



Quality, trade unions and selection: the impact of unionization

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Abstract

This paper investigates how changes in the bargaining power of trade unions affect firm-level innovation through their impact on quality investment, within a model of heterogeneous firms. In a closed economy setting, we find that an increase in union bargaining power reduces average quality investment at the industry level. However, the effect is heterogeneous across firms: more productive firms decrease their investment in quality, while less productive firms increase it. These findings are confirmed in an open economy context, where we also find that stronger union power reduces the quality of exported goods but leads to an increase in the quality of imported goods.

Keywords Firm heterogeneity · Monopolistic competition · Quality upgrading · Trade unions

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1 Introduction

Innovation is a fundamental driver of economic growth and long-term prosperity. It manifests in various forms, from the development of new technologies that lower pro-

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duction costs to the creation of new products and the adaptation of existing ones to better meet consumer needs. While extensive research has identified the institutional and market conditions that foster innovation—including competition, intellectual property rights, and international trade—much less is known about how labour market institutions shape firms' incentives to innovate.¹ This paper contributes to this literature by investigating the role of trade union activity in firm's innovation activities.

Unionization remains a central institution in the labour market across many developed and developing economies. In 2013, an average of 17% of employees in OECD countries were union members, while 33% were covered by collective bargaining agreements (OECD 2017). However, union density varies widely across countries and industries—for instance, only 4.5% of employees in Estonia belonged to a union, while this share increased to 65% in Sweden, Denmark and Finland and 92% in Iceland. Unions are especially prevalent in sectors such as manufacturing, construction, mining, education, and health. The heterogeneous presence of unions across countries and sectors raises the question of whether unionization favours or hinders innovation. This is the central question we address in this paper.

This paper explores how firm-level union bargaining affects firms' incentives to invest in innovation, where innovation takes the form of quality upgrading, in a right to manage model with heterogeneous firms. Existing theoretical models typically rely on representative firms or partial equilibrium settings and focus on average effects. By contrast, a large literature in international trade and industrial organization document the existence of substantial differences in productivity across firms within an industry.² These differences imply that changes in union bargaining power may affect innovation incentives very differently across firms, with important consequences for average innovation.

Our paper makes three main contributions. First, we develop a general equilibrium heterogeneous-firm model that combines firm-level wage bargaining with endogenous quality upgrading in the Melitz and Ottaviano (2008) framework of monopolistic competition with variable mark-ups. To the best of our knowledge, this is the first model to integrate these elements. Second, we identify a novel mechanism through which stronger unions affect innovation. Rather than influencing all firms uniformly, greater union bargaining power changes wages, prices and mark-ups asymmetrically across firms, reallocating market shares and affecting innovation incentives differently according to firms' productivity. These differences are not only in terms of size but also in terms of sign. This result arises from the presence of variable mark-ups—a characteristic that aligns well with empirical evidence—, which emerge endogenously from non-constant demand elasticities.³ Within this context, unionization affects wages and,

¹ The literature investigating the impact of competition on innovation is extensive. For a general overview see Griffith and Van Reenen (2021). Intellectual property rights are central in early contributions on endogenous growth literature Romer (1990), Grossman and Helpman (1991), Helpman (1993). See Migueles Chazarreta and Stepanok (2025) for a more recent contribution on this literature. The impact of trade openness on innovation has also been widely explored. See Grossman and Helpman (1991), Rivera-Batiz and Romer (1991) and Peretto (1999). Among the recent contributions see Atkeson and Burstein (2005).

² See the literature on heterogeneous firms that has been developed since the seminal paper by Melitz (2003).

³ See Mayer et al. (2021).

consequently, prices and mark-ups, leading to shifts in market shares across firms. These reallocations in turn influence firms' innovation incentives. Third, we show that these heterogeneous responses are amplified in an open economy, where changes in union bargaining power also affect international competitiveness and the allocation of innovation across firms and countries.

Our model features two sectors: a homogeneous sector and a differentiated sector. In the differentiated sector, each firm produces a unique variety under monopolistic competition and may invest in quality improvements. Both goods are produced using a unique production factor, labour. Workers are unionized in the differentiated sector and wages are determined through a firm-level bargaining process between the company and its union, allowing wage heterogeneity across firms. This firm-level bargaining process is in line with the trends in unionization observed from 1980s onwards in many OECD countries, which have moved from sector/industry collective bargaining to firm/establishment negotiations.⁴ In our framework, more productive firms will pay higher wages, consistent with the empirical evidence.⁵ We explore the implications of increased union bargaining power in both closed and open economy settings, and we investigate how different firms respond heterogeneously in terms of quality upgrading decisions to changes in unions' bargaining power.

Our analysis delivers several results. In a closed economy, an increase in an industry's union strength, reduce average R&D expenditures and quality investment in the industry, but this aggregate effect masks substantial heterogeneity. The result of the wage negotiation between firms and workers will lead to an increase in wages that is disproportionately higher in the most productive firms. More productive firms have a smaller wage elasticity of labour demand, which induce their unions to push up wages further. More productive firms reduce investment in quality because larger wage increases compress their mark-ups and reduce their market shares. Less productive firms, in contrast, benefit from the reallocation of demand and may increase their investment in quality. Average quality nevertheless declines because the reduction among the most productive firms dominates the increase among the least productive ones.

These results are reinforced in an open economy scenario where we consider the case of two countries (Home and Foreign), where firms can serve each other's market by incurring iceberg trade costs and decide how much to invest in quality that is market-specific. Importantly, production for the domestic and export markets takes place at separate establishments, a modeling choice that is analytically convenient as it allows to find closed-form solutions. Wage negotiation also occurs at the establishment/profit-centre level which allows wages to differ within the firm depending on the establishment as in Montagna and Nocco (2013) and Chang et al. (2023).

The same type of mechanism operates in an open economy. An increase in a country's union bargaining power will reduce the quality that the most productive firms offer in the domestic market while it will increase the one offered by the least productive ones. Additionally, quality investment for the foreign market will unambiguously

⁴ Examples of countries in which collective bargaining is at the establishment level are USA, Japan and Korea (Chang et al. 2023).

⁵ See Idson and Oi (1999), Card et al. (2013), Carlson et al. (2016) among others.

decrease following the increase in the union's strength. By contrast, foreign firm's investment in quality for the domestic market will increase. The increase in the bargaining power of the unions pushes up domestic wages creating a competitive disadvantage for the domestic firms in both the domestic and the foreign market. The latter effect induces them to lose market share and reduce their incentives to invest in quality for the foreign market. The gain in market share enjoyed by the foreign firms in the home market will induce them to invest more. The total investment in quality of the home market firms will increase or decrease depending on their productivity with the quality of the most productive ones decreasing on average.

Our paper contributes to four strands of the literature. First, it relates to the literature on unions and innovation. This relationship has been widely explored theoretically although not in the context of heterogeneous firms models in a general equilibrium setting, one of the main novelties of the paper.⁶ Existing theoretical contributions have emphasised hold-up problems (Grout 1984; Haucap and Wey 2004) or distinguished between labour-saving and labour-augmenting innovations (Desmet and Parente 2002; Lommerud et al. 2006).⁷ Unlike these studies, we focus on quality upgrading, which increases product demand rather than reducing labour requirements and, therefore, it does not pose a direct threat to workers.⁸ Nevertheless, we show that stronger unions still reduce average innovation, indicating that the adverse effect of unionization does not depend exclusively on the labour-saving nature of technological change or the existence of hold-up problems. In our work, an increase in the unions' bargaining power still reduces on average the firm's investment in quality, even if the hold-up problem is absent.

Second, our paper contributes to the literature on heterogeneous firms and labour market institutions.⁹ Previous work has incorporated unions into models of trade with firm heterogeneity to study firm selection, internationalisation and employment (Eckel and Egger 2009; Montagna and Nocco 2013, 2015; De Pinto and Michaelis 2019; Chang et al. 2023). Eckel and Egger (2009) show that, in a right-to-manage model, stronger local unions affect firms' incentives to relocate production abroad and, consequently, the share of multinational firms. Closer to our paper, Montagna and Nocco (2013, 2015) examine the impact of unionization on firm selection in heterogeneous firm models. In both cases, they show that an increase in domestic unions' bargaining power weakens firm selection in the domestic market and toughens selection for

⁶ See Menezes-Filho and Van Reenen (2003) for a survey of the theoretical and empirical literature.

⁷ See also Gust and Marquez (2004), Lommerud and Straume (2012), Bryson and Dale-Olsen (2021).

⁸ This is also a result of the fact that we are using only one type of labour engaged in production. Quality uses as input the homogenous good for simplicity, and indirectly non-unionized workforce. While the assumption helps us to keep tractability in the model, we recognize that there is substantial evidence suggesting that quality is correlated with skill intensity and it may induce skill upgrading (Bustos (2011), Verhoogen (2008)). Kugler and Verhoogen (2012) finds instead a positive correlation between quality and the use of skilled workforce. Considering different types of labour and different union structures to investigate the impact of unions on innovation could be an interesting avenue for future research. We thank an anonymous referee for pointing this out to us.

⁹ For the analysis of the effects of other types of labour market imperfections see the discussion of the literature in Montagna and Nocco (2013, 2015).

domestic exporters.¹⁰ De Pinto and Michaelis (2019) explores the impact of unionization on selection in a two-symmetric-country open economy model when the unions' bargaining power is firm-specific. Chang et al. (2023) analyses within the Melitz and Ottaviano (2008) framework the impact of unionization on unemployment considering that the differentiated sector is unionized while searching frictions are present in the homogeneous good sector. However, innovation is absent in all these models. By introducing endogenous quality upgrading, we show that union bargaining affects not only firm selection, having an impact on average productivity but also the allocation of innovation across firms, with further implications for average productivity and international competitiveness.¹¹

Third, our results help interpret the mixed empirical evidence on unions and innovation. Earlier studies generally found a negative relationship between unionization and innovative activity (Allen 1988; Hirsch 1991, 1992; Menezes-Filho and Van Reenen 2003), whereas more recent work reports effects that vary across countries, innovation outcomes and institutional settings. Using regression discontinuity design and union elections to control for endogeneity, Bradley et al. (2016) finds that an increase in unionization results in a decrease in both patent volume and patent quality for the case of the USA. Similar results have been found for the case of investment in physical capital for a panel of OECD countries from 1980–2000 (Cardullo et al. (2020)). In contrast, Bryson and Dale-Olsen (2021) found that for the case of the UK and Norway, union power increases product innovation. Finally, Cetrulo et al. (2025) finds for the case of Italy that while there is a non-statistically significant association between firm-level collective bargaining and investment in R&D at the extensive margin, the interaction of firm-level collective bargaining with the degree of unionization at the firm level becomes positive and statistically significant in affecting the likelihood of R&D investments. Our model suggests that considering heterogeneous firm-level responses identify new mechanisms that may help to interpret this mixed empirical evidence.

Finally, our paper is also related to the recent literature that explores the relationship between quality and international trade. Numerous studies explore the determinants of product quality in an international trade environment. Early contributions as Amiti and Khandelwal (2013) and Fan et al. (2015) show that lower trade barriers foster specialisation in higher quality products. More recent contributions have outlined other important factors. Navas et al. (2026) shows in a Heckscher-Ohlin environment how comparative advantage shapes firms' response to international trade in terms of quality investment. Picard and Tampieri (2021) shows in an Armington framework that larger population size also leads countries to specialize in higher quality products when the economies are open to trade. Our work instead analyses another important element that may shape the country's response to trade openness in terms of product quality. Indeed, we show that an increase in the union's strength in a country reduces,

¹⁰ The main difference between both frameworks is that in Montagna and Nocco (2013) the authors use a variable elasticity of demand framework as in Melitz and Ottaviano (2008), while in Montagna and Nocco (2015) they use a constant elasticity of substitution (CES) and implicitly constant mark-ups framework as in Melitz (2003). This has also implications for wages which are heterogeneous across firms in the former, and homogeneous in the latter.

¹¹ We thank an anonymous referee to point us on this direction.

on average, firms' investment in quality, while inducing firms in their trading partners to increase theirs.

The rest of the paper is organized as follows. Section 2 introduces the model and Sect. 3 analyses the impact of unions on firms' investment on quality in the closed economy scenario. Section 4 extends the analysis to an open economy setting and Sect. 5 analyses the impact of unionization on quality investments undertaken by firms active in the domestic and in the foreign markets. Section 6 concludes.

2 The model

2.1 Demand side

Preferences are defined over two goods: good 0 is a homogeneous good while the other good consists of a continuum of differentiated varieties indexed by $i \in \Omega_l$. Consumers derive utility from the consumption of each good and the quality of each variety within the differentiated sector is endogenously determined by the firm's investment in quality choice. The homogeneous good is used as the numeraire of the model.

Specifically, all L_l consumers in country l inelastically supply one unit of work and share the same utility function

$$U_l(q_{0l}^\varepsilon; q_l^\varepsilon(i), i \in \Omega_l) = q_{0l}^\varepsilon + \alpha \int_{i \in \Omega_l} q_l^\varepsilon(i) di + \beta_l \int_{i \in \Omega_l} z_l(i) q_l^\varepsilon(i) di - \frac{1}{2} \gamma \int_{i \in \Omega_l} (q_l^\varepsilon(i))^2 di - \frac{1}{2} \eta \left(\int_{i \in \Omega_l} q_l^\varepsilon(i) di \right)^2, \quad \alpha, \gamma, \eta, \beta_l > 0$$

where $q_l^\varepsilon(i)$ represents the individual consumption of the differentiated variety i and $z_l(i)$ the quality associated with it, while q_{0l}^ε represents the individual consumption of the homogeneous good. The positive parameters γ and β_l , respectively, represents 'love for variety' and 'preference for quality', with the latter potentially varying across countries. As usual, α and η represents the intensity of preferences for the differentiated good with respect to the homogeneous one.¹²

Solving the consumer's utility maximization problem and considering that the homogenous good is the numeraire, the individual demand of variety i in country l is

$$q_l^\varepsilon(i) = \frac{\alpha + \beta_l z_l(i) - p_l(i) - \eta Q_l^\varepsilon}{\gamma} \quad (1)$$

where $p_l(i)$ and $z_l(i)$ are, respectively, the price and the quality of variety i in country l , and $Q_l^\varepsilon = \int_{j \in \Omega_l} q_l^\varepsilon(j) dj$ is the total individual demand of the differentiated varieties.

¹² We assume that the income of consumers with the smallest income (I_l^ε) is big enough so that it covers the consumption of the differentiated varieties with $I_l^\varepsilon > \int_{i \in \Omega_l} p(i) q_l^\varepsilon(i) di$. If that is the case for consumers with the smallest income, that will be also the case for all consumers.

Aggregating over all varieties and considering the average quantity, quality and price in country l , the aggregate demand of variety i in country l , $q_l(i)$, can be expressed as

$$q_l(i) = \frac{\alpha L_l}{\gamma + N_l \eta} + \frac{\beta_l}{\gamma} z_l(i) L_l - \frac{1}{\gamma} p_l(i) L_l + \frac{\eta N_l L_l}{\gamma (\gamma + \eta N_l)} (\bar{p}_l - \beta_l \bar{z}_l)$$

where $\bar{z}_l$ and $\bar{p}_l$, respectively, represent the average quality and price of all N_l varieties sold in country l . Thus, the aggregate demand of a variety increases with its quality and the average price of all substitutable varieties and decreases with its price and the average quality of other differentiated varieties. An important property of this demand system is that the price elasticity of demand is not constant, but it depends on the price charged, and indirectly on the firm's productivity. In particular, the price elasticity of demand for each variety is $\epsilon_{q,p}(i) \equiv \left| \frac{\partial \log q_l(i)}{\partial \log p_l(i)} \right| = \left(\frac{p_l^{\max} + \beta_l z_l(i)}{p_l(i)} - 1 \right)^{-1}$, and, ceteris paribus, it increases with the price and it decreases with the quality¹³ the elasticity also depends negatively on the choke price, $p_l^{\max}$ (i.e. the maximum price the firm can charge for its products), and it becomes larger as $p_l^{\max}$ decreases, in a more competitive environment.¹⁴

2.2 Supply side

Let us first consider the autarky case, in which firms in country l only produce for the domestic market and consumers are able to buy only domestic varieties. Firms in the differentiated good sector behave monopolistically. They produce their variety with a technology that uses labour as a production factor. Quality upgrading is market-specific as discussed above. To improve the quality of an existing variety in a specific market, each firm uses a technology with the homogeneous good as the unique production factor. The total production cost of a firm i producing the quantity $q_{D,l}(i)$ for the domestic market l with unit labour requirement $c(i)$ is represented by

$$TC_{D,l}(i) = w_{D,l}(i) c(i) q_{D,l}(i) + \frac{\theta_l}{2} (z_{D,l}(i))^2$$

where $z_{D,l}(i)$ is the quality of the differentiated good for sales in the domestic market, $w_{D,l}(i)$ is the wage rate paid by firm i , and θ_l is a positive parameter indicating how costly or 'difficult' is in country l to innovate due to its specific characteristics.¹⁵ The cost function indicates that labour is used in the production of the differentiated good while, to invest in quality, the firm must incur in a variable cost $\frac{\theta_l}{2} (z_{D,l}(i))^2$ in terms of the homogeneous good that is invariant to output, but it can be country specific:

¹³ Specifically, for a given quality, $\frac{\partial \epsilon_{q,p}(i)}{\partial p_l(i)} = \frac{p_l^{\max} + \beta_l z_l(i)}{(p_l^{\max} + \beta_l z_l(i) - p_l(i))^2} > 0$.

¹⁴ Notice that $p_l^{\max} \equiv \frac{\alpha \gamma + \eta N_l (\bar{p}_l - \beta_l \bar{z}_l)}{\gamma + \eta N_l}$ that is smaller than in the standard model with no quality upgrading.

¹⁵ For the sake of simplicity, we assume that the cost of quality upgrading is common across firms, even though it could vary across countries in the open economy scenario.

countries with a marked ability to innovate or with a better technology of innovation are characterized by a smaller value of θ_l .

To develop the specific variety they produce, firms incur in a fixed entry cost $f_{E,l}$ in terms of the homogeneous good that is sunk, learning only afterwards how productive their technology is by drawing c from a cumulative distribution, $G(c)$ that is common to all potential entrants in the industry. Notice that, given that all firms with the same cost parameter c are symmetric, the index i will be dropped from now onward, and firms will be indexed by c alone. Once the productivity is revealed, firms decide whether to stay in the market and produce or exit. Then firms and unions bargain over the wage level, and, finally, firms choose the quality of their product and its price. The model is solved by backward induction.

Starting from the final stage, firms choose the price they charge in their (domestic) market l , $p_{D,l}(c)$, and the quality of their product $z_{D,l}(c)$, by maximizing profits obtained from their sales, that is

$$\max_{p_{D,l}(c), z_{D,l}(c)} (p_{D,l}(c) - w_{D,l}(c)c) q_{D,l}(c) - \frac{\theta_l}{2} (z_{D,l}(c))^2$$

s.t.

$$q_{D,l}(c) = \frac{\alpha L_l}{\gamma + N_l \eta} + \frac{\beta_l}{\gamma} z_{D,l}(c) L_l - \frac{1}{\gamma} p_{D,l}(c) L_l + \frac{\eta N_l L_l}{\gamma (\gamma + \eta N_l)} (\bar{p}_l - \beta_l \bar{z}_l) \quad (2)$$

The first order condition associated with the price, $p_{D,l}(c)$, yields the optimal quantity

$$q_{D,l}(c) = \frac{L_l}{\gamma} (p_{D,l}(c) - w_{D,l}(c)c) \quad (3)$$

Rearranging terms, we find that the optimal price is given by:

$$p_{D,l}(c) = \frac{\gamma}{L_l} q_{D,l}(c) + w_{D,l}(c)c \quad (4)$$

The first order condition for the optimal quality, $z_{D,l}(c)$, yields the following solution

$$z_{D,l}(c) = \frac{\beta_l L_l}{\theta_l \gamma} (p_{D,l}(c) - w_{D,l}(c)c) \quad (5)$$

Indeed, when firms decide how much to invest in quality they consider that an increase in the investment in quality, will increase their demand by $\frac{\beta_l L_l}{\gamma}$ units for which they will obtain an extra profit of $\frac{\beta_l L_l}{\gamma} (p_{D,l}(c) - w_{D,l}(c)c)$. The firm will invest in quality by equating the marginal profit to the marginal cost, which is given by $\theta_l z_{D,l}(c)$.

Note that substituting (4) into (5) we find that

$$z_{D,l}(c) = \frac{\beta_l}{\theta_l} q_{D,l}(c) \quad (6)$$

Equation (6), therefore, reveals the traditional complementarity between market size and R&D investment found in early innovation models (Aghion and Howitt (1992), Peretto (1999) and more recently Navas and Licandro (2011) and Impullitti and Licandro (2018)). Note that, holding constant firm's output, the demand for quality increases when the preference for quality is stronger (i.e. larger β_l) and decreases with the cost of quality improving, (i.e. smaller θ_l).

Substituting (3) and (5) into the profit function yields the maximized profits from domestic sales, net of investment costs for quality upgrading, that are given by

$$\pi_{D,l}(c) = \frac{\gamma}{L_l} \left(\frac{2\gamma\theta_l - \beta_l^2 L_l}{2\gamma\theta_l} \right) (q_{D,l}(c))^2 \quad (7)$$

2.3 Unions

Workers employed in the monopolistic competitive sector are unionized, and their wage is determined by a bargaining process. Following Montagna and Nocco (2013) and Chang et al. (2023) we assume that the wage is determined by decentralization, where each union meets with each firm with unit input requirement c to determine the wage $w_{D,l}(c)$ at the profit-center. We also consider that the resulting wage is the Nash-bargain solution where wage maximizes a weighted sum of the union's and firm's surplus. In particular, denoting $v_l \in (0, 1]$ the bargaining power of unions in country l , the equilibrium wage corresponds to the solution of the following Nash-bargaining problem

$$\max_{w_{D,l}(c)} v_l \ln [(w_{D,l}(c) - 1)l_{D,l}(c)] + (1 - v_l) \ln \pi_{D,l}(c) \quad (8)$$

where $(w_{D,l}(c) - 1)l_{D,l}(c)$, represents the union's surplus and $\pi_{D,l}(c)$ represents the firm's surplus. The first element is composed of two terms, the first one is the unionized worker's extra rents and this comes from the difference between the bargained wage $w_{D,l}(c)$ and the reservation wage that workers could obtain if employed in the homogeneous sector (i.e. $w_0 = 1$), where workers are non-unionized, and the second term is the level of employment in the differentiated sector, represented by the firm's labour demand, $l_{D,l}(c)$. The firm's surplus is given by the firm's profits as firms do not count on an alternative source of investment.

The firm's labour demand, $l_{D,l}(c)$, given by $q_{D,l}(c)c$, yields

$$l_{D,l}(c) = \frac{L_l}{\gamma} (p_{D,l}(c) - w_{D,l}(c))c$$

Note that the labour demand depends directly on the mark-up of the firm defined in this context as $\mu_{D,l}(c) = (p_{D,l}(c) - w_{D,l}(c))c$. As the mark-up of the firm is part of the union surplus, unions have an incentive to reduce their demands regarding their wage in order to boost employment in the sector. In order to obtain an expression for the mark-up, let us first substitute (6) into the demand function (2) and, rearranging

terms, we obtain that the demand can be rewritten as follows

$$q_{D,l}(c) = \frac{\theta_l L_l}{(\theta_l \gamma - \beta_l^2 L_l)} \left[\frac{\alpha \gamma + \eta N_l (\bar{p}_l - \beta_l \bar{z}_l)}{\gamma + N_l \eta} - p_{D,l}(c) \right] \quad (9)$$

Note that a necessary condition for these preferences to satisfy the law of demand (*i.e.* $\frac{\partial q_{D,l}}{\partial p_{D,l}} < 0$) is that $\theta_l > \frac{\beta_l^2 L_l}{\gamma}$. Our analysis will be focused in this case as this will provide us with interior solutions. The need to impose this parameter restriction comes because of the complementarity between quantity and quality in this framework. An increase in the price of the variety will have two effects in a variety's demand: A direct reduction in the quantity demanded through the standard demand forces and an indirect increase in demand through the increase in the variety's quality as firms will be interested in increasing the quality as the price increases. The condition above guarantees that this second effect is smaller than the first one so the law of demand is satisfied.¹⁶

When expressions (3) and (5) are substituted into the demand function (2) we obtain

$$p_{D,l}(c) - w_{D,l}(c)c = \lambda_{D,l} \left[\frac{\alpha \gamma + \eta N_l (\bar{p}_l - \beta_l \bar{z}_l)}{\gamma + N_l \eta} - w_{D,l}(c)c \right] \quad (10)$$

and, therefore, $q_{D,l}(c)$ in (3) can be rewritten as follows

$$q_{D,l} = \lambda_{D,l} \frac{L_l}{\gamma} \left[\frac{\alpha \gamma + \eta N_l (\bar{p}_l - \beta_l \bar{z}_l)}{\gamma + N_l \eta} - w_{D,l}(c)c \right]$$

where, $\lambda_{D,l} \equiv \frac{\gamma \theta_l}{2\gamma \theta_l - \beta_l^2 L_l}$. This parameter, novel to the analysis with quality improving, shapes the elasticity of demand with respect to the firm's wage. When the optimal investment in quality is equal to zero, either because $\beta_l = 0$ or θ_l tends to infinity, this parameter will take the value of $\frac{1}{2}$. As β_l increases or θ_l decreases, $\lambda_{D,l}$ increases. Thus, $\lambda_{D,l}$ can be interpreted as a measure of the *profitability of investing in quality upgrading*. Hence, in environments in which it is more profitable to invest in quality, the demand of a variety increases, and therefore the labour demand of a firm becomes more sensitive to changes in wages. This parameter is also upper bounded by the value of 1 (*i.e.* $\frac{1}{2} \leq \lambda_{D,l} < 1$).¹⁷

¹⁶ Substituting quality from (5) in (2), the demand becomes $q_{D,l}(c) = \frac{\alpha L_l}{\gamma + N_l \eta} + \frac{\beta_l^2 L_l^2}{\theta_l \gamma^2} (p_{D,l}(c) - w_{D,l}(c)c) - \frac{L_l}{\gamma} p_{D,l}(c) + \frac{\eta N_l L_l}{\gamma(\gamma + \eta N_l)} (\bar{p}_l - \beta_l \bar{z}_l)$, from which $\frac{\partial q_{D,l}}{\partial p_{D,l}} = \frac{\beta_l^2 L_l^2}{\theta_l \gamma^2} - \frac{L_l}{\gamma} = \frac{\beta_l^2 L_l^2 - \theta_l \gamma L_l}{\theta_l \gamma^2}$, that is negative when $\frac{\beta_l^2 L_l}{\gamma} < \theta_l$. Note that $-\frac{L_l}{\gamma}$ corresponds to the direct effect of prices on demand, while $\frac{\beta_l^2 L_l^2}{\theta_l \gamma^2}$ corresponds to the indirect effect on quality.

¹⁷ Note that when quality upgrading is not profitable either because $\beta_l = 0$ or θ_l tends to infinity, $\lambda_{D,l} = 1/2$. By contrast, when θ_l takes the minimum value for a solution to be interior, $\theta_l = \frac{\beta_l^2 L_l}{\gamma}$, $\lambda_{D,l} = 1$.

Hence, making use of (3) and (9), firm's labour demand can be rewritten as

$$l_{D,l}(c) = \frac{\theta_l L_l}{2\gamma\theta_l - \beta_l^2 L_l} \left[\frac{\alpha\gamma + \eta N_l (\bar{p}_l - \beta_l \bar{z}_l)}{\gamma + N_l \eta} - w_{D,l}(c)c \right] c \quad (11)$$

Substituting (7) into (8), and considering the labour demand (11), the bargaining problem becomes¹⁸

$$\max_{w_{D,l}(c)} v_l \ln(w_{D,l}(c) - 1) + v_l \ln(p_{D,l}(c) - w_{D,l}(c)c) + 2(1 - v_l) \ln(p_{D,l}(c) - w_{D,l}(c)c)$$

s.t.

$$p_{D,l}(c) - w_{D,l}(c)c = \lambda_{D,l} \left(\frac{\alpha\gamma + \eta N_l (\bar{p}_l - \beta_l \bar{z}_l)}{(\gamma + N_l \eta)} - w_{D,l}(c)c \right)$$

The first order condition is

$$\frac{v_l}{w_{D,l}(c) - 1} - v_l \frac{\lambda_{D,l} c}{p_{D,l}(c) - w_{D,l}(c)c} - \frac{2(1 - v_l) \lambda_{D,l} c}{p_{D,l}(c) - w_{D,l}(c)c} = 0$$

The first order condition above suggests that the equilibrium wage is the one in which the surplus gains of increasing the wage equates the costs of doing it. The first term represents the gain in worker's rents when the wage is increased. The second and the third term represent the losses of an increase in the wage. Specifically, the second term is the cost the union faces in terms of employment (as an increase in wage will reduce the firm's labour demand), while the third term captures the loss in profits, when the wage rises. These last two terms are more negative for the least productive firms (higher c).¹⁹ As firms become more productive (lower c) their goods demand becomes less sensitive to changes in prices, as explained at the end of section 2.1, and their labour demand becomes less sensitive to changes in wages, as explained below. The latter implies that as a result of wage bargaining more productive firms are expected to pay higher wages compared to the least productive ones. This result is consistent with the empirical evidence presented in Barth et al. (2020) for the case of Norwegian firms for which they find that stronger unions lead to a larger wage premium and this premium increases with the productivity of the firm. Note as well that the last two terms are also weighted by the profitability of investing in quality $\lambda_{D,l}$. This implies, for instance, that in those industries in which investing in quality is more profitable, unions will moderate their demand for higher wages as the costs of rising the wage in terms of employment become larger.

¹⁸ Notice that the sole relevant terms in the Nash maximand have been considered.

¹⁹ Note that ceteris paribus, $\frac{\partial \left(v_l \frac{\lambda_{D,l} c}{p_{D,l}(c) - w_{D,l}(c)c} \right)}{\partial c} = \frac{\lambda_{D,l} \frac{\gamma\theta_l}{2\gamma\theta_l - \beta_l^2 L_l} \frac{\alpha\gamma + \eta N_l (\bar{p}_l - \beta_l \bar{z}_l)}{(\gamma + N_l \eta)}}{(p_{D,l}(c) - w_{D,l}(c)c)^2} > 0$, and $\frac{\partial \left(\frac{2(1-v_l)\lambda_{D,l} c}{p_{D,l}(c) - w_{D,l}(c)c} \right)}{\partial c} = \frac{2(1-v_l)\lambda_{D,l} \frac{\gamma\theta_l}{2\gamma\theta_l - \beta_l^2 L_l} \frac{\alpha\gamma + \eta N_l (\bar{p}_l - \beta_l \bar{z}_l)}{(\gamma + N_l \eta)}}{(p_{D,l}(c) - w_{D,l}(c)c)^2} > 0$.

Rearranging terms gives the wage paid by a firm producing with unit labour requirement c , that is

$$w_{D,l}(c) = 1 + \frac{v_l}{(2 - v_l)\lambda_{D,l} + v_l} \left(\frac{p_{D,l}(c)}{c} - 1 \right) \quad (12)$$

Note that the equilibrium wage is composed of two terms. The first term of the right hand side is the workers' reservation wage, which is given by the one obtained in the non-unionized sector ($w_0 = 1$). The second term is a proportion of the firm's price markup evaluated at the reservation wage, $(p_{D,l}(c) - c)/c$. It is also important to notice that, *ceteris paribus*, wages in this framework are smaller than in an environment where quality upgrading is not possible like in Montagna and Nocco (2013), and an increase in $\lambda_{D,l}$ will reduce wages.²⁰ This result is linked to the impact of quality improvement on labour demand and, consequently, on employment in the differentiated sector, as discussed above. When firms can invest in quality improvement, an increase in the wage may reduce the firm's market size, reducing the incentives to quality upgrading having a more negative impact on employment. Both unions and firms take this into account when setting the firm's wage.

It can be readily derived from (7) that a firm will have positive profits as long as $q_{D,l} > 0$. From equation (9), we obtain that this is the case if and only if $p_l^{\max} \equiv \frac{\alpha\gamma + \eta N_l(\bar{p}_l - \beta_l \bar{z}_l)}{\gamma + \eta N_l} > p_{D,l}(c)$. Thus, there must exist a threshold level of the unit labour requirement $c_{D,l}$ such that $q_{D,l} = 0$, which corresponds to $p_{D,l}(c_{D,l}) = p_l^{\max}$. From equation (4) we have that the price set by marginal firms producing with unit labour requirement $c_{D,l}$ is $p_{D,l}(c_{D,l}) = w_{D,l}(c_{D,l})c_{D,l}$, and equation (5) reveals that these marginal firms do not invest in quality given that $z_{D,l}(c_{D,l}) = 0$. Then, the wage rule (12) implies that for marginal firms with $c_{D,l}$, the wage is equal to that obtained by non-unionized workers in the homogeneous good sector

$$w_{D,l}(c_{D,l}) = 1$$

The last expression implies that in equilibrium $p_{D,l}(c_{D,l}) = c_{D,l} = p_l^{\max}$. It is important to point out that $c_{D,l}$ marks a survival threshold. When $c > c_{D,l}$ the minimum price the firm can charge $p(c) > p_l^{\max}$ for which there is no positive demand as its marginal cost is too high (i.e. its productivity, $1/c$, is too low). Therefore, the demand function (2) can be rewritten as follows

$$q_{D,l}(c) = \frac{L_l}{\gamma} (c_{D,l} + \beta_l z_{D,l}(c) - p_{D,l}(c)),$$

This can be used together with expression (3) to obtain

$$p_{D,l}(c) = \frac{c_{D,l} + w_{D,l}(c)c + \beta_l z_{D,l}(c)}{2} \quad (13)$$

²⁰ From Montagna and Nocco (2013), we know that the wage equation in an environment in which quality upgrading is not allowed is given by, $w_{D,l}(c) = 1 + \frac{2v_l}{(2+v_l)} \left(\frac{p_{D,l}(c)}{c} - 1 \right)$. This is the wage equation that this framework will take when quality improvement is non-profitable (i.e. $\lambda_{D,l} = \frac{1}{2}$).

Substituting (13) into the wage rule (12), the wage can be rewritten as

$$w_{D,l}(c) = 1 + \frac{v_l}{2(2-v_l)\lambda_{D,l} + v_l} \left(\frac{c_{D,l} - c}{c} \right) + \frac{\beta_l v_l z_{D,l}(c)}{[2(2-v_l)\lambda_{D,l} + v_l]c} \quad (14)$$

Expression (14) shows that more productive firms, with smaller c , pay higher wages. Specifically, the second term reflects the higher wage that can be extracted from more productive firms, while the third term represents the contribution that can be ascribed at the higher quality of the produced good. This last term comes because quality allows firms to charge a higher price and then obtain a higher mark-up. Substituting (14) in the price rule (13), rearranging terms and substituting the optimal expression for $z_{D,l}(c)$ from (5), the mark-up can be rewritten as

$$p_{D,l}(c) - w_{D,l}(c)c = \frac{(2-v_l)\lambda_{D,l}}{2} (c_{D,l} - c) \quad (15)$$

As in Montagna and Nocco (2013) with no quality upgrading, the firm's mark-up is a decreasing function of the workers' bargaining power v_l and this effect is stronger for the most productive firms. Novel in the current framework, this effect is stronger in a more intensive innovation environment (i.e. larger $\lambda_{D,l}$).²¹ Ceteris paribus, more productive firms charge higher mark-ups and the mark-ups are larger when the profitability of quality investment increases.

Expression (15) can be used to obtain the final expression for the firm variables in (3), (5) and (7). Specifically, the quality is

$$z_{D,l}(c) = \frac{(2-v_l)\beta_l L_l}{2(2\gamma\theta_l - \beta_l^2 L_l)} (c_{D,l} - c) \quad (16)$$

where the final expression is obtained substituting the definition of $\lambda_{D,l}$. This is the expression for the investment in quality. It shows that a firm's quality choice decreases with the unit labour requirement c (i.e. more productive firms invest more in quality) and, ceteris paribus, for a given cutoff $c_{D,l}$, it increases with market size as $\frac{\partial z_{D,l}(c)}{\partial L_l} > 0$; it decreases with the cost of adoption θ_l , it increases with the preference for quality, β_l , and, more importantly, it falls with the union's bargaining power v_l . It is also important to realize that the last effect is also stronger for the most productive firms.

Substituting $z_{D,l}(c)$ from (16) into expression (14), the wage can be rewritten as

$$w_{D,l}(c) = 1 + \frac{v_l}{2} \left(\frac{c_{D,l} - c}{c} \right) \quad (17)$$

For a given cutoff $c_{D,l}$, any increase in the union's bargaining power v_l results in a direct increase in the wage $w_{D,l}(c)$ that is not homogeneous across firms, being larger

²¹ Note that, for a given $c_{D,l}$, $\frac{\partial(p_{D,l}(c) - w_{D,l}(c)c)}{\partial v_l} = -\frac{\lambda_{D,l}(c_{D,l} - c)}{2} < 0$, and that the fall is stronger the larger $\lambda_{D,l}$, (i.e. the larger the profits from quality upgrading) and the larger $(c_{D,l} - c)$. This means that the fall is stronger for the most productive firms.

for more productive firms (i.e. with lower c).²² Therefore, we state the following lemma:

Lemma *For a given cutoff $c_{D,l}$, the percentage increase in wages due to a percentage increase in the union's bargaining power v_l is larger for more productive firms (i.e. with lower c).*

Notice that this result depends on the fact that more productive firms have a more inelastic labour demand. In particular, the elasticity of labour demand $\epsilon_{l,w} \equiv \left| \frac{dl_{D,l}(c)}{dw_{D,l}(c)} \frac{w_{D,l}(c)}{l_{D,l}(c)} \right| = \frac{c}{(2-v_l)\lambda_{D,l}(c_{D,l}-c)} + \frac{v_l}{2} \frac{1}{(2-v_l)\lambda_{D,l}}$ is smaller for more productive firms, as $\frac{\partial \epsilon_{l,w}}{\partial c} = \frac{c_{D,l}}{(2-v_l)\lambda_{D,l}[(c_{D,l}-c)]^2} > 0$.²³ Hence, given that a more productive firm has a more inelastic labour demand, this gives its union a larger incentive to push up wages.

Making use of (15) and (17), gives the price set by a firm with unit labour requirement c , that is

$$p_{D,l}(c) = c + \frac{[v_l + (2 - v_l)\lambda_{D,l}](c_{D,l} - c)}{2} \quad (18)$$

Note that firms charge higher prices when quality investment is possible and the prices increase with the profitability of quality investment $\lambda_{D,l}$. Moreover, for a given $c_{D,l}$, $\frac{\partial(p_{D,l}(c))}{\partial v_l} = \frac{(1-\lambda_{D,l})(c_{D,l}-c)}{2} > 0$, and the price increase with v_l , with the increase that is larger the smaller $\lambda_{D,l}$, and the larger $(c_{D,l} - c)$. This means that the increase in prices is larger for the most productive firms. Finally, substituting (15) in (3) and (7), respectively, gives the quantity produced by a firm with unit labour requirement c

$$q_{D,l}(c) = \frac{(2 - v_l)\lambda_{D,l}L_l}{2\gamma} (c_{D,l} - c) = \frac{(2 - v_l)\theta_l L_l}{2(2\gamma\theta_l - \beta_l^2 L_l)} (c_{D,l} - c), \quad (19)$$

²² To see this, note that holding constant $c_{D,l}$ and differentiating $w_{D,l}(c)$ in (17) yields

$$\frac{d \ln w_{D,l}(c)}{d \ln v_l} = \frac{v_l}{2} \frac{\left(\frac{c_{D,l}-c}{c}\right)}{w_{D,l}(c)} = \frac{v_l \mu_l}{2 + v_l \mu_l} > 0$$

where $\mu_l = \frac{c_{D,l}-c}{c}$, showing that an increase in union's bargaining power v_l increases wages $w_{D,l}(c)$ for all firms. Moreover, since $\frac{d^2 \ln w_{D,l}(c)}{d \ln v_l d \mu_l} > 0$ and $\frac{d \mu_l}{dc} < 0$, the impact on the wage is stronger the most productive (i.e. lower c) the firm is (i.e. $\frac{d^2 \ln w_{D,l}(c)}{d \ln v_l dc} = \frac{d \ln w_{D,l}(c)}{d \ln v_l d \mu_l} \frac{d \mu_l}{dc} < 0$).

²³ Specifically, the elasticity of labour demand can be computed as follows. Making use of (13), we know that $\frac{dp_{D,l}(c)}{dw_{D,l}(c)} = \frac{c}{2}$ and from labour demand $l_{D,l}(c) = \frac{L_l}{\gamma} (p_{D,l}(c) - w_{D,l}(c)c)$ we can compute $\frac{dl_{D,l}(c)}{dw_{D,l}(c)} = \frac{L_l c}{\gamma} \left[\frac{dp_{D,l}(c)}{dw_{D,l}(c)} - c \right] = -\frac{L_l c^2}{2\gamma}$. Then, the elasticity of labour demand facing the union with respect to the wage is $\epsilon_{l,w} \equiv \left| \frac{dl_{D,l}(c)}{dw_{D,l}(c)} \frac{w_{D,l}(c)}{l_{D,l}(c)} \right| = \frac{c}{2} \frac{w_{D,l}(c)}{(p_{D,l}(c) - w_{D,l}(c)c)}$, which, making use of (15) and (17) can be rewritten as in the text.

the firm's level of employment is given by

$$l_{D,l}(c) = \frac{(2 - v_l) \lambda_{D,l} L_l}{2\gamma} (c_{D,l} - c) c,$$

and its profits are

$$\pi_{D,l}(c) = \frac{(2 - v_l)^2 \theta_l L_l}{8(2\gamma\theta_l - \beta_l^2 L_l)} (c_{D,l} - c)^2 \quad (20)$$

Holding constant the cutoff $c_{D,l}$, an increase in union's bargaining power v_l , reduces quantities, employment in the differentiated sector and profits for all firms as, respectively, $\frac{d \ln q_{D,l}(c)}{d \ln v_l} = -\frac{v_l}{(2-v_l)} < 0$, and $\frac{d \ln \pi_{D,l}(c)}{d \ln v_l} = -\frac{2v_l}{(2-v_l)} < 0$. Moreover, these effects are stronger for the most productive firms as $c_{D,l} - c$ is larger there. This direct effects of the increase in the bargaining power of the unions can be completed by the inclusion of the indirect effects that act through changes in the cutoff $c_{D,l}$ that will be determined in the next subsection.

2.4 Threshold cut-off

The survival threshold cut-off is determined by the standard free entry condition that ensures that in equilibrium the expected profits from entry are equal to the entry fixed costs

$$\int_0^{c_{D,l}} \pi_{D,l}(c) dG_l(c) = f_{E,l} \quad (21)$$

Focussing on the case of the Pareto distribution with

$$G_l(c) = \left(\frac{c}{c_{M,l}} \right)^k \quad \text{with } c \in (0, c_{M,l}] \text{ and } k \geq 1,$$

where $c_{M,l}$ represents the maximum realization of c in country l , and k is the shape parameter of the Pareto distribution, allows to obtain a closed form solution. Substituting (20) into (21), yields the cutoff

$$c_{D,l} = \left[\frac{4\gamma (k+2) (k+1) c_{M,l}^k f_{E,l}}{\lambda_{D,l} L_l (2 - v_l)^2} \right]^{\frac{1}{k+2}} \quad (22)$$

As in Montagna and Nocco (2013) the survival threshold $c_{D,l}$ increases with the union's bargaining power, v_l , softening selection. This result comes from the asymmetric impact on wages of an increase in the bargaining power of the union, discussed above, which hurts the most productive firms the most. Notice that this mechanism is

weaker when quality upgrading is introduced.²⁴ Indeed, our model predicts that the larger the profitability to invest in quality upgrading, $\lambda_{D,l}$, the weaker the softening selection effect of unionization. In our framework, the increase in wages as a result of an increase in the union's strength is smaller than in an environment where quality improvement is not allowed, and an increase in $\lambda_{D,l}$ will reduce wages (i.e. equations (14) and (17)). As the increase in wages is smaller, the disadvantage introduced to the most productive firms is smaller. Moreover, new to this context is the impact of variables that affect the profitability of quality upgrade. Indeed, a quick inspection of (22) reveals that the survival threshold is smaller in an environment in which firms can quality upgrade and this reduction is stronger when the profitability of investing in quality upgrading represented by $\lambda_{D,l}$ increases. Where the profitability of innovation is higher, firms will invest more in quality upgrading with the most productive firms investing more. This asymmetric increase in profit opportunities reallocates market shares towards the most productive firms and selection becomes tougher.

Note that the differential impact of an increase in union's strength on the survival threshold $c_{D,l}$ has implications for the average marginal cost (i.e. average productivity). In Montagna and Nocco (2013) the softening selection effect was increasing the average marginal cost, or equivalently, reducing the average productivity. As discussed above, the increase in union's strength leads to a smaller softening selection effect which weakens the impact on the average marginal cost/productivity.

Finally, given that the price set by marginal firms is $p_l^{\max} = c_{D,l}$, by substituting it together with the average price $\bar{p}_l = \bar{p}_{D,l} = \frac{2k+v_l+(2-v_l)\lambda_{D,l}}{2(k+1)}c_{D,l}$ and the average quality $\bar{z}_l = \bar{z}_{D,l} = \frac{(2-v_l)\beta_l L_l \lambda_{D,l}}{2\gamma\theta_l(k+1)}c_{D,l}$ into $p_l^{\max} = \frac{\alpha\gamma+\eta N_l(\bar{p}_l-\beta_l \bar{z}_l)}{\gamma+\eta N_l}$, we find that the number of firms that survive after entering the market and sell their variety is $N_l = \frac{2\gamma(k+1)(\alpha-c_{D,l})}{(2-v_l)\eta\lambda_{D,l}c_{D,l}}$, while the number of entrants is $N_{E,l} = N_l / (c_{D,l}/c_{M,l})^k$.

These results have profound implications on the firm's quality investment as discussed in the following section.

3 Quality and unions

Let us recall that the firm's investment in quality is given by (16). As commented above the effects of an increase in the union's bargaining power, v_l , on a firm's investment in quality, $z_{D,l}(c)$, are initially ambiguous. From (16), we observe that the increase in v_l has a direct negative impact on quality and it is captured by the element $(2 - v_l)$ in expression (16). The increase in the bargaining power of the unions increases the firm's marginal cost of production, reducing the firm's demand and, therefore, the quantity sold in the market. This is the market size effect identified in innovation models. However, the increase in the union's bargaining power v_l increases the cutoff $c_{D,l}$, having a positive impact in the firm's investment in quality. Thus, the increase in the union's bargaining power creates a disadvantage, which is stronger for the most productive firms, whose profits decrease, and the increase in the average price reduces

²⁴ This can be readily obtained from $\frac{\partial c_{D,l}}{\partial v_l} = \frac{2c_{D,l}}{(k+2)(2-v_l)} > 0$, from which we get $\frac{\partial^2 c_{D,l}}{\partial v_l \partial \lambda_{D,l}} = \frac{2}{(k+2)(2-v_l)} \frac{\partial c_{D,l}}{\partial \lambda_{D,l}} < 0$.

competition in the market. The latter increases the firm's demand and, therefore, the output sold for the least productive firms. These results, together with the results mentioned in the previous paragraph, are formally summarized and reflected in the following proposition:

Proposition 1 *An increase in the bargaining power of unions v_l reduces (increases) the firm's investment in quality, the firm's produced quantity, the firm's profits and the firm's mark-up for the most (least) productive firms with $c < \frac{k}{k+2}c_{D,l}$ ($c > \frac{k}{k+2}c_{D,l}$).*

Proof See Appendix. $\square$

This is an interesting result which suggests that the effect of an increase in unionization on a firm's quality investment within an industry is not homogeneous across all firms and benefits the least productive ones. As discussed after equation (22), the main reason behind this mixed effect is the fact that, in this setting, an increase in the bargaining power of the unions will have an impact on wages that is stronger for the most productive firms as their labour demand is less sensitive to changes in wages, as explained after the Lemma. This increase in production cost is passed on to consumers who reallocate their expenditures from the high-quality varieties, whose prices have increased by more, towards the low-quality varieties, whose prices have increased less. The latter effect increases relatively the market size for the least productive firms and decrease the market size of the most productive ones, affecting their incentives in quality investment.

This asymmetric effect across different firms is not only present when there is an increase in the union's bargaining power. Although the thresholds will vary with the parameter in question, we can conclude that a variation in market size, L_l , the preference strength, β_l , and the cost of innovation, θ_l , will have a different effect across most productive and less productive firms as the following proposition states.

Proposition 2 *i) An increase in market size L_l increases (decreases) the firm's quality investment when $c < \frac{k+1}{k+2}c_{D,l}$ ($c > \frac{k+1}{k+2}c_{D,l}$).*
ii) An increase in the preference parameter β_l increases (decreases) the firm's quality investment when $c < \frac{2\gamma\theta_l(k+2)+k\beta_l^2L_l}{(2\gamma\theta_l+\beta_l^2L_l)(k+2)}c_{D,l}$ ($c > \frac{2\gamma\theta_l(k+2)+k\beta_l^2L_l}{(2\gamma\theta_l+\beta_l^2L_l)(k+2)}c_{D,l}$).
iii) An increase in the cost of innovation θ_l decreases (increases) the firm's quality investment when $c < \frac{2\gamma\theta_l(k+2)-\beta_l^2L_l}{2\gamma\theta_l(k+2)}c_{D,l}$ ($c > \frac{2\gamma\theta_l(k+2)-\beta_l^2L_l}{2\gamma\theta_l(k+2)}c_{D,l}$).

Proof See Appendix. $\square$

Notice that the last result highlights that an increase in the cost of innovation has an ambiguous impact on innovation at the firm level. This is because the increase in θ_l affects unevenly the different firms in the industry being the effect negative for the most productive firms. As an increase in θ_l disproportionately affects the most productive firms, this generates a reallocation of market shares that increases the market size and the benefits for the least productive ones, compensating the initial effect of the increase in the innovation cost.

Given the firms' heterogeneous response in terms of quality investment, following an increase in the worker's bargaining power, assessing the aggregate effect becomes

paramount. The average investment in quality is given by:

$$\bar{z}_{D,l} = \frac{(2 - v_l) \beta_l L_l \lambda_{D,l}}{2\gamma\theta_l (k + 1)} c_{D,l}$$

Differentiating it with respect to v_l and rearranging terms yields the following proposition.

Proposition 3 *An increase in the bargaining power of unions v_l reduces the average investment in quality.*

Proof See Appendix. $\square$

When the union bargaining power increases, the average investment in quality in the industry is affected through different mechanisms. On the one hand, the most productive firms reduce their investment in quality. On the other hand, there is an increase in investment in quality by the least productive firms, which also experience an increase in market shares due to reallocation forces. However, the former effect on the most productive firms is strong enough to compensate the latter one leaving a negative net effect as a result.

A common measure of innovative activity at the firm and industry level is R&D intensity which can be defined as $\Lambda = \frac{R\&D \text{ expenditures}}{Sales}$. The following proposition points out that while R&D intensity at the firm level, $\Lambda(c)$, may increase or decrease with the degree of unionization depending on the size of the firm, R&D intensity at the industry level, Λ , is negatively related to the degree of unionization.

Proposition 4 *An increase in the bargaining power of unions reduces (increases) the R&D intensity at the firm level for more (less) productive firms, and reduces the overall R&D intensity at the industry level.*

Proof See Appendix $\square$

3.1 Welfare

The impact on welfare of changes in the bargaining power of unions v_l when quality matters can be analyzed through the effects on the indirect utility, even though the complexity of the expressions does not allow to have unambiguous results.

To point out the different channels that are in action, let us first point out that the average indirect utility $\bar{W}_l$ in country l can be expressed as

$$\bar{W}_l = \bar{I}_l + \bar{q}_{0l}^\varepsilon + B_l, \quad (23)$$

with $\bar{I}_l$ denoting the average individual income in country l , $\bar{q}_{0l}^\varepsilon$ the average individual endowment of the homogeneous good in the same country and B_l defined as $B_l \equiv \frac{N_l}{2\gamma} \left(\sigma_{p_l}^2 + \beta_l^2 \sigma_{z_l}^2 - 2\beta_l \text{cov}(p_l(i), z_l(i)) \right) + \frac{(\alpha - \bar{p}_l + \beta_l \bar{z}_l)^2 N_l}{2(\gamma + \eta N_l)}$. Note that this expression is equivalent to the one in Montagna and Nocco (2013) except for the fact that here

B_l depends on price-adjusted quality indexes (i.e. $(p_l - \beta_l z_l)$).²⁵ From the expression of B_l we can infer that, ceteris paribus, $\partial \bar{W}_l / \partial \bar{p}_l < 0$, $\partial \bar{W}_l / \partial N_l > 0$, $\partial \bar{W}_l / \partial \sigma_{p_l}^2 > 0$, $\partial \bar{W}_l / \partial \bar{z}_l > 0$, $\partial \bar{W}_l / \partial \sigma_{z_l}^2 > 0$ and $\partial \bar{W}_l / \partial \text{cov}(p_l(i), z_l(i)) < 0$. That is, ceteris paribus, welfare increases with the average quality ($\bar{z}_l$) - the more so, the larger is the value of the preference for quality β_l - and decreases with the average price ($\bar{p}_l$). Moreover, welfare increases with the number of varieties (N_l) and with the possibility to redistribute consumptions towards cheaper varieties when the variance of p increases (σ_p^2), or towards variety of higher quality when σ_z^2 increases, while it decreases with the covariance of prices and quality as a higher quality increases prices by a larger amount.

The impact of an increase in unionization on welfare goes through the effects on the average individual income and on the element B_l . In general, that is when quality is relevant, the average individual income in country l can be defined as $\bar{I}_l \equiv (\overline{VC}_l N_l + L_l - \bar{l}_l N_l) / L_l$, where $\overline{VC}_l$ and $\bar{l}_l$ are, respectively, the average variable cost and the average labour demand per-firm in the monopolistic sector in country l .

In the appendix we obtain an expression for each of the elements affecting the average individual income and B_l . From the point of view of the average individual income, as in Montagna and Nocco we can state that $\partial \bar{I}_l / \partial v_l \geq 0$ iff: $\alpha \geq c_{D,l} [2v_l + 4 + k(2 - v_l)] / [k(2 - v_l) + 4]$. Note that even though the results for $\partial \bar{I}_l / \partial v_l$ are the same as in Montagna and Nocco (2013), the cutoffs are different, and they imply that for a sufficiently large α , which implies strong pro-variety effects, $\partial \bar{W}_l / \partial v_l > 0$ cannot be ruled out. Moreover, when quality matters, we can state that $\frac{\partial \bar{I}_l}{\partial v_l} > 0$ is more likely when the preference for quality β_l increases as $\frac{\partial c_{D,l}}{\partial \beta_l} < 0$. Consequently, through the average income channel we can infer that it is more likely that an increase in unionization has a positive effect on welfare when technology upgrading is allowed.

The effects of an increase in unionization on B_l are more intricate though. Note that $\partial B_l / \partial v_l = -\Phi_l \frac{[2c_{D,l} + (\alpha - c_{D,l})k]c_{D,l}}{4\eta(k+2)^2} + \frac{2c_{D,l}(\alpha - c_{D,l})}{\eta(k+2)(2-v_l)}$. It is important to note that this expression is similar to Montagna and Nocco (2013), and it is identical where the variable Φ_l takes the value of one. However, comparing the effects of an increase in unionization on B_l across the two frameworks is challenging. These expressions differ not only in the value of Φ_l but also in the cost cut-offs that are different between both models.

This confirms that overall the effects of an increase in v_l on $\bar{W}_l$ are ambiguous, due to the different effects on quality, prices and varieties and that it becomes challenging to conclude in which way these are different when firms have the possibility to quality upgrade. To give a hint of the nature of such complex effects, it is sufficient to recall that the covariance in the closed economy is positive, and rising quality, which would by itself result in an increase in welfare, will also increase prices, and this tends to decrease welfare.

Unfortunately, the complexity of the interactions of many different effects at work does not allow us to give clear conclusions on whether the inclusion of endogenous

²⁵ Notice that when $\beta_l = 0$, that is when quality does not matter, the variance of the price adjusted quality collapses to the one of prices and the utility in (23) reduces to the expression in Montagna and Nocco (2013).

technology upgrading changes the desirability of stronger unions. While it is likely that endogenous technology upgrading makes stronger unions more likely to increase welfare through the average income channel, the changes in the number of varieties and prices are more cumbersome which makes the overall effect more intricate. However, the analysis so far conducted allows us at least to understand the nature of some of the main forces at work that generate this ambiguity, unveiling that, for instance, even if stronger unions would be able to increase the average income of workers in the economy, they would tend to lower average quality having contrasting effects on welfare.

In this section, we have unveiled how firms within an industry may respond differently in terms of quality investment, when the industry faces an increase in the power of unions. These surprising results are worth exploring within the context of an open economy. The following sections will investigate this.

4 Open economy

Consider now that country l can trade with another economy h . Trade is costly and, adopting the iceberg specification, we assume that $\tau_{lh} > 1$ units of a good have to be produced in country l in order to allow one unit of the good to be consumed in country h . In the simplest analytically solvable version of the model, the union wage is decided at each segmented market level, which can be interpreted as a branch level. Thus, wages are determined at the profit-centre level, as in Montagna and Nocco (2013) and in Chang et al. (2023).²⁶ Moreover, given that markets are segmented, firms not only set their price market by market, but they also choose the quality of their product adapting it to the country's characteristics. However, not all producers will export their products, as only the most productive firms will be able to export to foreign markets.

Starting from the final stage, firms in country l with unit labour requirement c choose the price and the quality for each market they sell to by solving the following optimisation problem:

$$\max_{p_{D,l}(c), p_{X,l}(c), z_{D,l}(c), z_{X,l}(c)} \left(p_{D,l}(c) - w_{D,l}(c)c \right) q_{D,l}(c) - \frac{\theta_l}{2} (z_{D,l}(c))^2 + \\ + \left(p_{X,l}(c) - \tau_{lh} w_{X,l}(c)c \right) q_{X,l}(c) - \frac{\theta_l}{2} (z_{X,l}(c))^2$$

with $p_{X,l}(c)$ denoting the price firms charge in the foreign market, and $z_{X,l}(c)$ the quality of their product for the foreign market, subject to both the domestic demand

²⁶ The tendency to bargaining decentralisation at the sub-firm or profit-centre level has been commonplace in many industrialized countries as pointed out by Chang et al. (2023) who refer to the works by Brown (1987) for the UK, Jürgens (2008) for Germany, Townsend et al. (2013) for Australia, and Tuman (2003) for Canada, Japan, Korea, the US, and New Zealand.

in (2) and to the foreign demand expressed by consumers in country h , that is

$$q_{X,l}(c) = \frac{\alpha L_h}{\gamma + N_h \eta} + \frac{\beta_h}{\gamma} z_{X,l}(c) L_h - \frac{1}{\gamma} p_{X,l}(c) L_h + \frac{\eta N_h L_h}{\gamma (\gamma + \eta N_h)} (\bar{p}_h - \beta_h \bar{z}_h) \quad (24)$$

Notice that the model will endogenously determine which of the firms investing in R&D in country l will survive and which of them will produce not only for the domestic market, but also for the foreign market.

Given market segmentation, the first order conditions of the maximization problem yield the quantity $q_{D,l}(c)$ and the quality $z_{D,l}(c)$ chosen for the domestic market l that, respectively, correspond to expressions (3) and (5) reported in previous section. The expression for the quantity sold in the foreign market h , is given by

$$q_{X,l}(c) = \frac{L_h}{\gamma} (p_{X,l}(c) - \tau_{lh} w_{X,l}(c) c) \quad (25)$$

and the optimal quality choice for products sold in the foreign market is given by

$$z_{X,l}(c) = \frac{\beta_h}{\theta_l \gamma} L_h (p_{X,l}(c) - \tau_{lh} w_{X,l}(c) c) \quad (26)$$

Substituting expressions (3), (5), (25) and (26) into the profit function, gives the profits obtained in the domestic market l , $\pi_{D,l}(c)$, analogous to that in expression (7), and those obtained in the foreign market h that are given by

$$\pi_{X,l}(c) = \frac{\gamma}{L_h} \left(\frac{2\gamma\theta_l - \beta_h^2 L_h}{2\gamma\theta_l} \right) (q_{X,l}(c))^2 \quad (27)$$

Then, the equilibrium wages paid to workers employed in the production for the domestic market and for the export branch of the firm need to be determined. The bargaining process that occurs at the division (branch) level, requires to consider labour demand for the production in the two branches, which are, respectively, given by $l_{D,l}(c)$ in expression (11) for the domestic division, and by $l_{X,l}(c)$ for the branch devoted to the production for the foreign market corresponding to

$$l_{X,l}(c) = \frac{L_h}{\gamma} (p_{X,l}(c) - \tau_{lh} w_{X,l}(c) c) \tau_{lh} c$$

As in the closed economy case, the labour demand for exports depends directly on the mark-up the firm charges in the exporting market (i.e., $\mu_{X,l}(c) = (p_{X,l}(c) - \tau_{lh} w_{X,l}(c) c)$ in (25)).

Making use of (25) and (26) to get $z_{X,l}(c) = \frac{\beta_h}{\theta_l} q_{X,l}(c)$, and substituting it into the demand function (24) allows us to rewrite $q_{X,l}(c)$ as follows

$$q_{X,l}(c) = \frac{\theta_l L_h}{\gamma \theta_l - \beta_h^2 L_h} \left[\frac{\alpha \gamma + \eta N_h (\bar{p}_h - \beta_h \bar{z}_h)}{(\gamma + N_h \eta)} - p_{X,l}(c) \right] \quad (28)$$

Following the same line of reasoning as in the closed economy we need to impose $\theta_l > \beta_h^2 L_h / \gamma$. Thus, the analysis will be focused on this case as this will give interior solutions.

Then, similar to the closed economy case we find that the quantity produced in country l for the foreign market h by firms with unit labour requirement c is

$$q_{X,l}(c) = \frac{L_h}{\gamma} \lambda_{X,l} \left[\frac{\alpha \gamma + \eta N_h (\bar{p}_h - \beta_h \bar{z}_h)}{(\gamma + N_h \eta)} - \tau_{lh} w_{X,l}(c) \right] c$$

where $\lambda_{X,l} \equiv \frac{\gamma \theta_l}{2\gamma \theta_l - \beta_h^2 L_h}$. This parameter exhibits the same properties as $\lambda_{D,l}$ and, therefore, $\lambda_{X,l}$ can be interpreted as a measure of the *profitability of investing in quality upgrading for the foreign market*, with $\frac{1}{2} \leq \lambda_{X,l} < 1$. Clearly, $\lambda_{D,l}$ represents the profitability of investing in quality upgrading for the domestic market.

The labour demand for exports is then

$$l_{X,l}(c) = \tau_{lh} \frac{L_h}{\gamma} \lambda_{X,l} \left[\frac{\alpha \gamma + \eta N_h (\bar{p}_h - \beta_h \bar{z}_h)}{(\gamma + N_h \eta)} - \tau_{lh} w_{X,l}(c) \right] c \quad (29)$$

Thus, if the firm is exporting, there are two profit-centres within the firms. The assumption of separability of the two markets implies that the resulting wage for workers performing in the domestic branch is qualitatively the same as that given by expression (12). Consequently, the expressions for the variables of the domestic market, such as quantity $q_{D,l}(c)$, quality $z_{D,l}(c)$, wages $w_{D,l}(c)$, and profits $\pi_{D,l}(c)$ are analogous to those obtained in the autarkic case, even though their values are different as the cutoff $c_{D,l}$ in the case of the open economy is different from that obtained in autarky, and its value will be computed in the following of the paper.

Analogously to the case of the production for the domestic market l , the determination of the wage paid in the branch devoted to the production for the foreign market h will be the result of the following Nash-bargaining problem the requires

$$\max_{w_{X,l}(c)} v_l \ln l_{X,l}(c)(w_{X,l}(c) - 1) + (1 - v_l) \ln \pi_{X,l}$$

subject to the labour demand in (29). Substituting profits $\pi_{X,l}(c)$ obtained from sales in the foreign market h from expression (27), and $l_{X,l}(c) = \tau_{lh} q_{X,l}(c) c = \tau_{lh} \frac{L_h}{\gamma} ((p_{X,l}(c) - \tau_{lh} w_{X,l}(c)) c$, and considering only the relevant terms, the problem can be rewritten as follows

$$\max_{w_{X,l}(c)} (2 - v_l) \ln (p_{X,l}(c) - \tau_{lh} w_{X,l}(c) c) + v_l \ln (w_{X,l}(c) - 1)$$

subject to

$$p_{X,l}(c) - \tau_{lh} w_{X,l}(c)c = \frac{\gamma \theta_l}{2\gamma \theta_l - \beta_h^2 L_h} \left[\frac{\alpha \gamma + \eta N_h (\bar{p}_h - \beta_h \bar{z}_h)}{\gamma + N_h \eta} - \tau_{lh} w_{X,l}(c)c \right]$$

The first order condition is given by

$$\frac{(v_l - 2) \tau_{lh} \lambda_{X,l} c}{p_{X,l}(c) - \tau_{lh} w_{X,l}(c)c} + \frac{v_l}{w_{X,l}(c) - 1} = 0$$

Rearranging terms we find that

$$w_{X,l}(c) = 1 + \frac{v_l}{(2 - v_l) \lambda_{X,l} + v_l} \left(\frac{p_{X,l}(c)}{\tau_{lh} c} - 1 \right) \quad (30)$$

Note that this wage rule is similar to that obtained for $w_{D,l}(c)$ in (12) with the exception of $\lambda_{X,l}$, which, unlike $\lambda_{D,l}$, depends on foreign demand parameters and the marginal cost of production that in this case coincides with the marginal cost of delivery, $\tau_{lh}c$.

The next step is to find the wage for marginal exporting firms, whose unit labour requirement is $c_{X,l}$. Note that a firm will have positive profits in the export market when $q_{X,l} > 0$. From equation (28), we obtain that this is the case if and only if $\frac{\alpha \gamma + \eta N_h (\bar{p}_h - \beta_h \bar{z}_h)}{\gamma + \eta N_h} > p_{X,l}(c)$. Thus, there must exist a threshold level of the unit labour requirement $c_{X,l}$ such that $q_{X,l} = 0$, which corresponds to $p_{X,l}(c_{X,l}) = \frac{\alpha \gamma + \eta N_h (\bar{p}_h - \beta_h \bar{z}_h)}{\gamma + \eta N_h}$. From equation (25) we have that the price set in the foreign country h by marginal firms exporting from country l is $p_{X,l}(c_{X,l}) = \tau_{lh} w_{X,l}(c_{X,l}) c_{X,l}$, and equation (26), reveals that these marginal firms exporting from l to h do not invest in quality for the foreign market given that $z_{X,l}(c_{X,l}) = 0$. Then, the wage rule (30) implies that for marginal firms with $c_{X,l}$, the wage is equal to that obtained by non-unionized workers in the homogeneous good sector

$$w_{X,l}(c_{X,l}) = 1$$

The last conditions implies that, in equilibrium, marginal firms exporting to h set their price equal to $p_{X,l}(c_{X,l}) = \tau_{lh} c_{X,l}$ and $\tau_{lh} c_{X,l} = \frac{\alpha \gamma + \eta N_h (\bar{p}_h - \beta_h \bar{z}_h)}{\gamma + \eta N_h}$. Therefore the demand function can be rewritten as

$$q_{X,l}(c) = \tau_{lh} c_{X,l} L_h + \frac{\beta_h}{\gamma} z_{X,l}(c) L_h - \frac{1}{\gamma} p_{X,l}(c) L_h$$

This can be used together with (25), (26) and (30) to obtain an expression for the firm's exporting mark-up which is equal to

$$p_{X,l}(c) - \tau_{lh} w_{X,l}(c)c = \frac{(2 - v_l) \lambda_{X,l}}{2} \tau_{lh} (c_{X,l} - c) \quad (31)$$

As in the closed economy model, mark-ups in the exporting market are also larger when firms have the choice of quality upgrading and they are also increasing with the

profitability of investing in quality for the foreign market $\lambda_{X,l}$. Making use of (26) and (30), we obtain a final expression for the exporting wage rule which is given by

$$w_{X,l}(c) = 1 + \frac{v_l}{2} \left(\frac{c_{X,l} - c}{c} \right) \quad (32)$$

Hence, it is interesting to point out that the wage rule that the exporting plant follows given in expression (32) is relatively similar to that of the domestic plant and of the firm in the autarkic scenario given in (17).

Moreover, exporting profits in (27) can be rewritten as follows

$$\pi_{X,l}(c) = \frac{\tau_{lh}^2 (2 - v_l)^2 \theta_l L_h}{8 (2\gamma\theta_l - \beta_h^2 L_h)} (c_{X,l} - c)^2 \quad (33)$$

Finally, since the wage rule the domestic plant follows is equivalent to the one the firm would follow in the autarkic scenario, then the domestic profits expression is the same as the one in autarky and, therefore, it is given by equation (20), even though the domestic cutoff $c_{D,l}$ is different as it will be shown in the following subsection.

4.1 Threshold cut-off

The free entry condition for each country l is given by

$$\int_0^{c_{D,l}} \pi_{D,l}(c) dG_l(c) + \int_0^{c_{X,l}} \pi_{X,l}(c) dG_l(c) = f_{E,l}$$

Substituting (20) and (33) into the previous expression and making use of $\tau_{lh}c_{X,l} = c_{D,h}$ we obtain the free entry conditions for both countries h and l , which are, respectively, given by

$$\frac{(2 - v_h)^2}{4\gamma c_{M,h}^k (k+2)(k+1)} \left[\lambda_{D,h} L_h (c_{D,h})^{k+2} + (\tau_{hl})^{-k} \lambda_{X,h} L_l (c_{D,l})^{k+2} \right] = f_{E,h} \quad (34)$$

and

$$\frac{(2 - v_l)^2}{4\gamma c_{M,l}^k (k+2)(k+1)} \left[\lambda_{D,l} L_l (c_{D,l})^{k+2} + (\tau_{lh})^{-k} \lambda_{X,l} L_h (c_{D,h})^{k+2} \right] = f_{E,l}, \quad (35)$$

Solving this system of two equations in two unknowns yields the domestic cutoff for country h

$$c_{D,h} = \left\{ \frac{4\gamma(k+2)(k+1) \left[(2-v_l)^2 f_{E,h} c_{M,h}^k \lambda_{D,l} - (2-v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h} \right]}{(2-v_l)^2 (2-v_h)^2 L_h \left[\lambda_{D,h} \lambda_{D,l} - (\tau_{hl} \tau_{lh})^{-k} \lambda_{X,h} \lambda_{X,l} \right]} \right\}^{\frac{1}{k+2}} \quad (36)$$

and a symmetric solution applies for country l .²⁷ Clearly, the domestic cutoff $c_{D,l}$ obtained in the case of the open economy is different from that obtained in the case of autarky in (22).²⁸

In order to understand how quality upgrading alters the properties of the economy, we consider the case in which the parameters related to the production and innovation technology are identical across countries (i.e., $\theta_l = \theta_h = \theta$, $f_{E,l} = f_{E,h} = f_E$ and $c_{M,l}^k = c_{M,h}^k = c_M^k$), and that the preference for quality and the market size of the two countries are equal (i.e. $\beta_l = \beta_h$ and $L_l = L_h$), then the cutoff in (36) can be rewritten as

$$(c_{D,h})^{k+2} = \frac{4\gamma(k+2)(k+1) c_M^k f_E \left[(2-v_l)^2 - (2-v_h)^2 (\tau_{hl})^{-k} \right]}{\lambda_{D,h} (2-v_l)^2 (2-v_h)^2 L_h \left[1 - (\tau_{hl} \tau_{lh})^{-k} \right]}$$

This is similar to the respective cutoff obtained in a model with no investment in quality (i.e. when quality has no value and $\beta_l = \beta_h = 0$, and $\lambda_{D,h} = 1/2$). Denoting this last threshold $c_{D,h}^{'N}$ we find that

$$(c_{D,h})^{k+2} = \frac{1}{2\lambda_{D,h}} (c_{D,h}^{'N})^{k+2} < (c_{D,h}^{'N})^{k+2}$$

as $\frac{1}{2} \leq \lambda_{D,h} < 1$. This shows that as in the case of the closed economy, when the possibility of quality upgrading is included in this framework, selection becomes stronger.

In the next section we analyse the impact on quality upgrading of strengthening the worker's bargaining power on wage negotiations in a country (i.e. v_l increases), which is easier to be understood starting from a situation in which the two countries are symmetric in terms of technology, market size and preference for quality.

²⁷ To be meaningful, the solution for the domestic cutoff needs to be positive for both countries. This, for instance, when the two countries are symmetric in terms of market size L , cost of quality upgrading θ and preference for quality β , requires the condition on the numerator $(2-v_l)^2 f_{E,h} c_{M,h}^k \lambda_{D,l} - (2-v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h} > 0$ to be satisfied to have $c_{D,h} > 0$, while the corresponding condition on the numerator of the other domestic cutoff $(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l} > 0$ is required to have $(c_{D,l})^{k+2} > 0$.

²⁸ See the Appendix for the derivation of the number of entrants ($N_{E,l}$), the number of sellers (N_l), producers ($N_{D,l}$) and exporters ($N_{X,l}$).

5 Quality and unions in the open economy

The investment in quality for the domestic and export markets of a firm producing in country l are, respectively, given by

$$z_{D,l}(c) = \frac{\beta_l L_l (2 - v_l)}{2\gamma\theta_l} \lambda_{D,l} (c_{D,l} - c)$$

and

$$z_{X,l}(c) = \frac{\beta_h L_h (2 - v_l)}{2\gamma\theta_l} \lambda_{X,l} \tau_{lh} (c_{D,l} - c)$$

where the last expression for $z_{X,l}(c)$ is obtained substituting the markup from (31) into (26). Notice that, substituting the value of $\lambda_{D,l}$ into the first expression for $z_{D,l}(c)$ gives $z_{D,l}(c) = \frac{(2-v_l)\beta_l L_l}{2(2\theta\gamma - \beta_l^2 L_l)} (c_{D,l} - c)$, and it shows that the relationship between the firm's investment in quality in the domestic market in a global environment and the relative efficiency of the firm (i.e. $(c_{D,l} - c)$), is the same as in the closed economy (16). However, the results of proposition 1 need to be modified in the global environment as the cutoff $c_{D,l}$ is different in the case of the open economy. Thus, we show in the related proof in the Appendix that there exists a $c^* < \frac{k}{k+2} c_{D,l}$, such that the following Proposition holds:

Proposition 5 *An increase in the bargaining power of unions v_l reduces the firm's investment in quality for the domestic market, $z_{D,l}(c)$, for the most productive firms (i.e. firms with $c < c^*$), increasing it for less productive firms (i.e. firms with $c > c^*$).*

Proof See Appendix. □

Corollary 1 *The reduction of the investment in quality $z_{D,l}(c)$ is larger in an autarkic environment, while the increase is larger in a free trade environment (i.e. Most productive firms reduce more innovation in an autarkic environment, while less productive firms increase more innovation in a free trade environment).*

Proof See Appendix

The increase in the bargaining power of the unions in the home economy creates an economic disadvantage to the domestic firms which need to pay higher wages and these increases in wages are larger for more productive firms. Interestingly, it is shown that this effect is shaped differently in the global economy compared to the autarkic economy. The mechanism explaining these results relies on the fact that for an open economy, the competition of foreign firms already exerts its pressures on domestic firms and, therefore, when increasing their quality they are induced to innovate more than if they were in autarky because they have to face the competition of foreign firms that is absent in the closed economy. At the same time, being the pressures of the competition of foreign firms present also for more productive firms, which innovate

less, they choose to reduce quality less in order to be able to better face the competition of foreign firms, which again is absent in the closed economy.²⁹

In a global environment, firms can also invest in improving the quality of their product offered in the foreign market. The following proposition discusses the effects of a change in the domestic worker's union bargaining power on the investment in quality for the foreign market.

Proposition 6 *When the economy is open to trade, an increase in v_l decreases the firm's investment in quality for the foreign market by a firm originating in the home market (i.e. $\frac{\partial z_{X,l}}{\partial v_l} < 0$).*

Proof See Appendix. □

The latter suggests that the results in the global economy are actually more intricate than in the autarkic case. In principle, most productive firms will reduce their investment in quality for both the domestic product and the foreign product. In the case of the least productive firms the results are less clear-cut.

It is important to realize here that there are two firm's investment in quality that could be relevant for the perspective of the home economy: On the one hand, we have the firm's investment in quality for the foreign market by the domestic firms $z_{X,l}$. This will not directly affect the consumer's utility since domestic consumers are not enjoying the improvements in quality in which local firms invest for the foreign market. On the other hand, consumer's utility in l will be directly affected by the foreign firms' investment in the quality of their exported goods towards market l , $z_{X,h}$. The next proposition highlights how these two measures at the firm level are affected by changes in v_l .

Proposition 7 *When the economy is open to trade, an increase in v_l increases the firm's investment in quality for market l by a firm producing in the foreign market h (i.e. $\frac{\partial z_{X,h}}{\partial v_l} > 0$).*

Proof See Appendix. □

Interestingly, in a global environment the increase in the bargaining power of the unions in the home economy will reduce the investment in quality that exporting firms offer in the foreign market, while it will increase the investment in quality that the foreign exporters are undertaking in the products served in the home market. Note that the increase in the bargaining power of the unions in the home economy creates an economic disadvantage to the domestic firms, which need to pay higher wages. The latter is translated in less sales abroad and therefore the investment in quality results to be less profitable. Instead, an increase in the home economy union's bargaining power will create an advantage in terms of sales to the foreign varieties offered in the home market. The latter is translated into bigger sales and larger profits for foreign firms exporting towards the home economy.

²⁹ This effect works through the cutoffs that are equal to the maximum price that firms can set for a specific market. Moreover, both cutoffs depend on the number of firms competing in a market as the definition of $p_l^{\max}$ shows. Clearly, in the open economy, the cutoff also encompasses the presence of foreign firms that are exporting in that specific country, and drives the results in the corollary.

The average quality for domestic firms selling in country l is

$$\bar{z}_{D,l} = \frac{(2 - v_l) \beta_l L_l \lambda_{D,l}}{2\gamma\theta_l (k + 1)} c_{D,l} = \frac{(2 - v_l)}{2(k + 1)} \frac{\beta_l L_l}{2\gamma\theta_l - \beta_l^2 L_l} c_{D,l}$$

while that of imported varieties is

$$\bar{z}_{X,h} = \frac{(2 - v_h) \beta_l L_l \lambda_{X,h}}{2\gamma\theta_h (k + 1)} c_{D,h} = \frac{(2 - v_h)}{2(k + 1)} \frac{\beta_l L_l}{2\gamma\theta_h - \beta_l^2 L_l} c_{D,h}$$

Hence, we can state that

Proposition 8 *An increase in the bargaining power of unions v_l :*

- a) *reduces the average investment in quality for the domestic market $\bar{z}_{D,l}$ by local firms when $\frac{(2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}}{(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h}} < \frac{k}{(k+2)}$, otherwise, it increases it;*
- b) *increases the average investment in quality by firms exporting in l ($\bar{z}_{X,h}$);*
- c) *reduces the average quality offered by domestic firms in the foreign market ($\bar{z}_{X,l}$).*

Proof See Appendix. $\square$

A natural question is then to reflect on what happens at the firm level with the total R&D investment. As R&D investment of foreign firms targeting the domestic market increases following an increase in the bargaining power of unions in the domestic country, while the R&D investment of domestic firms targeting the foreign market decreases, we wonder to what extent it is possible to observe that the total R&D investment of some exporting firms may increase. This depends on the relative value of a threshold value c^* with respect to that of the export cut-off $c_{X,l}$. For the cases in which $c_{X,l}$ lies below c^* , exporters to country h will reduce their overall R&D investments, as the increase in the bargaining power of the unions will reduce the investment in quality for both the domestic and the foreign market. However, for the cases in which $c_{X,l}$ lies above c^* , this could not happen for some of the least productive exporters (i.e. firms with $c \in (c^*, c_{X,l})$) when the increase of the R&D investments for the domestic market more than compensates the decrease in R&D for the foreign market as exports represent a very small amount of their market size (and, therefore, of their aggregate R&D expenditures). However, this does not happen when the economy starts from a situation in which the two countries are perfectly symmetric, as it is shown from the following Proposition.

Proposition 9 *An increase in the bargaining power of unions v_l starting from a situation in which the two countries are symmetric, decreases total R&D of exporting firms.*

Proof See Appendix. $\square$

Therefore, the total expenditure in R&D for an exporting firm always decline when v_l increases, with $\frac{\partial R\&D(c)}{\partial v_l} < 0$. Hence, the decline in the investment for quality

upgrading for the foreign market is always larger than the eventual increase in quality for the domestic market of the less productive exporting firms. Moreover, the decline in total expenditure in R&D for more productive exporting firms is the result of the decrease in the investment in quality for both the domestic and the foreign markets.

In summary, a general result obtained from this study is that more productive exporting firms decrease their investment in R&D, while the least productive domestic firms increase their investment in R&D.

6 Conclusions

Trade Unions are an important feature of labour markets in both developed and developing economies. However, the consequences of trade unions for innovation remains still a widely unexplored research area. In this paper, we aimed to fill this gap by proposing a general equilibrium model of heterogeneous firms and variable mark-ups to allow for a heterogeneous response at the firm level. To introduce innovation in our setting we consider that firms are investing in the quality of the products they offer which is market-specific. Trade unions negotiate wages with firms at the profit centre level, allowing for firms to pay different wages, consistent with the evidence. We explore this research question in both a closed and an open economy, allowing for asymmetries across different countries. We characterize the closed-form solution for both scenarios and in the case of the open economy we make comparative statics for the case in which countries are symmetric except for the union's bargaining power allowing for unilateral changes in this parameter.

In the closed economy we find that an increase in the bargaining power of unions reduces the average investment in quality, although the effect is heterogeneous across firms. More productive firms will reduce their investment in quality while the least productive firms will increase it. The increase in wages that results from an increase in the union's bargaining power is higher in the most productive firms as labour demand in these firms is less sensitive to changes in wages. This will push up prices more in these firms and will generate a reallocation of output from the most efficient firms to the least productive ones. The latter effect will benefit the least productive firms that will see their market size increased incentivising them to increase their investment in quality. More productive firms will see their market size decreased and will reduce their quality investment. These results are confirmed in an open economy scenario where we also find that the increase in the bargaining power of unions in one country will increase the quality offered by the firms from which the country is importing. The last result comes because the increase in the bargaining power of unions, rises the domestic wage creating a competitive disadvantage for them and reallocates demand from domestic firms to foreign firms. This increases the investment in innovation undertaken by foreign firms.

The results highlighted in this work clearly have testable implications that with the right data could offer an interesting avenue for future research. As young and small firms are somehow benefited from the increase in the bargaining power of unions, a more dynamic version of the model that explores the consequences of their role for firm growth could also be an interesting path for future investigations.

A Appendix

A.1 Proof of proposition 1

Proposition 1 *An increase in the bargaining power of unions v_l reduces (increases) the firm's investment in quality, the firm's produced quantity, the firm's profits and the firm's mark-up for the most (least) productive firms with $c < \frac{k}{k+2}c_{D,l}$ ($c > \frac{k}{k+2}c_{D,l}$).*

Proof Taking logs in (16) gives

$$\ln z_{D,l}(c) = \ln \frac{\beta_l L_l}{2(2\theta_l \gamma - \beta_l^2 L_l)} + \ln(2 - v_l) + \ln(c_{D,l} - c)$$

and differentiating it with respect to v_l yields

$$\frac{\partial \ln z_{D,l}(c)}{\partial v_l} = -\frac{1}{2 - v_l} + \frac{1}{c_{D,l} - c} \frac{\partial c_{D,l}}{\partial v_l} \quad (37)$$

From (22) we find that

$$\frac{\partial c_{D,l}}{\partial v_l} = \frac{2c_{D,l}}{(k+2)(2-v_l)} > 0$$

and then substituting it into (37) gives

$$\frac{\partial z_{D,l}(c)}{\partial v_l} = \frac{(k+2)c - c_{D,l}k}{(c_{D,l} - c)(2 - v_l)(k+2)} z_{D,l}(c) = \frac{\beta_l L_l [(k+2)c - c_{D,l}k]}{2(2\theta_l \gamma - \beta_l^2 L_l)(k+2)}$$

This expression is negative (positive) for $c < \frac{k}{k+2}c_{D,l}$ ($c > \frac{k}{k+2}c_{D,l}$).

Then, as $q_{D,l}(c) = \frac{\theta_l}{\beta_l} z_{D,l}(c)$ from (6) and $\pi_{D,l}(c) = \frac{2\gamma\theta_l - \beta_l^2 L_l}{2\theta_l L_l} (q_{D,l}(c))^2$ from (7), the sign of $\frac{\partial q_{D,l}(c)}{\partial v_l}$ and of $\frac{\partial \pi_{D,l}(c)}{\partial v_l}$ is the same as that of $\frac{\partial z_{D,l}(c)}{\partial v_l}$. Moreover, as the markup is $p_{D,l}(c) - w_{D,l}(c)c = \frac{(2-v_l)\lambda_{D,l}}{2} (c_{D,l} - c)$ from (15), making use of (37), it can be readily verified that $\frac{\partial \ln(p_{D,l}(c) - w_{D,l}(c)c)}{\partial v_l} = -\frac{1}{2-v_l} + \frac{1}{c_{D,l}-c} \frac{\partial c_{D,l}}{\partial v_l} = \frac{\partial \ln z_{D,l}(c)}{\partial v_l}$. $\square$

A.2 Proof of proposition 2

Proposition 2 *i) An increase in market size L_l increases (decreases) the firm's quality investment when $c < \frac{k+1}{k+2}c_{D,l}$ ($c > \frac{k+1}{k+2}c_{D,l}$)*

ii) An increase in the preference parameter β_l increases (decreases) the firm's quality investment when $c < \frac{2\gamma\theta_l(k+2)+k\beta_l^2 L_l}{(2\gamma\theta_l+\beta_l^2 L_l)(k+2)}c_{D,l}$ ($c > \frac{2\gamma\theta_l(k+2)+k\beta_l^2 L_l}{(2\gamma\theta_l+\beta_l^2 L_l)(k+2)}c_{D,l}$).

iii) An increase in the cost of innovation θ_l decreases (increases) the firm's quality investment when $c < \frac{2\gamma\theta_l(k+2)-\beta_l^2 L_l}{2\gamma\theta_l(k+2)}c_{D,l}$ ($c > \frac{2\gamma\theta_l(k+2)-\beta_l^2 L_l}{2\gamma\theta_l(k+2)}c_{D,l}$)

- Proof** i) Deriving quality from (16) and the cutoff from (22) with respect to L_l yields $\frac{\partial z_{D,l}(c)}{\partial L_l} = \frac{(2-v_l)\beta_l}{2(2\theta_l\gamma - \beta_l^2 L_l)} \left[\frac{2\gamma\theta_l}{2\theta_l\gamma - \beta_l^2 L_l} (c_{D,l} - c) + \frac{\partial c_{D,l}}{\partial L_l} L_l \right]$ and $\frac{\partial c_{D,l}}{\partial L_l} = -\frac{1}{k+2} \frac{2\theta_l\gamma}{L_l} \frac{1}{(2\theta_l\gamma - \beta_l^2 L_l)} c_{D,l}$. Substituting the latter into the former and rearranging terms gives $\frac{\partial z_{D,l}(c)}{\partial L_l} = \frac{(2-v_l)\beta_l\gamma\theta_l}{(2\theta_l\gamma - \beta_l^2 L_l)^2} \left(\frac{k+1}{k+2} c_{D,l} - c \right)$. Hence, $\frac{\partial z_{D,l}(c)}{\partial L_l} > 0$ ($\frac{\partial z_{D,l}(c)}{\partial L_l} < 0$) when $c < \frac{k+1}{k+2} c_{D,l}$ ($c > \frac{k+1}{k+2} c_{D,l}$).
- ii) Deriving quality from (16) and the cutoff from (22) with respect to β_l yields $\frac{\partial z_{D,l}(c)}{\partial \beta_l} = \frac{(2-v_l)L_l}{2(2\theta_l\gamma - \beta_l^2 L_l)} \left[\frac{2\theta_l\gamma + \beta_l^2 L_l}{(2\theta_l\gamma - \beta_l^2 L_l)} (c_{D,l} - c) + \beta_l \frac{\partial c_{D,l}}{\partial \beta_l} \right]$ and $\frac{\partial c_{D,l}}{\partial \beta_l} = -\frac{2\beta_l L_l c_{D,l}}{k+2} \left(\frac{1}{2\theta_l\gamma - \beta_l^2 L_l} \right)$. Substituting the latter into the former and rearranging terms gives $\frac{\partial z_{D,l}(c)}{\partial \beta_l} = \frac{(2-v_l)L_l}{2(2\theta_l\gamma - \beta_l^2 L_l)^2} \left[\frac{2\theta_l\gamma(k+2)c_{D,l} + \beta_l^2 L_l k c_{D,l} - (2\theta_l\gamma + \beta_l^2 L_l)(k+2)c}{(k+2)} \right]$. Hence, $\frac{\partial z_{D,l}(c)}{\partial \beta_l} > 0$ ($\frac{\partial z_{D,l}(c)}{\partial \beta_l} < 0$) when $c < \frac{2\gamma(k+2)\theta_l + k\beta_l^2 L_l}{(2\theta_l\gamma + \beta_l^2 L_l)(k+2)} c_{D,l}$ ($c > \frac{2\gamma(k+2)\theta_l + k\beta_l^2 L_l}{(2\theta_l\gamma + \beta_l^2 L_l)(k+2)} c_{D,l}$).
- iii) Deriving quality from (16) and the cutoff from (22) with respect to θ_l yields $\frac{\partial z_{D,l}(c)}{\partial \theta_l} = \frac{(2-v_l)\beta_l L_l}{2(2\theta_l\gamma - \beta_l^2 L_l)} \left[-2\gamma \frac{1}{(2\theta_l\gamma - \beta_l^2 L_l)} (c_{D,l} - c) + \frac{\partial c_{D,l}}{\partial \theta_l} \right]$ and $\frac{\partial c_{D,l}}{\partial \theta_l} = \frac{c_{D,l}}{k+2} \frac{1}{2\theta_l\gamma - \beta_l^2 L_l} \frac{\beta_l^2 L_l}{\theta_l}$. Substituting the latter into the former and rearranging terms gives $\frac{\partial z_{D,l}(c)}{\partial \theta_l} = \frac{(2-v_l)\beta_l L_l}{2(2\theta_l\gamma - \beta_l^2 L_l)^2} \left[\frac{(\beta_l^2 L_l - 2\gamma(k+2)\theta_l)c_{D,l} + 2\gamma(k+2)\theta_l c}{(k+2)\theta_l} \right]$. Hence, $\frac{\partial z_{D,l}(c)}{\partial \theta_l} > 0$ ($\frac{\partial z_{D,l}(c)}{\partial \theta_l} < 0$) when $c < \frac{2\gamma(k+2)\theta_l - \beta_l^2 L_l}{2\gamma(k+2)\theta_l} c_{D,l}$ ($c > \frac{2\gamma(k+2)\theta_l - \beta_l^2 L_l}{2\gamma(k+2)\theta_l} c_{D,l}$). $\square$

A.3 Proof of proposition 3

Proposition 3 *An increase in the bargaining power of unions v_l reduces the average investment in quality.*

Proof The average quality is given by

$$\bar{z}_{D,l} = \frac{(2-v_l)\beta_l L_l \lambda_{D,l}}{2\gamma\theta_l(k+1)} c_{D,l} = \frac{(2-v_l)^{\frac{k}{k+2}} \beta_l L_l^{\frac{k+1}{k+2}} \lambda_{D,l}}{2\gamma\theta_l(k+1)} \left[\frac{4(2\gamma\theta_l - \beta_l^2 L_l)(k+2)(k+1)c_{M,l}^k}{\theta_l} f_{E,l} \right]^{\frac{1}{k+2}}$$

that can be used to derive

$$\frac{\partial \bar{z}_{D,l}}{\partial v_l} = -\frac{k}{(k+2)(2-v_l)} \bar{z}_{D,l} < 0$$

A.4 Proof of proposition 4

Proposition 4 *An increase in the bargaining power of unions reduces (increases) the R&D intensity at the firm level for more (less) productive firms, and reduce the overall R&D intensity at the industry level.*

Proof R&D intensity is defined as $\Lambda \equiv \frac{\text{R\&D expenditures}}{\text{Sales}}$, and at the firm level is given by $\Lambda(c) = \frac{\theta_l(z_{D,l}(c))^2}{2p_{D,l}q_{D,l}}$. Making use of (6), (19) and (18), it can be rewritten as follows $\Lambda(c) = \frac{\beta_l^2 L_l}{2(2\gamma\theta_l - \beta_l^2 L_l)} \frac{(2-v_l)(c_{D,l}-c)}{2c+[v_l+(2-v_l)\lambda_{D,l}](c_{D,l}-c)}$. Then the sign of $\frac{\partial \Lambda(c)}{\partial v_l}$ depends on that of $2c_{D,l} \frac{3c-c_{D,l}}{\{2c+[v_l+(2-v_l)\lambda_{D,l}](c_{D,l}-c)\}^2}$. Then, $\frac{\partial \Lambda(c)}{\partial v_l} > 0$ when $c > \frac{c_{D,l}}{3}$. Viceversa, $\frac{\partial \Lambda(c)}{\partial v_l} < 0$ when $c < \frac{c_{D,l}}{3}$.

R&D intensity at the industry level is given by $\Lambda = \frac{\theta_l N \int_0^{c_{D,l}} (z_{D,l}(c))^2 \frac{dG_l(c)}{G(c_{D,l})}}{2N \int_0^{c_{D,l}} p_{D,l}(c)q_{D,l}(c) \frac{dG_l(c)}{G(c_{D,l})}}$. Substituting $z_{D,l}(c)$ from (16), the integral in the numerator can be rewritten as follows $\int_0^{c_{D,l}} (z_{D,l}(c))^2 \frac{dG_l(c)}{G(c_{D,l})} = \left[\frac{(2-v_l)\beta_l L_l}{2(2\gamma\theta_l - \beta_l^2 L_l)} \right]^2 (c_{D,l})^2 \frac{2}{(k+2)(k+1)}$, while the integral in the denominator, making use of (19) and (18), becomes $\int_0^{c_{D,l}} p_{D,l}(c)q_{D,l}(c) \frac{dG_l(c)}{G(c_{D,l})} = \frac{(2-v_l)\theta_l L_l}{2(2\gamma\theta_l - \beta_l^2 L_l)} \frac{k+v_l+(2-v_l)\lambda_{D,l}}{(k+1)(k+2)} (c_{D,l})^2$.

Substituting them above in Λ we find that $\Lambda = \frac{\beta_l^2 L_l (2-v_l)}{2(2\gamma\theta_l - \beta_l^2 L_l)[k+v_l+(2-v_l)\lambda_{D,l}]}$, and that $\frac{\partial \Lambda}{\partial v_l} = -\frac{\beta_l^2 L_l (k+v_l)}{2(2\gamma\theta_l - \beta_l^2 L_l)[k+v_l+(2-v_l)\lambda_{D,l}]^2} < 0$. □

A.5 Entrants, producers, exporters and sellers in the open economy.

The analysis for the open economy can be completed with the computations of the number of firms entering, producing, exporting and selling in each country. Specifically, recall that the number of firms entering in country l is $N_{E,l}$, while the number of firms selling in country l is N_l , which is given by the sum of domestic firms $N_{D,l}$ and firms exporting from the other country $N_{X,h}$ (i.e. $N_l = N_{D,l} + N_{X,h}$).

Making use of the Pareto distribution, the number of domestic producers is $N_{D,l} = G_l(c_{D,l})N_{E,l} = \left(\frac{c_{D,l}}{c_{M,l}}\right)^k N_{E,l}$, and the number of producers exporting from h in l is $N_{X,h} = G_h(c_{X,h})N_{E,h} = \left(\frac{c_{X,h}}{c_{M,h}}\right)^k N_{E,h}$, where $c_{X,h} = \frac{c_{D,l}}{\tau_{hl}}$. Previous expressions, can be used together with the expressions for the average price in country l

$$\bar{p}_l = \left\{ \frac{N_{D,l}}{N_{D,l}+N_{X,h}} [2k+v_l+(2-v_l)\lambda_{D,l}] + \frac{N_{X,h}}{N_{D,l}+N_{X,h}} [2k+v_h+(2-v_h)\lambda_{X,h}] \right\} \frac{c_{D,l}}{2(k+1)},$$

the average quality in l

$$\bar{z}_l = \left\{ \frac{N_{D,l}}{N_{D,l} + N_{X,h}} \frac{(2-v_l)\lambda_{D,l}}{\theta_l} + \frac{N_{X,h}}{N_{D,l} + N_{X,h}} \frac{(2-v_h)\lambda_{X,h}}{\theta_h} \right\} \frac{\beta_l L_l c_{D,l}}{2\gamma(k+1)}$$

and $p_l^{\max} = c_{D,l} = \frac{\alpha\gamma + \eta N_l(\bar{p}_l - \beta_l \bar{z}_l)}{\gamma + \eta N_l}$ to find the following equation for country l

$$\frac{(2-v_l)\lambda_{D,l}}{(c_{M,l})^k} N_{E,l} + (\tau_{hl})^{-k} \frac{(2-v_h)\lambda_{X,h}}{(c_{M,h})^k} N_{E,h} = \frac{2\gamma(k+1)(\alpha - c_{D,l})}{\eta(c_{D,l})^{k+1}} \quad (38)$$

Equation (38), together with the analogous expression for country h , gives a system of two equations in two unknowns $N_{E,l}$ and $N_{E,h}$ that can be solved to find the number of entrants in country l

$$N_{E,l} = \frac{2(k+1)\gamma(c_{M,l})^k}{\eta(2-v_l)} \frac{\left[\frac{(\alpha - c_{D,l})}{(c_{D,l})^{k+1}} \lambda_{D,h} - \frac{(\alpha - c_{D,h})}{(c_{D,h})^{k+1}} (\tau_{hl})^{-k} \lambda_{X,h} \right]}{[\lambda_{D,l} \lambda_{D,h} - (\tau_{lh} \tau_{hl})^{-k} \lambda_{X,l} \lambda_{X,h}]} \quad (39)$$

and the number of entrants in country h

$$N_{E,h} = \frac{2(k+1)\gamma(c_{M,h})^k}{\eta(2-v_h)} \frac{\left[\frac{(\alpha - c_{D,h})}{(c_{D,h})^{k+1}} \lambda_{D,l} - (\tau_{lh})^{-k} \frac{(\alpha - c_{D,l})}{(c_{D,l})^{k+1}} \lambda_{X,l} \right]}{[\lambda_{D,l} \lambda_{D,h} - (\tau_{lh} \tau_{hl})^{-k} \lambda_{X,l} \lambda_{X,h}]} \quad (40)$$

The number of domestic producers $N_{D,l}$ and $N_{D,h}$, and of exporters $N_{X,l}$ and $N_{X,h}$ in both countries can be readily derived from the results above.

A.6 Welfare. Extra calculations

In this small section we provide additional calculations supporting the statements made in the main manuscript regarding welfare. Note that the following expressions for the different elements of the utility function can be obtained: the average income is $\bar{I}_l = 1 + \frac{N_l(2-v_l)\lambda_{D,l}(c_{D,l})^2 v_l}{2(k+2)(k+1)\gamma}$, the variance of prices is $\sigma_{p_l}^2 = \left[\frac{(2-v_l)(1-\lambda_{D,l})}{2} \right]^2 \frac{k}{(k+2)(k+1)^2} (c_{D,l})^2$, the variance of quality is $\sigma_{z_l}^2 = \left[\frac{(2-v_l)\lambda_{D,l}\beta_l L_l}{2\theta_l\gamma} \right]^2 \frac{k}{(k+2)(k+1)^2} (c_{D,l})^2$, the covariance of prices and quality is $cov(p_l(i), z_l(i)) = \frac{(2-v_l)^2(1-\lambda_{D,l})\lambda_{D,l}\beta_l L_l k}{4\theta_l\gamma(k+2)(k+1)^2} (c_{D,l})^2 > 0$ as $\frac{1}{2} \leq \lambda_{D,l} < 1$, and the term B_l can be rewritten as follows $B_l = \frac{(\alpha - c_{D,l})^2}{2\eta} + \frac{\Phi_l(2-v_l)(\alpha - c_{D,l})c_{D,l}}{4\eta(k+2)} > 0$, since

$\alpha > c_{D,l}$ and $\Phi_l \equiv \frac{k(\theta_l\gamma - 2\beta_l^2 L_l)^2 + (\theta_l\gamma)^2(k+2)}{(k+1)\theta_l\gamma(2\theta_l\gamma - \beta_l^2 L_l)} > \lambda_{D,l} > 0$.³⁰ Moreover, notice that $\Phi_l = 1$ in Montagna and Nocco (2013) when quality is not relevant with $\beta_l = 0$.

Taking the expression above for the average income we obtain that $\frac{\partial \bar{I}_l}{\partial v_l} = \frac{\frac{1}{\eta} c_{D,l} \{\alpha[k(2-v_l)+4] - c_{D,l}[2v_l+4+k(2-v_l)]\}}{(k+2)^2(2-v_l)}$, which is used in the main text of the paper to assess when $\partial \bar{I}_l / \partial v_l \gtrless 0$.

A.7 Proof of proposition 5 and of its corollary

Proposition 5 *An increase in the bargaining power of unions v_l reduces the firm's investment in quality for the domestic market, $z_{D,l}(c)$, for the most productive firms (i.e. firms with $c < c^*$), increasing it for less productive firms (i.e. firms with $c > c^*$).*

Proof Differentiating the cutoff for the open economy

$$c_{D,l} = \left\{ \frac{4\gamma(k+2)(k+1)}{L_l} \frac{[(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}]}{(2-v_l)^2 (2-v_h)^2 [\lambda_{D,h} \lambda_{D,l} - (\tau_{hl})^{-k} (\tau_{lh})^{-k} \lambda_{X,l} \lambda_{X,h}]} \right\}^{\frac{1}{k+2}}$$

gives

$$\frac{\partial c_{D,l}}{\partial v_l} = \frac{2(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} c_{D,l}}{(2-v_l)(k+2) [(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}]} > 0 \quad (41)$$

Considering that the expression for quality in the open economy is the same as that in the closed economy, (41) can be substituted into (37) to get

$$\begin{aligned} \frac{\partial \ln z_{D,l}(c)}{\partial v_l} &= -\frac{1}{2-v_l} + \frac{1}{c_{D,l}-c} \frac{\partial c_{D,l}}{\partial v_l} = \\ &= \frac{-(c_{D,l}-c)(k+2) [(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}] + 2(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} c_{D,l}}{(2-v_l)(c_{D,l}-c)(k+2) [(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}]} \end{aligned}$$

This expression can be rewritten as

$$\frac{\partial z_{D,l}(c)}{\partial v_l} = \frac{[-kc_{D,l}+c(k+2)](2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} + (c_{D,l}-c)(k+2)(2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}}{(2-v_l)(c_{D,l}-c)(k+2) [(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}]} z_{D,l}(c)$$

³⁰ Specifically, Φ_l is a bundle of parameters, and it can be readily shown that $\Phi_l > \lambda_{D,l}$. To show this,

Φ_l can be rewritten as follows $\Phi_l = \frac{k\lambda_{D,l}}{(k+1)} \left[\left(\frac{\theta_l\gamma - 2\beta_l^2 L_l}{\theta_l\gamma} \right)^2 + \frac{(k+2)}{k} \right]$. Then, it can be pointed out that

$\Phi_l > \lambda_{D,l}$ when $\left(\frac{\theta_l\gamma - 2\beta_l^2 L_l}{\theta_l\gamma} \right)^2 + \frac{k+2}{k} > \frac{k+1}{k}$, and this is always the case given that $k > 1$.

where the sign of the numerator determines the sign of the derivative that is negative when $\frac{kc_{D,l}-c(k+2)}{(c_{D,l}-c)} > \frac{(k+2)(2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}}{(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h}}$ ³¹ viceversa, it will be positive when the opposite inequality holds. Since the left hand side of the inequality is decreasing in c in its relevant range ($c \leq c_{D,l}$), and given that the right hand side is positive (while it is zero in the autarky case when $(\tau_{lh})^{-k} = 0$), there exists c^* such that $z_{D,l}(c)$ decreases for more productive firms with $c < c^*$, and it increases for less productive firms with $c > c^*$, as in autarky.³² Finally, notice that $c^* < \frac{k}{k+2} c_{D,l}$, where $c_{D,l}$ refers to the open economy cutoff. $\square$

Corollary *The reduction of the investment in quality $z_{D,l}(c)$ is larger in an autarkic environment, while the increase is larger in a free trade environment (i.e. Most productive firms reduce more innovation in an autarkic environment, while less productive firms increase more innovation in a free trade environment).*

Proof Note that in both cases we have that

$$\frac{\partial \ln z_{D,l}(c)}{\partial v_l} = -\frac{1}{2-v_l} + \frac{1}{c_{D,l}-c} \frac{\partial c_{D,l}}{\partial v_l}$$

Let us distinguish with $c_{D,l}^*$ the threshold cutoff in free trade and $c_{D,l}$ the threshold cutoff in autarky. $\frac{\partial \ln z_{D,l}^*(c)}{\partial v_l} > \frac{\partial \ln z_{D,l}(c)}{\partial v_l}$ when $\frac{1}{c_{D,l}^*-c} \frac{\partial c_{D,l}^*}{\partial v_l} > \frac{1}{c_{D,l}-c} \frac{\partial c_{D,l}}{\partial v_l}$.

Therefore, from the computations the last inequality is

$$\frac{2(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h}}{(2-v_l)(k+2) \left[(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l} \right]} \frac{c_{D,l}^*}{c_{D,l}^*-c} > \frac{c_{D,l}}{c_{D,l}-c} \frac{2}{(k+2)(2-v_l)}.$$

Simplifying

$$\frac{(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h}}{\left[(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l} \right]} \frac{c_{D,l}^*}{c_{D,l}^*-c} > \frac{c_{D,l}}{c_{D,l}-c}.$$

Note that the second element of the left hand side is larger than the first element of the right hand side when $\frac{c_{D,l}^*}{c_{D,l}^*-c} > \frac{c_{D,l}}{c_{D,l}-c}$, which simplifying implies that $c_{D,l}^* < c_{D,l}$. But this is the case as shown before.

The first element of the left hand side is bigger than one as the denominator is always positive.

Consequently we do have that $\frac{\partial \ln z_{D,l}^*(c)}{\partial v_l} > \frac{\partial \ln z_{D,l}(c)}{\partial v_l}$ which means that when $\frac{\partial \ln z_{D,l}^*(c)}{\partial v_l} > 0$ this is larger than the one in autarky so for the least productive firms the increase in innovation is larger in a free trade environment and when $\frac{\partial \ln z_{D,l}^*(c)}{\partial v_l} < 0$ it means that the reduction in innovation is larger in the autarkic environment. $\square$

³¹ Notice that if the two countries are symmetric in terms of market size L , cost of quality upgrading θ and preference for quality β , and, therefore, the denominator of the cutoff is positive, the numerator of the cutoff has to be positive. This requires that $\left[(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l} \right] > 0$. This would hold for continuity for small changes in the relevant parameters.

³² More precisely, c^* is implicitly defined as the value of c such that the right hand side is equal to the left hand side (i.e. $\frac{kc_{D,l}-c(k+2)}{(c_{D,l}-c)} = \frac{(k+2)(2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}}{(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h}}$).

A.8 Proof of propositions 6 and 7

Proposition 6 When the economy is open to trade, an increase in v_l decreases the firm's investment in quality for the foreign market by a firm originating in the home market (i.e. $\frac{\partial z_{X,l}}{\partial v_l} < 0$).

Proposition 7 When the economy is open to trade, an increase in v_l increases the firm's investment in quality for market l by a firm producing in the foreign market h (i.e. $\frac{\partial z_{X,h}}{\partial v_l} > 0$).

Proof Proposition 6) Note that the quality offered by a domestic firm in the foreign market is given by

$$z_{X,l}(c) = \frac{\beta_h L_h (2 - v_l)}{2\gamma\theta_l} \lambda_{X,l} \tau_{lh} (c_{X,l} - c)$$

Taking logs and differentiating the expression above with respect to v_l we find that

$$\frac{\partial \ln z_{X,l}(c)}{\partial v_l} = -\frac{1}{2 - v_l} + \frac{1}{(c_{X,l} - c)} \frac{\partial c_{X,l}}{\partial v_l}$$

and then, noting that $c_{X,l} = c_{D,h}/\tau_{lh}$, this can be rewritten as

$$\frac{\partial \ln z_{X,l}(c)}{\partial v_l} = -\frac{1}{2 - v_l} + \frac{1}{(c_{X,l} - c)} \frac{c_{D,h}}{\tau_{lh}} \frac{\partial \ln c_{D,h}}{\partial v_l}$$

and, given the cutoff in (36),

$$\begin{aligned} \frac{\partial \ln c_{D,h}}{\partial v_l} &= \frac{1}{k+2} \left[\frac{2}{2 - v_l} - \frac{2(2 - v_l) f_{E,h} c_{M,h}^k \lambda_{D,l}}{(2 - v_l)^2 f_{E,h} c_{M,h}^k \lambda_{D,l} - (2 - v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h}} \right] = \\ &= -\frac{2(2 - v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h}}{(k+2) \left[(2 - v_l)^2 f_{E,h} c_{M,h}^k \lambda_{D,l} - (2 - v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h} \right]} < 0 \end{aligned}$$

Then

$$\begin{aligned} \frac{\partial z_{X,l}(c)}{\partial v_l} &= - \left\{ \frac{1}{2 - v_l} + \frac{2(2 - v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h} c_{D,h}}{(c_{X,l} - c) \tau_{lh} (k+2) \left[(2 - v_l)^2 f_{E,h} c_{M,h}^k \lambda_{D,l} - (2 - v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h} \right]} \right\} \\ z_{X,l}(c) &< 0 \end{aligned}$$

Proof of Proposition 7) Note that the quality offered by a foreign firm producing in h in the domestic market l is given by

$$z_{X,h}(c) = \frac{\beta_l L_l (2 - v_h)}{2\gamma\theta_h} \lambda_{X,h} \tau_{hl} (c_{X,h} - c)$$

Therefore taking logs and differentiating with respect to v_l gives

$$\frac{\partial \ln z_{X,h}(c)}{\partial v_l} = \frac{\partial \ln (c_{X,h} - c)}{\partial v_l} = \frac{1}{(c_{X,h} - c)} \frac{\partial c_{X,h}}{\partial v_l}$$

and, considering $c_{X,h} = c_{D,l}/\tau_{hl}$, yields

$$\frac{\partial z_{X,h}(c)}{\partial v_l} = \frac{z_{X,h}(c)}{(c_{X,h} - c)} \frac{\partial c_{D,l}}{\partial v_l} > 0$$

□

A.9 Proof of proposition 8

Proposition 8 *An increase in the bargaining power of unions v_l :*

- reduces the average investment in quality for the domestic market $\bar{z}_{D,l}$ by local firms when $\frac{(2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}}{(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h}} < \frac{k}{(k+2)}$, otherwise, it increases it;*
- increases the average investment in quality by firms exporting in l ($\bar{z}_{X,h}$);*
- reduces the average quality offered by domestic firms in the foreign market ($\bar{z}_{X,l}$).*

Proof a) The average quality for the domestic market is

$$\begin{aligned} \bar{z}_{D,l} &= \frac{(2-v_l) \beta_l L_l \lambda_{D,l}}{2\gamma \theta_l (k+1)} c_{D,l} = \\ &= \frac{(2-v_l)^{\frac{k}{k+2}} \beta_l L_l \lambda_{D,l}}{2\gamma \theta_l (k+1)} \left\{ \frac{4\gamma (k+2)(k+1)}{L_l} \frac{[(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}]}{(2-v_h)^2 [\lambda_{D,h} \lambda_{D,l} - (\tau_{hl})^{-k} (\tau_{lh})^{-k} \lambda_{X,l} \lambda_{X,h}]} \right\}^{\frac{1}{k+2}} \end{aligned}$$

Taking log and differentiating it with respect to v_l gives

$$\begin{aligned} \frac{\partial \bar{z}_{D,l}}{\partial v_l} &= \frac{\bar{z}_{D,l}}{(k+2)} \\ &\left\{ -\frac{k}{(2-v_l)} + \frac{2(2-v_l) f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}}{[(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}]} \right\} \end{aligned}$$

that is negative when

$$\begin{aligned} &-k \left[(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l} \right] \\ &+ 2(2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l} < 0; \\ &\frac{(2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}}{(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h}} < \frac{k}{(k+2)} \end{aligned}$$

otherwise, it is positive.

b) The average investment in quality by firms exporting in l is

$$\bar{z}_{X,h} = \frac{(2 - v_h)}{2(k+1)} \frac{\beta_l L_l}{2\gamma\theta_h - \beta_l^2 L_l} c_{D,l}$$

and, making use of (41), it can be readily verified that

$$\frac{\partial \ln \bar{z}_{X,h}}{\partial v_l} = \frac{\partial \ln c_{D,l}}{\partial v_l}$$

c) The average quality offered by domestic firms in the foreign market is given by

$$\bar{z}_{X,l} = \frac{\beta_h L_h (2 - v_l) \lambda_{X,l}}{2\gamma\theta_l (k+1)} \tau_{lh} c_{X,l}$$

Taking logs and differentiating the expression above with respect to v_l we find that

$$\frac{\partial \ln \bar{z}_{X,l}}{\partial v_l} = -\frac{1}{2 - v_l} + \frac{\partial \ln c_{X,l}}{\partial v_l}$$

and then, noting that $c_{X,l} = c_{D,h}/\tau_{lh}$, this can be rewritten as

$$\frac{\partial \ln \bar{z}_{X,l}(c)}{\partial v_l} = -\frac{1}{2 - v_l} + \frac{\partial \ln c_{D,h}}{\partial v_l} < 0$$

given that in proof 4 it is shown that $\frac{\partial \ln c_{D,h}}{\partial v_l} < 0$. Given that $c_{X,l} = c_{D,h}/\tau_{lh}$, it is also the case that $\frac{\partial \ln c_{X,l}}{\partial v_l} < 0$. □

A.10 Proof of proposition 9

Proposition 9 *An increase in the bargaining power of unions v_l starting from a situation in which the two countries are symmetric, decreases total R&D of exporting firms.*

Proof Let us first compute the effect on $z_{D,l}(c)$:

$$\frac{\partial \ln z_{D,l}(c)}{\partial v_l} = -\frac{1}{2 - v_l} + \frac{1}{c_{D,l} - c} \frac{\partial c_{D,l}}{\partial v_l} = -\frac{1}{2 - v_l} + \left\{ \frac{[(k+2)(c_{D,l} - c) - 2c_{D,l}](2 - v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (k+2)(c_{D,l} - c)(2 - v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}}{(c_{D,l} - c)(k+2)[(2 - v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2 - v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}]} \right\}$$

, obtained making use of

$$\frac{\partial c_{D,h}}{\partial v_l} = \frac{1}{k+2} \frac{-2(2-v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h} c_{D,h}}{(2-v_l) \left[(2-v_l)^2 f_{E,h} c_{M,h}^k \lambda_{D,l} - (2-v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h} \right]}$$

, which can be rewritten as

$$\begin{aligned} & \frac{\partial z_{D,l}(c)}{\partial v_l} \\ &= -\frac{1}{2-v_l} \left\{ \frac{[kc_{D,l} - (k+2)c] (2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (k+2) (c_{D,l} - c) (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}}{(c_{D,l} - c) (k+2) [(2-v_h)^2 f_{E,l} c_{M,l}^k \lambda_{D,h} - (2-v_l)^2 f_{E,h} c_{M,h}^k (\tau_{lh})^{-k} \lambda_{X,l}]} \right\} z_{D,l}(c). \end{aligned}$$

Then, let us compute the effect on $z_{X,l}(c) = \frac{\beta_h L_h (2-v_l)}{2\gamma \theta_l} \lambda_{X,l} \tau_{lh} (c_{X,l} - c) =$
 $= \frac{\beta_h L_h (2-v_l)}{2\gamma \theta_l} \lambda_{X,l} \tau_{lh} \left(\frac{c_{D,h}}{\tau_{lh}} - c \right)$, that is

$$\begin{aligned} \frac{\partial \ln z_{X,l}(c)}{\partial v_l} &= -\frac{1}{2-v_l} + \frac{1}{(c_{X,l} - c)} \frac{\partial c_{X,l}}{\partial v_l} = -\frac{1}{2-v_l} + \frac{1}{(c_{X,l} - c)} \frac{1}{\tau_{lh}} \frac{\partial c_{D,h}}{\partial v_l} = \\ &= -\left(\frac{1}{2-v_l} + \frac{1}{(c_{X,l} - c)} \frac{1}{k+2} \frac{2(2-v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h} c_{D,h}}{(2-v_l)^2 f_{E,h} c_{M,h}^k \lambda_{D,l} - (2-v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h}} \right) \end{aligned}$$

, which can be rewritten as

$$\begin{aligned} \frac{\partial z_{X,l}(c)}{\partial v_l} &= -\left\{ \frac{1}{2-v_l} + \frac{2(2-v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h} c_{D,h}}{(2-v_l)(c_{X,l} - c) \tau_{lh} (k+2) [(2-v_l)^2 f_{E,h} c_{M,h}^k \lambda_{D,l} - (2-v_h)^2 f_{E,l} c_{M,l}^k (\tau_{hl})^{-k} \lambda_{X,h}]} \right\} \\ z_{X,l}(c) &< 0. \end{aligned}$$

The R&D expenditures of a firm in l , serving countries l and h , with a marginal cost c are given by $R\&D(c) = \frac{\theta_l}{2} \left[z_{D,l}^2(c) + z_{X,l}^2(c) \right]$, where $z_{X,l}(c) > 0$ if and only if $c < \frac{c_{D,l}}{\tau}$.

Then,

$$\begin{aligned} \frac{\partial R\&D(c)}{\partial v_l} &= \frac{\theta_l}{2} \left[2z_{D,l}(c) \frac{\partial z_{D,l}(c)}{\partial v_l} + 2z_{X,l}(c) \frac{\partial z_{X,l}(c)}{\partial v_l} \right] \\ &= \theta_l \left[z_{D,l}(c) \frac{\partial z_{D,l}(c)}{\partial v_l} + z_{X,l}(c) \frac{\partial z_{X,l}(c)}{\partial v_l} \right]. \end{aligned}$$

Evaluating these derivatives at the full symmetric case, which implies that $c_{D,l} = c_{D,h} = c_D$ and $\lambda_{D,l} = \lambda_{D,h} = \lambda_{X,l} = \lambda_{X,h} = \lambda$, we find that

$$\frac{\partial z_{D,l}(c)}{\partial v_l} = -\frac{1}{2-v} \left(\frac{[kc_D - c(k+2)] - (k+2)(c_D - c)(\tau)^{-k}}{(c_D - c)(k+2)[1 - (\tau)^{-k}]} \right) z_D(c),$$

and, since $z_{D,l}(c) = \frac{\beta_l L_l(2-v_l)}{2\gamma\theta_l} \lambda_{D,l}(c_{D,l} - c)$, it can be rewritten as

$$\frac{\partial z_{D,l}(c)}{\partial v_l} = \frac{[-kc_D + c(k+2) + (c_D - c)(k+2)(\tau)^{-k}]}{(k+2)[1-(\tau)^{-k}]} \frac{\beta L}{2\gamma\theta} \lambda.$$

Moreover, multiplying previous expression for $z_{D,l}(c)$ gives

$$z_{D,l}(c) \frac{\partial z_{D,l}(c)}{\partial v_l} = \frac{[-kc_D + c(k+2) + (c_D - c)(k+2)(\tau)^{-k}]}{(k+2)[1-(\tau)^{-k}]} \frac{\beta^2 L^2}{4\gamma^2 \theta^2} \lambda^2 (2-v) (c_D - c).$$

Now let us take $\frac{\partial z_{X,l}(c)}{\partial v_l}$ from above and rewrite it in the case of symmetry as follows

$$\frac{\partial z_{X,l}(c)}{\partial v_l} = - \left\{ \frac{\beta L}{2\gamma\theta} \lambda \tau (c_X - c) + \frac{\beta L}{\gamma\theta} \lambda \frac{(\tau)^{-k} c_D}{(k+2)[1-(\tau)^{-k}]} \right\} < 0.$$

Then, multiplying previous expression for $z_{X,l}(c)$ gives

$$\begin{aligned} z_{X,l}(c) \frac{\partial z_{X,l}(c)}{\partial v_l} \\ = - \frac{\beta^2 L^2 (2-v)}{2\gamma^2 \theta^2} \lambda^2 \tau \left(\frac{c_D}{\tau} - c \right) \left\{ \frac{1}{2} \tau \left(\frac{c_D}{\tau} - c \right) + \frac{(\tau)^{-k} c_D}{(k+2)[1-(\tau)^{-k}]} \right\} < 0. \end{aligned}$$

Hence, making use of these results, we get that the sign of $\frac{\partial R \& D(c)}{\partial v_l}$ depends on that of

$$\begin{aligned} z_{D,l}(c) \frac{\partial z_{D,l}(c)}{\partial v_l} + z_{X,l}(c) \frac{\partial z_{X,l}(c)}{\partial v_l} &= \\ &= \frac{[-kc_D + c(k+2) + (c_D - c)(k+2)(\tau)^{-k}]}{(k+2)[1-(\tau)^{-k}]} \frac{\beta^2 L^2}{4\gamma^2 \theta^2} \lambda^2 (2-v) (c_D - c) + \\ &\quad - \frac{\beta^2 L^2 (2-v)}{2\gamma^2 \theta^2} \lambda^2 \tau \left(\frac{c_D}{\tau} - c \right) \left\{ \frac{1}{2} \tau \left(\frac{c_D}{\tau} - c \right) + \frac{(\tau)^{-k} c_D}{(k+2)[1-(\tau)^{-k}]} \right\} = \\ &= \frac{\beta^2 L^2 (2-v)}{2\gamma^2 \theta^2} \lambda^2 \\ &\quad \left\{ \frac{[-kc_D + c(k+2) + (c_D - c)(k+2)(\tau)^{-k}]}{(k+2)[1-(\tau)^{-k}]} \frac{1}{2} (c_D - c) - (c_D - \tau c) \left[\frac{1}{2} (c_D - \tau c) + \frac{(\tau)^{-k} c_D}{(k+2)[1-(\tau)^{-k}]} \right] \right\} = \\ &= \frac{\beta^2 L^2 (2-v)}{2\gamma^2 \theta^2} \lambda^2 \\ &\quad \left\{ \frac{[-kc_D + c(k+2) + (c_D - c)(k+2)(\tau)^{-k}]}{(k+2)[1-(\tau)^{-k}]} \frac{1}{2} (c_D - c) - (c_D - \tau c) \frac{(\tau)^{-k} c_D}{(k+2)[1-(\tau)^{-k}]} - \frac{1}{2} (c_D - \tau c)^2 \right\} = \\ &= \frac{\beta^2 L^2 (2-v)}{4\gamma^2 \theta^2} \lambda^2 \end{aligned}$$

$$\left\{ \frac{[-kc_D + c(k+2)](c_D - c) + (c_D - c)^2(k+2)(\tau)^{-k} - 2(\tau)^{-k}c_D(c_D - \tau c)}{(k+2)[1 - (\tau)^{-k}]} - (c_D - \tau c)^2 \right\} =$$

$$= \frac{\beta^2 L^2 (2 - v)}{4\gamma^2 \theta^2} \lambda^2$$

$$\left\{ \frac{[-kc_D + c(k+2)](c_D - c) + (c_D - c)^2(k+2)(\tau)^{-k} - 2(\tau)^{-k}c_D(c_D - \tau c) - (c_D - \tau c)^2(k+2)[1 - (\tau)^{-k}]}{(k+2)[1 - (\tau)^{-k}]} \right\}$$

Previous expression is negative, if its numerator in the curly brackets is negative, that is if

$$[-kc_D + c(k+2)](c_D - c) + (c_D - c)^2(k+2)(\tau)^{-k} - 2(\tau)^{-k}c_D(c_D - \tau c) - (c_D - \tau c)^2(k+2)[1 - (\tau)^{-k}] < 0$$

. Viceversa, it is positive, if the numerator is positive. In what follows, we show that it is always negative.

This can be achieved by rewriting the last expression as follow

$$\left\{ \left[(\tau)^{-k} - 1 \right] (k+2) \tau^2 - k + (\tau)^{-k} (k+2) - 2 \right\} c^2 +$$

$$+ \left[c_D (k+2) + kc_D + 2c_D (\tau)^{1-k} - 2c_D (\tau)^{-k} (k+2) \right. \\ \left. - 2c_D \tau \left((\tau)^{-k} - 1 \right) (k+2) \right] c$$

$$+ \left\{ (c_D)^2 (\tau)^{-k} (k+2) - 2(c_D)^2 (\tau)^{-k} - k(c_D)^2 + (c_D)^2 \left[(\tau)^{-k} - 1 \right] (k+2) \right\}$$

Previous expression is a downward opening parabola in c , which is negative for $c \geq 0$ given that it has the following properties:

1) When $c = 0$, the value of the parabola is negative, because

$$\left\{ (c_D)^2 (\tau)^{-k} (k+2) - 2(c_D)^2 (\tau)^{-k} - k(c_D)^2 + (c_D)^2 \left[(\tau)^{-k} - 1 \right] (k+2) \right\} =$$

$$= 2(c_D)^2 \left[(\tau)^{-k} - 1 \right] (k+1) < 0.$$

This, the intercept of the parabola with the vertical axis is negative.

2) The coefficient of c^2 is negative and, consequently, the parabola is concave and it opens downward because $\left\{ \left[(\tau)^{-k} - 1 \right] (k+2) \tau^2 - k + (\tau)^{-k} (k+2) - 2 \right\} = (\tau^2 + 1) \left[(\tau)^{-k} - 1 \right] (k+2) < 0$.

3) The coefficient of c is

$$\left[c_D (k+2) + kc_D + 2c_D (\tau)^{1-k} - 2c_D (\tau)^{-k} (k+2) - 2c_D \tau \left((\tau)^{-k} - 1 \right) (k+2) \right] =$$

$$= -2c_D \left[-k - 2\tau + 2(\tau)^{-k} - k\tau + k(\tau)^{-k} + (\tau)^{1-k} + k(\tau)^{1-k} - 1 \right]$$

Moreover, the maximum value of the parabola is obtained when

$$2(\tau^2 + 1) \left[(\tau)^{-k} - 1 \right] (k + 2) c - 2c_D \left[-k - 2\tau + 2(\tau)^{-k} - k\tau + k(\tau)^{-k} + (\tau)^{1-k} + k(\tau)^{1-k} - 1 \right] = 0;$$

that is in

$$\begin{aligned} c &= -\frac{2c_D \left[-k - 2\tau + 2(\tau)^{-k} - k\tau + k(\tau)^{-k} + (\tau)^{1-k} + k(\tau)^{1-k} - 1 \right]}{2(\tau^2 + 1) \left[(\tau)^{-k} - 1 \right] (k + 2)} \\ &= \frac{c_D \left[-k - 2\tau + 2(\tau)^{-k} - k\tau + k(\tau)^{-k} + (\tau)^{1-k} + k(\tau)^{1-k} - 1 \right]}{(\tau^2 + 1) \left[1 - (\tau)^{-k} \right] (k + 2)} \end{aligned}$$

, which is negative because the denominator is positive and, as it will be shown below, the numerator is negative. Indeed, the expression in the brackets in the numerator can be rewritten as follows:

$$\begin{aligned} &\left[-k - 2\tau + 2(\tau)^{-k} - k\tau + k(\tau)^{-k} + (\tau)^{1-k} + k(\tau)^{1-k} - 1 \right] = \\ &= \left[(\tau)^{-k} - \tau + (\tau)^{1-k} - 1 \right] k + \left[2(\tau)^{-k} - 2\tau + (\tau)^{1-k} - 1 \right] \end{aligned}$$

In what follow, we show that the coefficient of k and the other term both expressed as fuctions of τ are negative for $\tau \geq 1$ for different values of $k \geq 1$.

Indeed, it can be readily shown that:

$$\left[(\tau)^{-k} - \tau + (\tau)^{1-k} - 1 \right] = (\tau)^{-k} (1 + \tau) - (1 + \tau) = \left[(\tau)^{-k} - 1 \right] (1 + \tau) < 0$$

Moreover, it can also be shown that: $\left[2(\tau)^{-k} - 2\tau + (\tau)^{1-k} - 1 \right] = (\tau)^{-k} (2 + \tau) - (2\tau + 1) < 0$ iff $(\tau)^{-k} < \frac{(2\tau+1)}{(2+\tau)}$ which is always the case for $\tau > 1$ because $\frac{(2\tau+1)}{(2+\tau)} > 1$.

Therefore, the value of the parabola for $c \geq 0$ is negative, and, thus, the total expenditure in R&D for an exporting firm always decline when v_l increases, with $\frac{\partial R\&D(c)}{\partial v_l} < 0$. $\square$

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