7\,ijk}$)	1.365	0.217	6.290	0.000***	3.916
Network of museums ($x_{8\,ijk}$)	0.530	0.231	2.294	0.022**	1.699
Type of institution (public/private) ($x_{9\,ijk}$)	0.238	0.119	2.000	0.046**	1.269
Regional-level covariates					
Gross domestic product per capita, current prices (x_{10k})	0.198	0.093	2.129	0.033**	1.219
Expenditure for recreation, culture and religion (x_{11k})	0.067	0.031	2.161	0.031**	1.069
Province-level covariate					
Number of tourist accommodation establishments (x_{12jk})	0.093	0.038	2.447	0.014**	1.097
Random parameters					
$\mathbf{\Omega}_v = \begin{bmatrix} 0.015(0.017) & \\ 0.038(0.018) & 0.099(0.029) \end{bmatrix}$					
$\mathbf{\Omega}_u = \begin{bmatrix} 1.086(0.578) & \\ 0.074(0.048) & 0.223(0.077) \end{bmatrix}$					

$*$*p-value* < 0.1 $**$*p-value* < 0.05 $***$*p-value* < 0.01

4.1 Results of multilevel binary logit model

Table 2 shows the estimates of the significant covariates' coefficients obtained by the maximum-likelihood method. As previously mentioned, the covariates which were not statistically significant have been removed from the model. In order to evaluate the positive or negative effect of covariates on the probability to include digital innovations, the ORs have been calculated.
The ORs, reported in the last column of Table 2, highlight that:

- a free "Access" to the museum, with respect to a paid visit, decreases the probability to introduce avant-garde digital innovations by −73%; this effect could be motivated by the consideration that museums might be poorly encouraged to implement digital innovations, which are often very expensive, and do not benefit of profits from visits, that would allow to cover the costs needed for the digitalization;
- proposing "Research activities" increases the probability to move towards digital innovations by +235% compared with not guaranteeing research activities in the museums; indeed, promoting research is crucial for museums that invest in digitalization, since the ability of museums to offer advanced digital cultural contents can contribute to an effective cultural enhancement of visitors;
- the presence of "Guided tours" increases the probability to experiment digital innovations by +102%, compared with the absence of guided visits; indeed, proposing guided tours can inspire museums to introduce innovative digital solutions, for example audio-guides or dedicated platforms capable of creating 3D tour itineraries with interactive maps and commentary podcasts;
- regarding the "Exhibition space", the availability of a medium (between 201 and 500 square meters) or large space (with more than 500 square meters), compared to a smaller space (not greater than 201 square meters), increases the probability to move towards digital innovations, respectively, by +76% and +292%. This is justified by taking into account that a greater exhibition space can be a characteristic of museums with adequate financial resources and economies of scale, thanks to which promoting digital transformation is easier;
- the existence of a formal "Partnership" with other public or private cultural institutions over the territory increases the probability of adopting digital innovations by +118% with respect to the absence of partnership; this is probably due to the fact that joining networks of integrated cultural services can increase visibility and push museums to renew their image by promoting digital innovations;
- belonging to a "Network of museums" (through formal acts), compared with not being a part of a network of museums, increases the probability of adopting digital innovations by +70%; in fact, belonging to a network gives the opportunity to share human, technological and/or financial resources, which can surely push towards a digitalization policy;
- being a private museum increases the probability of including digital innovations by +27% instead of a public institution; indeed, private museums have better realized the benefit of adopting digitalization policies and can find almost partially private contributions and funding (sponsorships, liberal disbursements, donations, legacies, benefits from former banking foundations, Art Bonus), which can support the digital transformation process;
- a "Percentage of Italians versus not Italian visitors greater than or equal to 50%" increases the probability of investing in digital innovations by +19% with respect to the presence of a higher percentage of foreign visitors; this result could be explained by taking into account that a greater turnout of Italian visitors compared to foreigners can stimulate

the propensity to include digital innovations, allowing to cover the costs of digital technologies, thanks to the more regular flow of Italian visitors compared to the presence of foreign visitors, which is mostly seasonal or occasional.

Regarding the contextual factors it is worth pointing out that:

- the “Number of tourist accommodation establishments”, at provincial level, clearly generates a positive effect on the probability of adopting digital innovations, by increasing the probability to include digital innovations of +10%; in fact, the presence of a larger number of tourist accommodation facilities is indicative of tourist attractiveness for the provinces and therefore can stimulate museums located in these areas to invest in digital technologies. The need for digital transformation is a challenge but also an opportunity to approach new audiences and to enhance the tangible and intangible assets that institutions preserve and produce;
- supporting “Expenditure for recreation, culture and religion”, at regional level, produces a positive effect on the probability of investing in digitalization, by increasing the probability to move towards digitalization of +7%; this indicator is symptomatic of the propensity of spending on cultural activities at regional level and may stimulate museums to invest in digital technologies;
- a high level of the “Gross domestic product per capita”, at regional level, greater than (or equal to) 0.028 generates a beneficial influence on the probability of adopting digital innovations, with an increment of +22%; this positive percentage reflects the growth of the regional economy and represents a leverage for museum institutions, which can be encouraged to invest in digital technologies in a more stable economic context.

In addition, by focusing on the random part of the model, it is evident that the variation in the probability to invest in digital innovations, regarding the third and second levels, concerns the covariates:

- “Research activities” and “Partnership”, with an influence of the regional-group effect which is greater on the second covariate than on the first one, as also confirmed by the estimated intra-class correlation coefficient corresponding to 4.36% and 23.13%, respectively;
- “Network of museums” and “Type of institution”, with an influence of the provincial-group effect which is greater on the first covariate than on the second one, as further highlighted by the value assumed by the intraclass correlation coefficient equal to 25% and 6%, respectively.

4.2 Estimated probabilities for province and region levels

In order to explain the above mentioned results referred to the model (3), the estimated probabilities have been calculated with respect to the province and region levels (Table 3). From Table 3 it is highlighted that the probability to move towards some forms of digital innovations for museums is high in all Italian regions (with estimated probabilities ranging from 0.799 to 0.959); in particular, it is slightly higher for some regions in the North (Emilia-Romagna, Trentino-Alto Adige, Lombardy), the center (Lazio, Tuscany, Umbria), the island of Sardinia and slightly lower for some regions of North-West and South (Calabria, Liguria, Abruzzo and Aosta Valley) among others.

By analysing the estimated probabilities with respect to the Italian provinces, it is possible to identify the highest probabilities of including some forms of digital innovations especially for the largest provinces characterized by the greatest museum densities, such as

Table 3 Estimated probabilities ($\hat{\pi}_{ijk}$) for multilevel binary logit model, classified by region and province

Region/Province	$\hat{\pi}_{ijk}$	Region/Province	$\hat{\pi}_{ijk}$	Region/Province	$\hat{\pi}_{ijk}$
Liguria	**0.892**	**Trentino-Alto Adige**	**0.959**	**Basilicata**	**0.952**
Genova	0.894	Bolzano	0.966	Matera	0.982
Imperia	0.894	Trento	0.953	Potenza	0.921
La Spezia	0.881	**Veneto**	**0.941**	**Molise**	**0.919**
Savona	0.898	Belluno	0.915	Campobasso	0.888
Lombardy	**0.950**	Padova	0.942	Isernia	0.950
Bergamo	0.947	Rovigo	0.939	**Calabria**	**0.908**
Brescia	0.961	Treviso	0.948	Catanzaro	0.894
Como	0.979	Venice	0.974	Cosenza	0.883
Cremona	0.973	Verona	0.949	Crotone	0.872
Lecco	0.990	Vicenza	0.922	Reggio Calabria	0.969
Lodi	0.924	**Lazio**	**0.953**	Vibo Valentia	0.923
Mantova	0.966	Frosinone	0.908	**Campania**	**0.910**
Milan	0.980	Latina	0.978	Avellino	0.877
Monza Brianza	0.873	Rieti	0.932	Benevento	0.864
Pavia	0.930	Roma	0.979	Caserta	0.917
Sondrio	0.928	Viterbo	0.966	Naples	0.970
Varese	0.953	**Marche**	**0.927**	Salerno	0.923
Piedmont	**0.909**	Ancona	0.940	**Apulia**	**0.950**
Alessandria	0.902	Ascoli Piceno	0.930	Bari	0.944
Asti	0.893	Fermo	0.910	Barletta-Andria-Trani	0.973
Biella	0.955	Macerata	0.914	Brindisi	0.990
Cuneo	0.929	Pesaro-Urbino	0.942	Foggia	0.880
Novara	0.925	**Tuscany**	**0.958**	Lecce	0.942
Torino	0.941	Arezzo	0.966	Taranto	0.971
Verbano-Cusio-Ossola	0.892	Firenze	0.969	**Sardinia**	**0.957**
Vercelli	0.833	Grosseto	0.966	Cagliari	0.975
Emilia-Romagna	**0.959**	Livorno	0.889	Nuoro	0.955
Bologna	0.972	Lucca	0.946	Oristano	0.938
Ferrara	0.989	Massa-Carrara	0.975	Sassari	0.952
Forlì-Cesena	0.927	Pisa	0.977	Sud Sardegna	0.962
Modena	0.973	Pistoia	0.955	**Sicily**	**0.942**
Parma	0.953	Prato	0.968	Agrigento	0.928
Piacenza	0.938	Siena	0.973	Caltanissetta	0.908
Ravenna	0.978	**Umbria**	**0.954**	Catania	0.977
Reggio nell'Emilia	0.931	Perugia	0.959	Enna	0.919
Rimini	0.970	Terni	0.948	Messina	0.939

Table 3 continued

Region/Province	$\hat{\pi}_{ijk}$	Region/Province	$\hat{\pi}_{ijk}$	Region/Province	$\hat{\pi}_{ijk}$
Friuli-Venezia Giulia	**0.941**	**Abruzzo**	**0.898**	Palermo	0.947
Gorizia	0.926	Chieti	0.939	Ragusa	0.952
Pordenone	0.930	L'Aquila	0.868	Siracusa	0.941
Trieste	0.973	Pescara	0.888	Trapani	0.966
Udine	0.933	Teramo	0.898		
Aosta Valley	**0.799**				

Bold indicate the mean value of the probability for each region

- Milan (denoted by 66 museums widespread throughout the territory, i.e. 4.2 museums per 100 km^2), in the Lombardy region, with an estimated probability equal to 0.980; this high predicted value could be justified by taking into account that Milan is prone to digitalization; it is useful to underline that since 2018 the municipality of Milan organizes the *Milan Digital Week event*, dedicated to the production and dissemination of knowledge and innovation through digital;
- Rome (characterized by 213 museums widespread throughout the territory, i.e. 4 museums per 100 km^2), in the Lazio region, with an estimated probability equal to 0.979; indeed, in the municipality of Rome the digitalization policy has involved up to now 8 museums reconstructed in virtual reality.

However, besides to the well-known provinces of Milan and Rome, it is also interesting to mention some other provinces with a relative smaller museum density, but with high estimated probabilities of digitalization, such as

- Ferrara (characterized by 38 museums widespread throughout the territory, i.e. 1.4 museums per 100 km^2), in the Emilia-Romagna region, with an estimated probability equal to 0.989; indeed, this high predicted value could be justified by considering that Ferrara was included, in 2014, in the European research project *Inception*, with the scope to implement the digitalization of various european cultural centers through the most modern systems of 3D modeling;
- Brindisi (denoted by 19 museums widespread throughout the territory, i.e. 1 museum per 100 km^2), in the region of Apulia, with a predicted probability corresponding to 0.990; this high estimated value can be due to the involvement of one of its municipalities (i.e. Fasano) in a project of the National Operational Programme (PON) Culture and Development 2014-2020, devoted to digitalize of the cultural heritage in the National Museum and its Archaeological Park, as well as to a 3D reconstruction of the most significant architectural structures. In addition, the province of Brindisi has been chosen in the year 2021 as the venue for the *Wired Digital Day*, an event that aims to disclose that digital, technological and scientific research are transforming the world of entrepreneurship and promoting the development of new skills;
- Como (characterized by 20 museums widespread throughout the territory, i.e. 1.6 museums per 100 km^2), in the region of Lombardy, with an estimated probability equal to 0.979; this high predicted value could be justified by considering that one of its municipalities (i.e. Barni) in 2018 has joined the *Digital Invasions* initiative, a game-event developed at national and international level for the promotion of culture, the discovery of the territory and the positive use of digital tools, as a means to share the historical and cultural heritage of Italy; moveover, in the year 2020, the province of Como has

activated *Smart Como*, that is an online communication channel, for the discovery of the collections stored in the museums;

- Lecco (characterized by 17 museums widespread throughout the territory, i.e. 2.1 museums per 100 km^2), in the Lombardy region, with an estimated probability equal to 0.990; this high estimated value can be explained by remembering that, starting from 2009 Lecco has been recognized the most effective and active *local museum network* in the Lombardy region;
- Matera (characterized by 20 museums widespread throughout the territory, i.e. 0.6 museums per 100 km^2), in the region of Basilicata, with an estimated probability of 0.982; this high value could be justified by considering that Matera has been awarded of the title of European Capital of Culture in 2019 (granted in the fall of 2014).

On the other hand, the lowest probabilities is measured for the province of Aosta Valley with an estimated probability of 0.799.

5 Modeling the probability of increasing levels of digitalization

Let $Y_{ijk}^{(s)}$ be a multinomial ordered response variable concerning the level of digitalization, which takes values $s = 1, 2, 3$ (response categories), where $s = 3$ represents the reference category "no digitalization", consistently with the binary model (Sect. 4.2). On the other hand, $s = 2$ corresponds to "medium/low digitalization" and $s = 1$ denotes "high digitalization", and let $\pi_{ijk}^{(3)}$ the probability of being without digital innovations, $\pi_{ijk}^{(2)}$ the probability of including a medium/low level of digitalization, $\pi_{ijk}^{(1)}$ the probability of having a high level of digitalization, with the index i $(i = 1, \ldots, n_{jk})$ representing the italian museums (level 1 unit), the index j $(j = 1, ..., N_k)$ corresponding to the provinces (level 2 unit) and the index k $(k = 1, ..., K)$ indicating the regions (level 3 unit). As previously mentioned, $Y_{ijk}^{(s)}$ has been derived by taking into account the presence of at most 15 significant digital innovations, specifically digital inventory, audio-guides, dedicated website, online ticketing service, virtual tours online, social media accounts (Facebook, Twitter, Instagram, Pinterest, Foursquare, etc.), link to digital maps and/or to GPS coordinates for the location of the museum, scientific digital catalogue and digital catalogue accessible online for the visitors, applications dedicated for smartphones and tablets, interactive installations and/or virtual reconstructions (touch screen tables, videos), QR Code and/or proximity devices (Bluetooth, Wi-Fi, etc.), tablets available for the visitors, immersion video/multimedia rooms, free Wi-Fi connection. In particular, this variable has been coded by considering three levels of digitalization, that is high, medium/low and no digitalization, with values 1, 2, 3, respectively, as specified below:

- $s = 3$ includes "0 digital innovations" (for consistency with the binary model) and corresponds to no digital innovations,
- $s = 2$ includes "from 1 to 7 digital innovations" and is associated to a medium/low level of digitalization,
- $s = 1$ includes "from 8 to 15 digital innovations" and is associated to a level of digitalization.

Moreover, let $\mathbf{X}_{.} = \{X_{1.}, X_{2.}, \ldots, X_{H.}\}$ be a set of covariates, which influences the dependent response variable (Table 5).

By taking into consideration the ordering, the chosen model is based on the cumulative response probabilities defined as follows:

$$\psi_{ijk}^{(s)} = \sum_{h=1}^{s} \pi_{ijk}^{(h)}, \qquad s = 1, 2.$$

Thus, the category probabilities can be expressed in terms of the cumulative probabilities, as clarified below:

$$\psi_{ijk}^{(1)} = \pi_{ijk}^{(1)}, \qquad \psi_{ijk}^{(2)} = \pi_{ijk}^{(1)} + \pi_{ijk}^{(2)}, \qquad \psi_{ijk}^{(3)} = 1.$$

This is followed by two sub-equations, one for each category ("high digitalization" and "medium/low-high digitalization"):

$$\begin{aligned} \eta_{ijk}^{(1)} = \log it(\psi_{ijk}^{(1)}) = \beta_0^{(1)} + \beta_1^{(1)} x_{1ijk} + \beta_2^{(1)} x_{2ijk} + \beta_{3k}^{(1)} x_{3ijk} + \beta_4^{(1)} x_{4ijk} + \\ + \beta_5^{(1)} x_{5ijk} + \beta_6^{(1)} x_{6ijk} + h_{ijk}, \end{aligned} \tag{4}$$

$$\begin{aligned} \eta_{ijk}^{(2)} = \log it(\psi_{ijk}^{(2)}) = \beta_0^{(2)} + \beta_1^{(2)} x_{1ijk} + \beta_2^{(2)} x_{2ijk} + \beta_{3k}^{(2)} x_{3ijk} + \beta_4^{(2)} x_{4ijk} + \\ + \beta_5^{(2)} x_{5ijk} + \beta_6{}^{(2)} x_{6ijk} + h_{ijk}, \end{aligned} \tag{5}$$

with

- $h_{ijk} = \beta_7 x_{7k} + \beta_8 x_{8k} + \beta_9 x_{9jk} + \beta_{10} x_{10ijk} + \beta_{11j} x_{11ijk} + \beta_{12j} x_{12ijk}$
- $\beta_{3k}^{(1)} = \beta_3^{(1)} + \nu_{3k}^{(1)}$
- $\beta_{3k}^{(2)} = \beta_3^{(2)} + \nu_{3k}^{(2)}$
- $\beta_{11j} = \beta_{11} + \nu_{11jk}$
- $\beta_{12j} = \beta_{12} + \nu_{12jk}$
- $\begin{bmatrix} \nu_{3k}^{(1)} \\ \nu_{3k}^{(2)} \end{bmatrix} \sim N\Big(\mathbf{0}, \mathbf{\Omega}_\nu\Big), \quad \mathbf{\Omega}_\nu = \begin{bmatrix} \sigma_{\nu 3^{(1)}}^2 & \\ \sigma_{\nu 3^{(1)} 3^{(2)}} & \sigma_{\nu 3^{(2)}}^2 \end{bmatrix};$
- $\begin{bmatrix} u_{11jk} \\ u_{12jk} \end{bmatrix} \sim N\left(\mathbf{0}, \mathbf{\Omega}_u\right), \quad \mathbf{\Omega}_u = \begin{bmatrix} \sigma_{u11}^2 & \\ \sigma_{u11\,12} & \sigma_{u12}^2 \end{bmatrix}.$

  It is worth pointing out that, after testing the proportional odds assumption by a Wald test (Sect. 5.1), a partial proportional odds model has been selected. In particular:
- the common coefficients of the covariates "Network of museums" and "Type of institution" have been assumed to vary across the 2nd level;
- the common coefficient of the covariate "Percentage of Italians vs Foreigners" has been assumed to be constant;
- the separate coefficient of the covariate "Partnership" has been assumed to vary across the 3rd level.

On the other hand, the coefficients of the contextual factors ("Expenditure for recreation", "Gross domestic product per capita, current prices", "Number of tourist accommodation establishments") included as common coefficients, as well as the slopes of the remaining covariates, implemented into the model as separate coefficients, have been assumed to be constant.

The reliability of this model has been confirmed by the residual diagnostics performed both for the third and second level of aggregation.

Table 4 Test for non-proportionality of the covariates' categories

Covariate's category	Wald statistic	*p-value*
Access	84.958	0.000***
Research activities	91.060	0.000***
Percentage of Italians vs Foreigners	3.058	0.217
Guided tours	29.913	0.000***
Exhibition space between 201 e 500 square meters	27.922	0.000***
Exhibition space of more than 500 square meters	86.007	0.000***
Network of museums	4.883	0.087*
Partnership	45.894	0.000***
Type of institution	3.494	0.174

**p*-value < 0.1 ***p*-value < 0.05 ****p*-value < 0.01

5.1 Testing the proportional odds assumption

In order to evaluate the adequacy of the fitted partial proportional odds model, a Wald Chi-square test can be applied (DeMaris, 1992; Williams, 2006; Soon, 2010). In other terms, the Wald Chi-square test is used to verify the partial proportional odds model's overall goodness-of-fit. It checks for the validity of the null hypothesis that the coefficients of some covariates added in the model are not significantly different across the two sub-equations. According to the rejection of the null hypothesis, the coefficients of some independent variables can be assumed different in the two sub-equations (Cole et al., 2004; Long & Freese, 2014). Note that this test is crucial to determine which covariates violate the proportionality assumption in order to avoid the selection of an inadequate model which might drive to misleading inferences.

Table 4 shows the results of the Wald Chi-square test.

By analysing the *p-values* in Table 4 it is highlighted that the proportional odds assumption holds only for the covariates "Percentage of Italians vs Foreigners", "Network of museums" and "Type of institution" at a significance level of 5%. Thus, the other covariates reported in the Table 4 have been added as separate coefficients to each of the two sub-equations of the model, since the proportional odds hypothesis is violated (rejected).

Therefore, one can conclude that a partial proportional odds model represents a suitable choice for fitting the data, since it allows separate effects for each cumulative logit only for some predictors.

5.2 Results of multilevel multinomial ordered logit model

Table 5 shows the estimates of fixed and random parameters of the multinomial ordered logit model.

The ORs given in the last column of Table 5 highlight that:

- a free "Access" to the museum, with respect to a paid visit, decreases the probability to introduce at least a medium/low digitalization by −72% or decreases the probability to have a high level of digitalization by −51%; the effect of this covariate is negative on the probability of both high and medium/low level of digitalization, with a stronger incidence on the probability of high digitalization than on medium/low digitalization

Table 5 Estimates of fixed and random parameters, together with the standard errors (SE), the Wald statistic, the *p-value* and the ORs for the ordered logit model

Fixed parameters (Covariate's category)	$\hat{\beta}$	$SE(\hat{\beta})$	Wald statistic	*p-value*	$OR = \exp(\hat{\beta})$
Separate coefficients					
High digitalization					
Individual-level covariates					
constant	−4.740	0.433	−10.947	0.000***	0.009
Access ($x_{1\,ijk}$)	−0.717	0.131	−5.473	0.000***	0.488
Research activities ($x_{2\,ijk}$)	0.962	0.123	7.821	0.000***	2.617
Partnership ($x_{3\,ijk}$)	0.976	0.200	4.880	0.000***	2.654
Guided tours ($x_{4\,ijk}$)	1.245	0.366	3.402	0.001***	3.473
Exhibition space between 201 e 500 square meters ($x_{5\,ijk}$)	0.742	0.188	3.947	0.000***	2.100
Exhibition space of more than 500 square meters ($x_{6\,ijk}$)	1.266	0.177	7.153	0.000***	3.547
Medium/low-High digitalization					
Individual-level covariates					
constant	1.300	0.245	5.306	0.000***	3.669
Access ($x_{1\,ijk}$)	−1.269	0.161	−7.882	0.000***	0.281
Research activities ($x_{2\,ijk}$)	1.264	0.204	6.196	0.000***	3.540
Partnership ($x_{3\,ijk}$)	0.889	0.164	5.421	0.000***	2.433
Guided tours ($x_{4\,ijk}$)	0.692	0.150	4.613	0.000***	1.998
Exhibition space between 201 e 500 square meters ($x_{5\,ijk}$)	0.565	0.148	3.818	0.000***	1.759
Exhibition space of more than 500 square meters ($x_{6\,ijk}$)	1.378	0.214	6.439	0.000***	3.967

Table 5 continued

Fixed parameters (Covariate's category)	$\hat{\beta}$	$SE(\hat{\beta})$	Wald statistic	*p-value*	$OR = \exp(\hat{\beta})$
Common coefficients					
Regional-level covariates					
Gross domestic product per capita, current prices (x_{7k})	0.290	0.129	2.248	0.025**	1.336
Expenditure for recreation, culture and religion (x_{8k})	0.078	0.037	2.108	0.035**	1.081
Province-level covariate					
Number of tourist accommodation establishments (x_{9jk})	0.117	0.058	2.017	0.044**	1.124
Individual-level covariates					
Percentage of Italians vs Foreigners ($x_{10\,ijk}$)	0.128	0.059	2.169	0.030**	1.137
Network of museums ($x_{11\,ijk}$)	0.491	0.018	27.278	0.000***	1.634
Type of institution (public/private) ($x_{12\,ijk}$)	0.257	0.003	85.667	0.000***	1.293
Random parameters					
$\mathbf{\Omega}_v = \begin{bmatrix} 0.146(0.075) & \\ 0.031(0.066) & 0.115(0.108) \end{bmatrix}$					
$\mathbf{\Omega}_u = \begin{bmatrix} 0.384(0.148) & \\ -0.125(0.082) & 0.227(0.078) \end{bmatrix}$					

*p-value < 0.1 **p-value < 0.05 ***p-value < 0.01

(which provides a further reduction of the OR of −21%). Thus, it is reasonable to confirm that museums cannot invest in digitalization, since it is very expensive, without having a profit from visits;

- proposing “Research activities”, compared with do not guaranteeing them, increases the probability to have at least a medium/low digitalization by +254% or increases the probability to have a high level of digitalization by +162%; indeed, the effect of this covariate is positive on the probability of both high and medium/low level of digitalization, with a major contribution on the probability of high digitalization than on medium/low digitalization (which entails an increase of the OR of +92%). Hence, research activities can make museum collections more attractive, allowing visitors to enjoy digital advanced cultural contents;
- offering “Guided tours”, compared with do not offer guided visits, increases the probability to reach at least a medium/low level of digitalization by +100% or to have a high degree of digitalization by +247%. Although the effect of this covariate is positive, the OR of the cumulative option “medium/low-high digitalization” is lower than the OR associated to the high level of digitalization: this means that offering guide tours produces a dampening impact on the medium/low digitalization (with an OR of +100%), on the other hand it generates a great positive influence on the high level of digitalization, maybe by stimulating the adoption of digital technologies with 3D virtual tours and others;
- regarding the “Exhibition space”, the availability of a medium (exhibition space between 201 and 500 square meters) or large space (exhibition space with more than 500 square meters), compared to a small exhibition space (exhibition space not greater than 201 square meters) increases the probability to include at least a medium/low digitalization by +76% for a medium exhibition space and +297% for a large exhibition space or increases the probability to move towards a high digitalization by +110% for a medium exhibition space and +255% for a large exhibition space. This is explained by considering that a greater exhibition space is reasonably referred to museums with solid financial resources, which decide to promote a high level of digital innovations (much more than investing on medium/low level of digitalization) for improving their visibility and image;
- the presence of “Partnership” with the territory, with respect to the absence of formal relationships with other public or private cultural institutions located in the territory, increases the probability of adopting at least a medium/low degree of digitalization by +143% or having a high degree of digitalization by +165%, with respect to the absence of partnership; this result is justified by the idea that joining networks of integrated cultural services can push the museums to promote a medium/low or high process of digitalization, in order to enhance their visibility and attractiveness. This contribution is larger on the high level of digitalization than on the medium/low level of digitalization;
- belonging to a “Network of museums” increases the probability to adopt a medium/low or high level of digitalization by +63%, compared with not being part of a network of museums; this is motivated by the idea that the participation in a network governed by formal acts, thanks to the sharing of human, technological or financial resources, can support and stimulate the museums to invest in a medium/low or high level of digital innovations;
- being a private museum increases the probability to have a medium/low or high level of digitalization by +29% instead of a public museum; this could be reasonably due to the fact that private museums can obtain private contributions and funding, which allows to sustain the development of a medium/low or high degree of digitalization;
- the presence of a “Percentage of Italians versus not Italian visitors greater than (or equal to) the 50%” increases the probability to include a medium/low or high level of digital-

ization by +14% compared to a higher percentage of foreign visitors; this is probably due to a larger and continuous presence of Italian visitors with respect to the seasonal or occasional presence of foreigners, which could lead to invest in a medium/low or high process of digitalization, allowing more revenue thanks to the more regular flow of Italian visitors compared to the presence of foreign visitors; another potential motivation could be that Italians are particularly attracted by high or medium/low technological innovations in museums.

With reference to the contextual factors, the results obtained from the multinomial ordered model confirm the same conclusion of the binary model, that is:

- a high provincial "Number of tourist accommodation establishments" produces a positive effect on the probability to have a medium/low or high level of digitalization, with an increment of the +12%;
- an available regional financial support to cover the expenditure for recreation, culture and religion generates a positive effect on the probability to have a medium/low or high level of digitalization of the +8%;
- a high regional "Gross domestic product per capita" increases the probability to have a medium/low or high degree of digitalization of the +34%.

Moreover, by focusing on the random part of the model, it is evident that the variation in the probability to invest in medium/low or high digital innovations occuring between the third and second level concerns the covariates:

- "Partnership" with a low variability at level three, slightly higher for high digitalization than in the case of medium/low digitalization, as confirmed by the estimated intra-class correlation coefficient corresponding to 4.25% and 3.38%, respectively;
- "Network of museums" and "Type of institution" with a greater influence of the provincial-group effect on the first covariate than on the second one, as confirmed by the value assumed by the intraclass correlation coefficient equal to 10.5% and 6.5%, respectively.

Note that the ORs' results are consistent with the information obtained from the binary model (first step analysis), in which the key factors influencing the digital innovation process have been identified. Furthermore, the implementation of the multinomial ordered model (second step analysis) has permitted to investigate the positive or negative effects of the selected covariates on the cumulative estimated probability of having a medium/low or high level of digitalization. Moreover, the effects produced by almost all the covariates included in the multinomial ordered model are always higher in the case of high digitalization than for medium/low digitalization. Only the covariate "Guided tours" represents an exception, since it produces a dampening effect on the medium/low digitalization, on the other hand it generates a major positive incidence on the high level of digitalization; as a consequence this could be a driving force to invest in advanced digitalization.

5.3 Estimated probabilities for province and region levels

In order to clarify the modeling findings, the estimated probabilities for the museums to move towards a medium/low or high degree of digitalization have been computed with respect to the regional and provincial level.

By focusing on the regional level, it is much more likely that museums in Italy are willing to adopt a medium/low level of digitalization (with predicted values ranging from 0.497 to

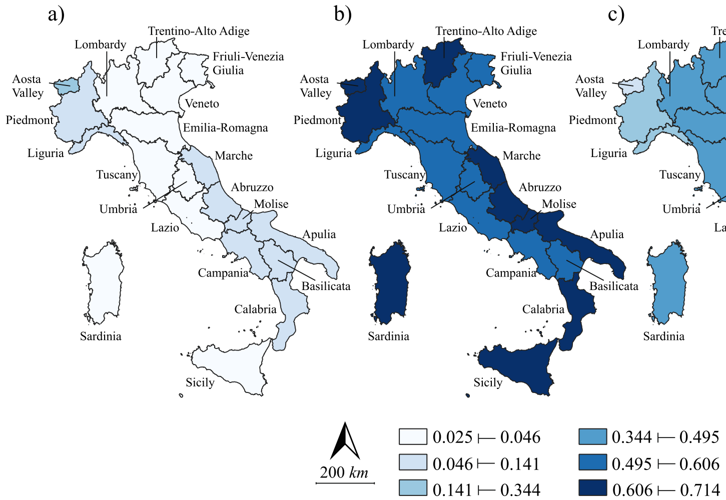

Fig. 1 colormaps of the **a** estimated probability of being without digital innovations ($\hat{\pi}_{ijk}^{(3)}$), **b** estimated probability of having a medium/low digitalization ($\hat{\pi}_{ijk}^{(2)}$), **c** estimated probability of having high digitalization ($\hat{\pi}_{ijk}^{(1)}$), classified by region

0.713) than a high level of digitalization (with estimated values ranging from 0.138 to 0.477), as shown in the Fig. 1.

This is confirmed, at the provincial level, from the color maps illustrated in Fig. 2, which reveal a greater likelihood for italian museums to invest in a medium/low degree of digitalization (with predicted values spanning from 0.403 to 0.773) than a high level of digitalization (with estimated values ranging from 0.124 to 0.587). Moreover, from Table 6 it is highlighted that among the Italian regions with the greatest cumulative estimated probability of adopting at least a medium/low level of digital innovations, besides to the well-known regions characterized by a major number of structures (ranging from 260 to 553 museums), such as Lombardy, Emilia-Romagna, Lazio, Tuscany and Sicily, there are also Apulia and Basilicata, with fewer than 170 museum structures widespread throughout their areas.

In particular, by analysing the Table 6, it is evident that the provinces with the greatest cumulative estimated probability of adopting at least a medium/low level of digitalization are the following:

- Lecco in the region of Lombardy, with a cumulative estimated probability of adopting at least a medium/low digital trasformations equal to 0.994 (out of which 0.557 concerning a medium/low level of digitalization and 0.437 for a high level of digitalization);
- Brindisi in the region of Apulia, with a cumulative estimated probability of integrating at least a medium/low digital trasformations equal to 0.993 (out of which 0.532 referred to a medium/low level of digitalization and 0.461 for a high level of digitalization);
- Ferrara in the region of Emilia-Romagna, with a cumulative estimated probability of including at least a medium/low digital trasformations equal to 0.993 (out of which 0.406 referred to a medium/low level of digitalization and 0.587 for a high level of digitalization);

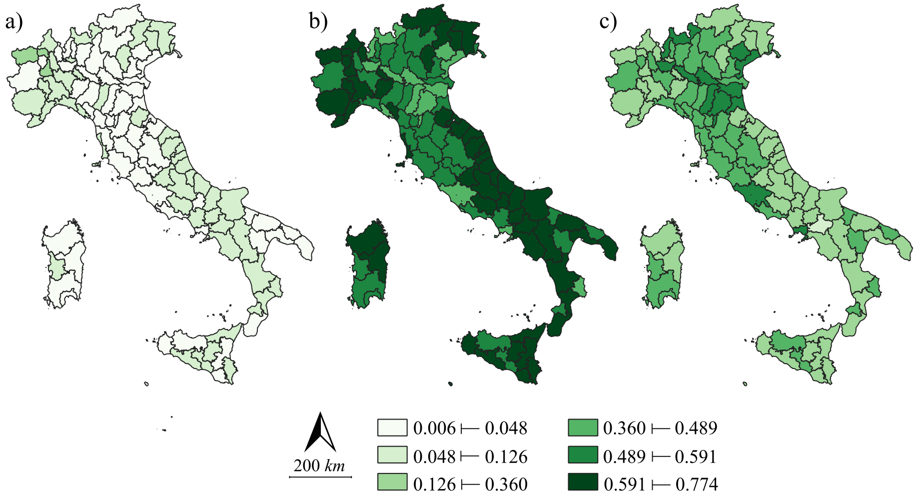

Fig. 2 colormaps of the **a** estimated probability of being without digital innovations ($\hat{\pi}_{ijk}^{(3)}$), **b** estimated probability of having a medium/low digitalization ($\hat{\pi}_{ijk}^{(2)}$), **c** estimated probability of having high digitalization ($\hat{\pi}_{ijk}^{(1)}$), classified by province

- Matera in the region of Basilicata, with a cumulative estimated probability of adopting at least a medium/low digital trasformations equal to 0.985 (out of which 0.518 referred to a medium/low level of digitalization and 0.467 for a high level of digitalization).

On the other hand, Aosta Valley represents the province with a slightly lower cumulative estimated probability of investing in at least a medium/low level of digitalization.

In addition, the largest estimated probability of having a high level of digitalization is observed for the following provinces:

- Milan (with 66 museums and a predicted probability of 0.583), Mantova (with 32 museums and an estimated probability of 0.570), Cremona (with 22 museums and a predicted probability of 0.551), Como (with 20 museums and a predicted probability of 0.508) in the Lombardy region. Note that, besides Milan and Como (for which a justification concerning the high level of digitalization has already been discussed in Sect. 4.2), it is worth underlining that the municipality of Mantova in 2018 (the European Year of Cultural Heritage) has coordinated a calendar of initiatives for enhancing and promoting the tangible and intangible, natural and digital heritage; moreover, the municipality of Cremona, similarly to Como, in 2015 has joined the *Digital Invasions* project;
- Ferrara (with 38 museums and a predicted probability of 0.587) in the Emilia-Romagna region, which was included, in 2014, in the research project *Inception*, as in Sect. 4.2;
- Prato (with 18 museums and a predicted probability of 0.543) in the Tuscany region, which in 2018 hosted the final event of the week dedicated to Places of Digital Culture, named *The future of culture: storytelling, marketing and digital*;
- Rome (with 213 museums and a predicted probability of 0.537) in the Lazio region, with the development of digitalization in 8 civic museums for the municipality of Rome, as already remembered;

Table 6 Estimate probabilities of being without digital innovations ($\hat{\pi}_{ijk}^{(3)}$), of having a medium/low digitalization ($\hat{\pi}_{ijk}^{(2)}$) or a high digitalization ($\hat{\pi}_{ijk}^{(1)}$) for multilevel ordered logit model, classified by regions and provinces

Region/Province	$\hat{\pi}_{ijk}^{(3)}$	$\hat{\pi}_{ijk}^{(2)}$	$\hat{\pi}_{ijk}^{(1)}$	Region/Province	$\hat{\pi}_{ijk}^{(3)}$	$\hat{\pi}_{ijk}^{(2)}$	$\hat{\pi}_{ijk}^{(1)}$
Liguria	**0.085**	**0.576**	**0.338**	**Tuscany**	**0.031**	**0.562**	**0.407**
Genova	0.084	0.512	0.405	Arezzo	0.024	0.571	0.405
Imperia	0.084	0.677	0.239	Firenze	0.023	0.512	0.464
La Spezia	0.095	0.456	0.449	Grosseto	0.025	0.575	0.400
Savona	0.079	0.660	0.261	Livorno	0.088	0.642	0.271
Lombardy	**0.038**	**0.509**	**0.452**	Lucca	0.040	0.589	0.371
Bergamo	0.041	0.522	0.437	Massa-Carrara	0.017	0.617	0.365
Brescia	0.030	0.493	0.477	Pisa	0.016	0.539	0.445
Como	0.016	0.476	0.508	Pistoia	0.034	0.587	0.379
Cremona	0.020	0.429	0.551	Prato	0.024	0.432	0.543
Lecco	0.006	0.557	0.437	Siena	0.020	0.558	0.422
Lodi	0.059	0.644	0.297	**Umbria**	**0.039**	**0.554**	**0.407**
Mantova	0.027	0.403	0.570	Perugia	0.034	0.571	0.396
Milan	0.014	0.403	0.583	Terni	0.044	0.537	0.419
Monza Brianza	0.100	0.559	0.341	**Abruzzo**	**0.087**	**0.665**	**0.248**
Pavia	0.054	0.564	0.381	Chieti	0.049	0.597	0.353
Sondrio	0.058	0.452	0.490	L'Aquila	0.114	0.727	0.159
Varese	0.036	0.607	0.357	Pescara	0.098	0.680	0.222
Piedmont	**0.071**	**0.619**	**0.310**	Teramo	0.087	0.655	0.258
Alessandria	0.078	0.631	0.291	**Basilicata**	**0.041**	**0.558**	**0.401**
Asti	0.085	0.626	0.289	Matera	0.015	0.518	0.467
Biella	0.034	0.600	0.366	Potenza	0.068	0.597	0.335
Cuneo	0.053	0.678	0.269	**Molise**	**0.070**	**0.701**	**0.230**
Novara	0.058	0.575	0.368	Campobasso	0.097	0.647	0.256
Torino	0.045	0.549	0.407	Isernia	0.042	0.754	0.204
Verbano-Cusio-Ossola	0.084	0.668	0.248	**Calabria**	**0.079**	**0.601**	**0.320**
Vercelli	0.132	0.623	0.245	Catanzaro	0.092	0.653	0.255
Emilia-Romagna	**0.031**	**0.538**	**0.431**	Cosenza	0.099	0.641	0.260
Bologna	0.020	0.481	0.499	Crotone	0.110	0.450	0.440
Ferrara	0.007	0.406	0.587	Reggio Calabria	0.024	0.698	0.278
Forlì-Cesena	0.056	0.614	0.330	Vibo Valentia	0.068	0.565	0.367
Modena	0.021	0.489	0.491	**Campania**	**0.076**	**0.632**	**0.292**
Parma	0.036	0.567	0.398	Avellino	0.104	0.772	0.124
Piacenza	0.048	0.617	0.335	Benevento	0.120	0.664	0.216
Ravenna	0.016	0.492	0.492	Caserta	0.070	0.575	0.355
Reggio nell'Emilia	0.052	0.560	0.387	Naples	0.024	0.463	0.513
Rimini	0.021	0.619	0.360	Salerno	0.064	0.684	0.252

Table 6 continued

Region/Province	$\hat{\pi}_{ijk}^{(3)}$	$\hat{\pi}_{ijk}^{(2)}$	$\hat{\pi}_{ijk}^{(1)}$	Region/Province	$\hat{\pi}_{ijk}^{(3)}$	$\hat{\pi}_{ijk}^{(2)}$	$\hat{\pi}_{ijk}^{(1)}$
Friuli-Venezia Giulia	**0.046**	**0.586**	**0.369**	**Apulia**	**0.042**	**0.620**	**0.338**
Gorizia	0.059	0.537	0.404	Bari	0.048	0.631	0.321
Pordenone	0.055	0.650	0.295	Barletta-Andria-Trani	0.021	0.568	0.410
Trieste	0.020	0.500	0.480	Brindisi	0.007	0.532	0.461
Udine	0.049	0.656	0.295	Foggia	0.103	0.690	0.207
Aosta Valley	**0.161**	**0.701**	**0.138**	Lecce	0.048	0.602	0.350
Trentino-Alto Adige	**0.031**	**0.611**	**0.358**	Taranto	0.023	0.699	0.279
Bolzano	0.026	0.668	0.306	**Sardinia**	**0.036**	**0.590**	**0.373**
Trento	0.036	0.554	0.410	Cagliari	0.020	0.520	0.460
Veneto	**0.045**	**0.536**	**0.420**	Nuoro	0.037	0.622	0.341
Belluno	0.066	0.610	0.323	Oristano	0.054	0.541	0.405
Padova	0.044	0.541	0.415	Sassari	0.039	0.688	0.274
Rovigo	0.045	0.546	0.409	Sud Sardegna	0.032	0.580	0.388
Treviso	0.040	0.452	0.507	**Sicily**	**0.049**	**0.651**	**0.300**
Venice	0.019	0.448	0.533	Agrigento	0.060	0.676	0.263
Verona	0.038	0.546	0.416	Caltanissetta	0.080	0.530	0.390
Vicenza	0.059	0.609	0.333	Catania	0.017	0.666	0.316
Lazio	**0.037**	**0.554**	**0.409**	Enna	0.069	0.724	0.207
Frosinone	0.073	0.635	0.292	Messina	0.051	0.670	0.280
Latina	0.016	0.524	0.460	Palermo	0.043	0.546	0.411
Rieti	0.054	0.674	0.272	Ragusa	0.041	0.773	0.185
Roma	0.015	0.448	0.537	Siracusa	0.051	0.629	0.320
Viterbo	0.025	0.490	0.486	Trapani	0.027	0.644	0.329
Marche	**0.061**	**0.659**	**0.280**				
Ancona	0.050	0.627	0.323				
Ascoli Piceno	0.059	0.641	0.300				
Fermo	0.077	0.700	0.224				
Macerata	0.071	0.680	0.249				
Pesaro-Urbino	0.048	0.646	0.306				

Bold indicate the mean value of the probability for each region

- Venice (with 61 museums and a predicted probability of 0.533) in the Veneto region, which in 2019 launched the Civil Service project called *Civic Museums and the Municipality of Venice: the inventory and digitalization of heritage to communicate and design*;
- Naples (with 89 museums and a predicted probability of 0.513) in the Campania region. Note that Naples, capital of the international community of museums and cultural centers, has launched in september 2021 the platform for digital use named *EDI - Global Forum on Education and Integration*, in order to create an important collective experience of exchange and synergies for supporting the educational and integration function of art, through the use of technological intelligence.

6 Concluding remarks

The Italian museum system is characterized by a large and geographically well-distributed heritage. In an era in which the success of organizations is determined by the degree of digitalization in all activities, it is interesting to evaluate the prominent factors which might stimulate the propensity of Italian museums to invest in digital technologies.

In this context, a multilevel binary logit model aimed at modeling the probability for the museums to introduce digital innovations and to define the specific determinants which inspire to invest in digital innovations (including also some forms of digitalization, even mild) was first implemented. Then a multilevel multinomial ordered logit model was provided, in order to estimate the probability for the museum of having different levels of digitalization (graduated in increasing order). For both steps, three levels were considered: museums (units of level 1), provinces (units of level 2) and regions (units of level 3).

From the first step, it was highlighted that the predicted probability to move towards digital innovations depends on the individual covariates "Access", "Research activities", "Guided tours", "Exhibition space", "Partnership", "Type of institution", "Percentage of Italians vs Foreigners", as well as on the contextual factors "Number of tourist accommodation establishments", "Expenditure for recreation, culture and religion" and "Gross domestic product per capita". In particular, apart from the covariate " Access", which produces a negative repercussion on the probability to introduce avant-garde digital innovations, the other above mentioned covariates have a positive impact on the probability to include digital innovations. The selected covariates was used also for assessing their effects on the probability of capturing ever-increasing levels of digitalization for museums and it was emphasized that their incidence is always higher in the case of high digitalization than for medium/low digitalization. However, it is interesting to underline the peculiar influence of the covariate "Guided tours", which produces a strong dampening impact on the medium/low digitalization, on the other hand it generates a major positive incidence on the high level of digitalization. Other covariates ("Medium exhibition space", "Partnership") highlight the same dampening effect on the medium/low digitalization, but the corresponding entity is toned down. In addition, from the estimates of the multinomial ordered logit model, it was underlined that it is much more likely that museums in Italy are willing to adopt a medium/low level of digitalization (with predicted values ranging from 0.495 to 0.714) than a high level of digitalization (with estimated values ranging from 0.141 to 0.495). By considering the regional-level, among the Italian regions with the greatest cumulative estimated probability of adopting at least a medium/low level of digital innovations, besides to the well-known regions characterized by a major number of structures (ranging from 201 to 553 museums), such as Lazio, Tuscany, Emilia-Romagna, Trentino-Alto Adige, Lombardy and the islands of Sardinia and Sicily, there are also Umbria, Apulia and Basilicata, with fewer than 170 museum structures in their territories.

This is also confirmed by focusing on the provincial-level, indeed among the Italian provinces with the greatest cumulative estimated probability of adopting at least a medium/low level of digitalization, it is possible to cite Lecco in the region of Lombardy; Brindisi in the region of Apulia; Ferrara in the region of Emilia-Romagna and Matera in the region of Basilicata. On the contrary, Aosta Valley is the province with a slightly lower cumulative estimated probability of investing in at least a medium/low level of digitalization.

Moreover, the biggest predicted probability of having especially high digitalization is observed for the provinces of Milan, Mantova, Cremona and Como in the region of Lombardy, as well as for Ferrara in the Emilia-Romagna region, Venice in the Veneto region, Naples in the Campania region, Prato in the Tuscany region, and Rome in the Lazio region.

The obtained empirical findings are useful to support optimal management strategies aimed at investments in increasingly digital innovations, in order to promote and consolidate the image of the museums and to improve their performance and attractiveness. This is especially important for the current historical context in which the COVID-19 pandemic has shown the vulnerability of the museum system, and this could push museums to move towards high levels of digitalization.

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