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Comparison among variation models of the hydraulic conductivity with the effective porosity in confined aquifer

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Abstract. This paper presents the experimental investigation results from the modalities of variation of the hydraulic conductivity scaling law for a confined aquifer, varying the porous medium that constitutes it. In four subsequent stages, different confined aquifers were built up, each with a different typological configuration of a porous medium. For each of the aquifers considered, various hydraulic conductivity (*K*) measurements were performed by slug tests. The effective porosity (n_e) was set as a scale parameter, therefore the scaling laws $K = K(n_e)$, already determined and reported in previous studies, were taken into consideration for each of the four artificial aquifers considered. The same variation law of *K* vs n_e was also determined by means of some of the well-known empirical and semi-empirical relationships. The latter are based on the particle size distribution and are suitable for application to the porous media considered here, which can be classified as coarse sand. The comparison between the different scaling laws mentioned above allowed us to discuss, through graphical analysis, the reliability of the models considered here. This will facilitate researchers and practitioners working in the field, in the methodological choice of the most appropriate model that should be used for this type of porous media.

1. Introduction

Among the hydraulic parameters that allow the description of water flow and mass transport phenomena in porous media, one of the most representative and meaningful is certainly the hydraulic conductivity. Its scaling behavior has been verified under different conditions and, as shown in numerous researches, this parameter has a clear tendency to increase according to the scale [1-3]. This behavior is mainly due to the influence exerted by the heterogeneity of the porous medium. On small scales, this manifests itself mainly via the geometry and size of the pores. On the larger scales, it presents via the tortuosity, the continuity, and the interconnection of the pores and canaliculi [4]. The intergranular spaces of the porous medium, characterized by a specific size, shape, and granulometric assortment can generally be defined through parameters summarizing the textural characteristics of the medium. This can be done also through other parameters such as porosity (*n*) which is frequently considered to be a representative one. Many authors, to consider the interconnection of the voids, refer to the effective porosity (n_e) [5]. In some studies [6,7] the effective porosity is considered as a scale parameter. Some researchers [8,9] verified the existence of a scaling behavior, which showed a decline in the trend of porosity value with increasing scale. Thus, the definition of the scaling law that describes the behavior of hydraulic conductivity as the porosity varies, represents an important alternative to the use of semi-empirical and empirical formulas based on Grain Size Distribution [10-14]. Knowing the scaling law of a given parameter means being able to identify its spatial distribution in the domain considered, avoiding the use of traditional geostatistical methods [15].

The present experimental study aims to investigate the modalities of variation of the hydraulic conductivity scaling law for a confined aquifer. To this end, a series of slug tests were carried out in the laboratory and the hydraulic conductivity (K) values were measured. The same variation law of K vs n_e has also been determined, for these configurations, through some of the well-known relationships based on the grain size distribution and suitable to be applied to porous media of coarse sand, such as those considered in this investigation.

2. Materials and Methods

2.1. Experimental set-up

Several slug tests were performed by mean of an experimental device constituted by a metal box, with dimensions of 2 m x 2 m x 1 m (L x W x H)(Figure 1a) [16-18]. Ten PVC wells with a diameter D of 2.8 cm were located in the device, as shown in Figure 1b.

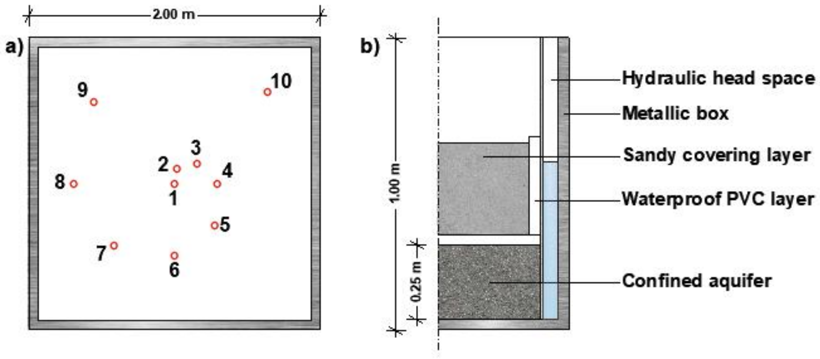

Figure 1. (**a**) Planimetric view of the confined aquifer with the location of the wells. (**b**) Stratigraphic scheme and section of the artificial aquifer.

Well No. 1 (Figure 1a) is the injection well, while the others are defined as observation wells. A perimeter chamber obtained along the internal perimeter of the device and connected to two external loading reservoirs, made it possible to verify compliance with the hydraulic head condition set for each test. At the end of each test and before the next, it was verified that the hydraulic head conditions returned to their initial undisturbed state. More details are given in [16, 17].

Water volumes ranging from 0.03 L to 0.09 L were injected into the central well for the execution of the slug tests, for each of the soil configurations considered. The measure of the variations of the hydraulic load values during the tests, at a data acquisition frequency equal to 100Hz, were obtained by means of transducers [18,19] placed on the bottom of each well. As proposed in the study conducted by Aristodemo et al. [16], the data was then filtered through the Mexican Hat Wavelet Transform.

2.2. Soil configurations

To determine the main granulometric and texture characteristics, a careful granulometric analysis has been made for each of the four porous media considered in the experiments. Table 1 shows the percentage values relating to the particle size composition of the materials, the diameter value d_{10}, the uniformity coefficient ($U=d_{60}/d_{10}$), the total porosity, and the effective porosity ($U=d_{60}/d_{10}$).
In Table 1 the percentages of the granulometric components, the diameter value d_{10}, the uniformity coefficient ($U=d_{60}/d_{10}$), and the total and effective porosity, are shown [16,17]. Figure 2 shows the granulometric curves of the considered porous media and shows that they are predominantly coarse-grained. In particular, the porous medium of type I can be defined as sandy, while those of type I, II, and III are predominantly sandy, with an amount of gravel that cannot be neglected.

Table 1. Granulometric components for each configuration [16,17]

Textural parameters and porosity	Porous media			
	Type I (%)	Type II (%)	Type III (%)	Type IV (%)
Gravel	12.01	27.70	23.90	22.50
Sand	87.39	71.00	61.00	56.10
Silt	0.60	1.30	15.10	16.40
Clay	---	---	---	5.00
d_{10} (mm)	0.19	0.16	0.02	0.0055
$U = d_{60}/d_{10}$	5,21	8.125	51.5	163.63
n	37.60	27.30	29.30	27.50
n_e	5.60	8.60	13.00	19.00

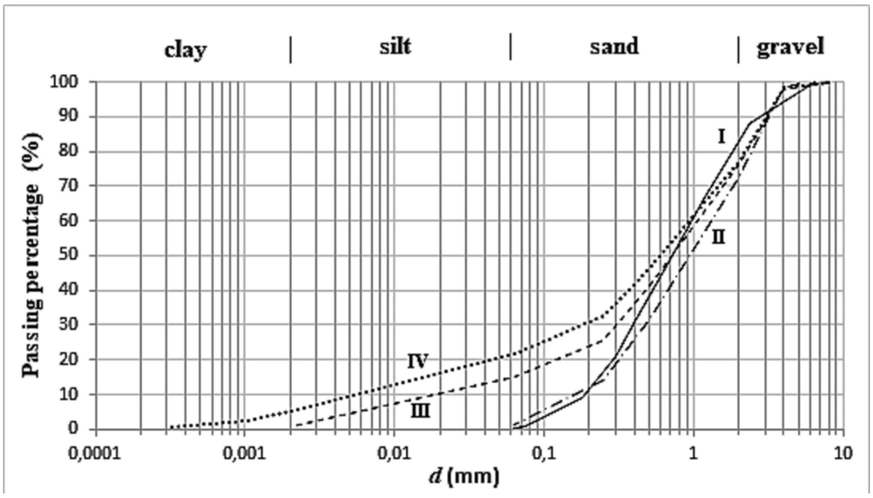

Figure 2. Granulometric curve. [16, 17].

2.3. Grain size analysis

The representation of the scaling behavior of a given hydraulic parameter characterizing an aquifer can be carried out through different types of law. Certainly the most suitable for this type of investigation is the power-type law, defined by the following relationship (1):

$$P = a \cdot x^{b} \tag{1}$$

where P [LT^{-1}] is the hydraulic parameter (i.e. hydraulic conductivity), x [L] refers to the scale, a [conguence] portends the ratio of structure to homogeneity, b is scaling index (crowding index).

The definition of the parameters obtained through the granulometric analysis of the porous media has allowed the determination of the hydraulic conductivity values by applying the most suitable formulas for the specific case considered [20]. The use of equation (1) allows the determination of K through the power-type scale law which uses n_e as a scaling parameter. In addition, both semi-empirical and empirical formulas generally follow the model proposed by Vuković and Soro [21], reported here in equation (2):

$$K = \frac{g}{\nu} C f(n_e) d_e^2 \tag{2}$$

where K is the hydraulic conductivity referred to the saturated porous media [LT^{-1}], g the gravity acceleration [LT^{-2}], ν the kinematic viscosity [L^2T^{-1}], C represents a general coefficient [-], n_e is the effective porosity [-], $f(n_e)$ the porosity function that relates the modeled porous medium to the real one, and d_e the effective diameter of the grain [L].

Where P (LT-1) is the hydraulic parameter, x (L) refers to the scale, a (congruence) portends ratio of structure to homogeneity, b is scaling index (crowding index)

2.4. Empirical Formulae

Several empirical equations used to calculate hydraulic conductivity by mean of grains size distribution have been evaluated. Kozeny [22] recommends the following empirical formula (3) for calculation of hydraulic conductivity when assuming the common ground-water temperature of 10°C:

$$K = \frac{g}{v} \times 8.3 \times 10^{-3} \left(\frac{n^3}{(1-n)^2}\right) d_{10}^2 \tag{3}$$

where K is the hydraulic conductivity t 10°C and d_{10} is the effective grain-size diameter.

The Terzaghi [23] empirical formula (4), for the determination of hydraulic conductivity, is:

$$K = \beta_0 \frac{\mu_{10}}{\mu_t} \left(\frac{n-0.13}{\sqrt[3]{1-n}}\right) d_{10}^2 \tag{4}$$

where β_0 is an empirical coefficient that depends on the nature of the grain surface, μ_{10} and μ_t are coefficients of dynamic viscosity at 10°C and 0°C respectively d_{10} is effective grain diameter in mm, which depends on all fractions of the analyzed porous medium.

Boonstra and de Ridder [24] presented the following equation (5), determined gave the best results when compared with results from aquifer tests:

$$K = 54000 \frac{1}{U^2} C_{SO} C_{cl} C_{gr} \tag{5}$$

where U is the specific surface of the main sand fraction, C_{SO} is a correction factor for sorting of sand, C_{cl} is a correction factor for the presence of particles and C_{gr} is a correction factor for the presence of gravel [24].

Slichter [25] developed a formula (6) expressed as:

$$K = \frac{g}{v} 0.01(n^{3.287}) d_{10}^2 \tag{6}$$

That formula is applicable for grain sizes between 0.01mm and 5mm.

Beyer suggested the empirical formula (7) for the determination of hydraulic conductivity in the form:

$$K = \frac{g}{v} 6.0 \times 10^{-4} log\left(\frac{500}{C}\right) d_{10}^2 \tag{7}$$

where C is an uniformity coefficient. This formula is recommended for materials with grain diameter $0.06 < d_{10} < 0.6$ mm, when the uniformity coefficient is $1 < C < 20$ [21].

Based on 300 experiments, Pavchich [26] proposed the following empirical formula (8):

$$K = \frac{g}{v} \left(\frac{n^3}{(1-n)^2}\right) d_{17}^2 \tag{8}$$

the domain of applicability is 0.06 mm $< d_{17} <$ 1.5 mm. In addition, some other formulae were considered, that did not follow the model of Vuković and Soro [21]were considered, as the empirical formula proposed by Seelheim [27] and that of Kaubish [28]. The Seelheim formula (9) is the following:

$$K = 0.0036 \times d_{50}^2 \tag{9}$$

where d_{50} is the diameter of the 50 percentile grain size. This formula is commonly used for sands, clay, and elutriated chalk. The Kaubish formula is shown by equation (10):

$$K = 10^{0.0005P^2 - 0.12P - 3.59} \tag{10}$$

where $P < 0.06$ mm in %, for application range $60\% < P < 10\%$. Kaubish derived this formula (10) from permeameter tests, and by power law it relates K to the percentual quantity of grain sizes <0.06 mm in the sample.

3. Results

For each of the four aquifer configurations considered, the hydraulic conductivity values were obtained from a precise analysis of the variation of the hydraulic head values deriving from the execution of the slug tests. It is important to highlight how the overall composition relating to each soil configuration remains uniform throughout the volume of the aquifer, although each of them consists of a different porous medium. Table 2 shows the K values obtained together with the relative values of the radius of influence that were experimentally determined during the execution of the tests.

Table 2. Hydraulic conductivity and radii of influence values [16, 17]

V (L)	Type I		Type II		Type III		Type IV	
	k (m/s)	*R* (m)	*k* (m/s)	*R* (m)	*k* (m/s)	*R* (m)	*k* (m/s)	*R* (m)
0.03	$2.15{\cdot}10^{-4}$	0.590	$1.36{\cdot}10^{-4}$	0.600	$7.13{\cdot}10^{-4}$	0.820	$1.07{\cdot}10^{-3}$	0.840
0.04	$2.38{\cdot}10^{-4}$	0.720	$2.20{\cdot}10^{-4}$	0.750	$7.20{\cdot}10^{-4}$	0.840	$1.09{\cdot}10^{-3}$	0.870
0.06	$2.79{\cdot}10^{-4}$	0.835	$2.60{\cdot}10^{-4}$	0.840	$7.38{\cdot}10^{-4}$	0.899	$1.30{\cdot}10^{-3}$	0.910
0.07	$2.82{\cdot}10^{-4}$	0.850	$2.47{\cdot}10^{-4}$	0.860	$7.53{\cdot}10^{-4}$	0.910	$1.31{\cdot}10^{-3}$	0.914
0.08	$2.67{\cdot}10^{-4}$	0.870	$2.98{\cdot}10^{-4}$	0.906	$7.40{\cdot}10^{-4}$	0.909	$1.31{\cdot}10^{-3}$	0.916
0.09	$2.88{\cdot}10^{-4}$	0.930	$3.10{\cdot}10^{-4}$	0.936	$7.86{\cdot}10^{-4}$	0.940	$1.34{\cdot}10^{-3}$	0.950

Overall results showed (Figure 3) that, for each type of porous media considered, the hydraulic conductivities calculated employing the Kozeny-Carman formula [22] are quite accurate, although a lower slope is observed for the same range. The hydraulic conductivity varies in a small interval.

As regarding the other empirical and semi-empirical formulae considered, it has been noted that Boonstra and de Ridder [24] formula, gives values lower than the experimental ones, the differences increase with the increase of n_e. Concerning Pavchich [26] formula, which is valid in the domain of applicability 0.06 mm < d_{17} < 1.5 mm, it should be noted that types III and IV of porous media considered in this study are out of the field of applicability. Pavchich [25] formula, for types I and II of porous media considered, provided an attentive representation of the K=K(n_e) law.

As regarding Terzaghi's empirical law [23], assuming the value of the sorting coefficient $C_t = \beta_0 \frac{\mu_{10}}{\mu_t} = 6{,}1 \times 10^{-3}$, it is noted that, for the first three types of porous media considered, it provides an acceptable description of the variation of K with n_e. Instead, this formula is not representative of the porous medium of the IV type, as it contains a greater quantity of fine fractions. However, this formula provides values lower than the experimental ones with a growing difference with the increase of n_e. As regards Slichter's empirical law [25], K values were considerably lower than the experimental ones, for this reason, it was preferred not to report them in Figure 3. Seelheim [27] empirical formula, for types I and II of porous media considered, provided an attentive representation of the K=K(n_e) law, although the reliability of the results is lower than the other methods, giving a slightly higher value. The Kaubish [28] formula, taking into account the applicability limits, can only be used for porous media of the III and IV type, for which it provides a description very close to the experimental one, as shown in figure 3.

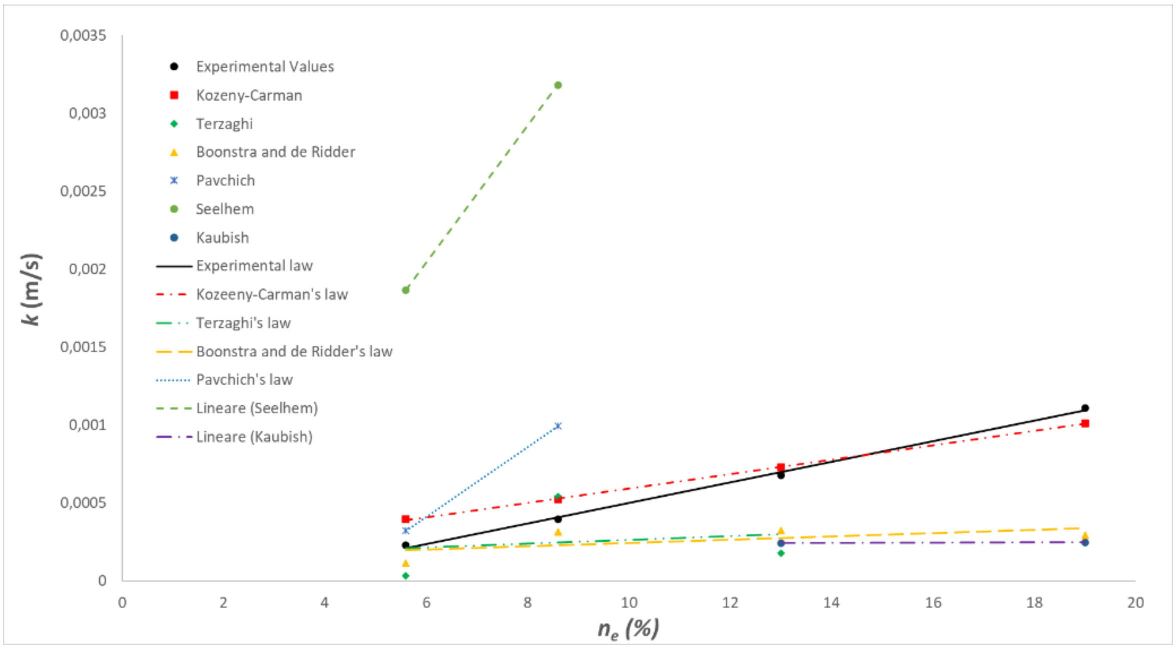

Figure 3. Comparison between the different empirical formulae used.

4. Conclusions

The present study shows, for the types of soils here considered, that both the empirical and semi-empirical formulas and the experimental scaling law are well suited to represent the scaling behavior of hydraulic conductivity. However, the experimental law seems to be preferable as it requires a smaller number of variables to be considered for its applicability. It is also important to point out that the experimental scaling law of the present study is to be considered valid for the particular types of soil here considered and can be extended only to soils of the same type, i.e. coarse sand media.

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