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The effects of cross-sensory attentional demand on subitizing and on mapping number onto space

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ABSTRACT

Various aspects of numerosity judgments, especially *subitizing* and the mapping of number onto space, depend strongly on attentional resources. Here we use a dual-task paradigm to investigate the effects of cross-sensory attentional demands on visual subitizing and spatial mapping. The results show that subitizing is strongly dependent on attentional resources, far more so than is estimation of higher numerosities. But unlike many other sensory tasks, visual subitizing is equally affected by concurrent attentionally demanding auditory and tactile tasks as it is by visual tasks, suggesting that subitizing may be amodal. Mapping number onto space was also strongly affected by attention, but only when the dual-task was in the visual modality. The non-linearities in numberline mapping under attentional load are well explained by a Bayesian model of central tendency.

1. Introduction

Most adult humans can count. However, we also share an approximate non-verbal system with infants and other animals: a direct visual sense of number (Burr & Ross, 2008). When verbal counting is prevented, we can still see and estimate the numerosity of large sets of items, although with a margin of error (Whalen, Gallistel, & Gelman, 1999), which increases with increasing set size. Small sets of items (up to 4 or 5) are perceived quickly and errorlessly by a system that is at least partially separate from estimation termed "subitizing" (from the Latin *subitus* meaning immediately). A good deal of evidence shows that both subitizing and estimation depend on attention (Railo et al., 2008; Raymond, Shapiro, & Arnell, 1992; Vetter, Butterworth, & Bahrami, 2008). However, it is not clear whether the attentional effects are modality specific, or whether they transfer across modalities. This question is particularly relevant to recent work showing that subitizing is not strictly visual, but also seems to operate in audition (Camos & Tillmann, 2008; Repp, 2007) and touch (Plaisier, Bergmann Tiest, & Kappers, 2009; Riggs et al., 2006).

1.1. Cross-modal attentional effects

Concurrent perceptual tasks of the same sensory modality interfere with each other to degrade performance (Pashler, 1992, 1994). However, evidence for cross-modal interference is conflicting. Bonnel and Hafter (1998) found evidence for audio–visual cross-modal interference for detecting the sign of a magnitude change (luminance in vision and intensity in audition). Spence, Ranson, and Driver (2000) found that selecting an auditory stream of words presented concurrently with a second (distractor) stream is more difficult if a video of moving lips mimicking the distracting sounds it is also displayed. These psychophysical findings are not only consistent with some of the cognitive literature of the 1970s and 1980s (Taylor, Lindsay, & Forbes, 1967; Tulving & Lindsay, 1967), but also with recent neurophysiological and imaging results. For example, Joassin et al. (2004) examined the electrophysiological correlates of auditory interference on vision in an identification task of non-ambiguous complex stimuli, such as faces and voices, and showed that cross-modal interactions occur at various different stages, involving brain areas such as the fusiform gyrus, associative auditory areas (BA 22), and the superior frontal gyri. Hein et al. (2007) showed with a functional magnetic resonance (fMRI) study, that even without competing motor responses, a simple auditory decision interferes with visual processing at neural levels including prefrontal cortex, middle temporal cortex, and other visual regions. Taken together these results imply that limitations on resources for vision and audition operate at a central level of processing, rather than in the auditory and visual peripheral senses.

However, much evidence also suggests independence of attentional resources for vision and audition. For example, Larsen et al. (2003) compared subject accuracy for identifying two concurrent stimuli (such as a visual and spoken letter) relative to

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performance in a single-task. They found that the proportion of correct responses was almost the same for all experimental conditions, either single-task or divided-attention. Similarly, Bonnel and Hafter (1998) used an audio–visual dual-task paradigm to show that when identification of the direction of a stimulus change is capacity-limited, simple detection of visual and auditory patterns is governed by ‘‘capacity-free’’ processes, as in the detection task there was no performance drop compared with single-task controls. Alais, Morrone, and Burr (2006) measured discrimination thresholds for visual contrast and auditory pitch, and showed that visual thresholds were unaffected by concurrent pitch discrimination of chords and *vice versa,* while when two tasks were performed within the same modality, thresholds increased by a factor of around two for visual discrimination and four for auditory discrimination. Also for sustained attentional tasks (such as 4 s of the *Moving-Objects-Tracking* task of Pylyshyn and Storm (1988) separate attentional resources seem to be allocated to vision and audition (Arrighi, Lunardi, & Burr, 2011). Many of these results are in line with imaging studies suggesting that attention can act at early levels, including primary cortices A1 and V1 (Jancke, Mirzazade, & Shah, 1999; Posner & Gilbert, 1999; Somers et al., 1999).

1.2. The effect of attention on numerosity perception

It is well established that even when verbal counting is prevented, humans can estimate the numerosity of large sets of items, albeit with error (usually about 25%). Smaller sets of numbers, up to about four, are enumerated quickly, effortlessly and accurately, termed *subitizing* (Kaufman & Lord, 1949). There has been a long-standing debate as to whether perception in the subitizing range invokes different processes than for larger *estimation* ranges, with evidence for and against (Atkinson, Campbell, & Francis, 1976; Balakrishnan & Ashby, 1992; Mandler & Shebo, 1982; Piazza et al., 2002; Sathian et al., 1999). One reason to suggest that different mechanisms may be involved is that the subitizing and estimation ranges seem to depend on attentional resources in a different fashion. Although subitizing is often thought to be pre-attentive, or at least makes use of pre-attentive information (Trick & Pylyshyn, 1994), several recent studies suggest that subitizing is in fact vulnerable to manipulations of attentive load (Olivers & Watson, 2008; Railo et al., 2008; Raymond, Shapiro, & Arnell, 1992; Vetter, Butterworth, & Bahrami, 2008). Our own studies also go in this direction, showing that for both dual-task and attentional-blink paradigms, precision in the subitizing range is far more affected than in the higher estimation range (Burr, Turi, & Anobile, 2010). We suggested that subitizing and estimation are not identical processes and that a relatively attention-free estimation mechanism could operate over both high and low number ranges, but small numbers, within the subitizing range, can call on an additional attentive mechanism that operates when attentional resources permit over a range of up to four items. In line with this idea, the ERP component P2p, a signature of numerosity processing, emerges in the subitizing range under dual-task conditions (Hyde & Wood, 2011).

Further evidence for this comes from the fact that, like many sensory attributes, numerosity is susceptible to adaptation: prolonged exposure to a more numerous visual stimulus makes the current stimulus appear less numerous, and *vice versa* (Burr & Ross, 2008). With normal free viewing, this effect is limited to numerosity estimation outside the subitizing range. However, under high attention load, numerosities with the subitizing range are also adapted (Burr, Anobile, & Turi, 2011). This suggests that when the supplementary attentive-mechanism for small numbers is impaired (by the dual task), only the estimation mechanism remains, which adapts as it does for high numerosities.

Interestingly, a body of research suggests that the capacity to rapidly enumerate low numbers of items many not be restricted to vision, but could reflect a general perceptual mechanism shared between different senses; subitizing has been shown to operate in audition (Camos & Tillmann, 2008; Repp, 2007), and also with haptic stimuli (Plaisier, Bergmann Tiest, & Kappers, 2009; Riggs et al., 2006). fMRI data also point to amodal representation of numbers. When subjects are asked to estimate numerosities of visual or auditory stimuli, both result in increased activity of a right lateralized fronto-parietal cortical network, independently of the modality of the stimuli (Piazza et al., 2006). Cross-modal interactions in subitizing have also been revealed in a study by Cordes et al. (2001), who showed that precision in tactile number production is affected by a concurrent verbal task.

1.3. Mapping numbers onto space

An interesting aspect of numerosity perception is our ready capacity to map numbers into space, pointing to intrinsic interconnections between number and space (Burr et al., 2010; Butterworth, 1999; Dehaene, 1997). Experimentally, this is studied with the so-called ‘‘numberline’’, where subjects are asked to position appropriately on the line numeric digits, or clouds of dots. Educated adults have no difficulty in doing this accurately, whereas the mapping of young children, children with dyscalculia and unschooled adults show distinct compressive, logarithmic-like non-linearities (Ashkenazi & Henik, 2010; Booth & Siegler, 2006; Dehaene et al., 2008; Geary et al., 2007, 2008; Siegler & Booth, 2004; Siegler & Opfer, 2003). Recently, we showed that limiting attentional resources by a dual-task also results in logarithmic-like numberline mapping (Anobile, Cicchini, & Burr, 2012).

However, the fact that the function follows a logarithmic form does not necessarily imply an intrinsic logarithmic representation of numerosity (Gallistel & Gelman, 1992; Karolis, Iuculano, & Butterworth, 2011). Several alternate explanations have also been put forward, including proportional judgments relative to the ends and centres of the numberline (Barth & Paladino, 2011), related to the well known *central tendency of judgment* (Hollingworth, 1910). We (Anobile et al.) have also explained the non-linearities in numberline-mapping caused by attention deprivation as a Bayesian model of central tendency, similar to that introduced by Jazayeri and Shadlen (2010) to model interval reproduction judgments. The results were well fit by a simple Bayesian model of central tendency, where *central tendency* is a *prior* of variable width, that effectively pulls the higher numbers towards the centre of the numberline (while the lower number remain anchored). We use this model again in this study (see Section 2 for details).

1.4. Goals of this study

The current study was designed to examine the role of cross-modal attentional competition in visual numerosity estimation, using dual-tasks with visual, auditory and haptic distractors on several number paradigms. We had three specific aims: (1) to test the effects of cross-modal attention on numerosity perception for both small (subitizing) and large item sets; (2) study the effects of cross-modal attention on mapping of numbers onto space; and (3) model the mapping effects within a Bayesian framework. We confirm our previous results, showing that high numbers are less affected by attentional demands, while the subitizing range is far more vulnerable. In the low subitizing range, the auditory and haptic distractors were as effective as visual distractors in decreasing precision. The results reinforce other studies in suggesting that subitizing may be an amodal capacity, not restricted to vision. We also replicate our previous results showing that dual-task attention to a concurrent visual task affects numberline mapping (well-modelled by a

Bayesian model), but further show that there is little cross-modal attentional effects from a concurrent auditory task to the visual numberline mapping.

2. Methods

Stimuli were presented in a dimly lit room on a 23-in. liquid crystal monitor (ACER) with 1280×1024 resolution, mean luminance 60 cd/m^2, refresh rate 60 Hz. Subjects viewed the screen binocularly at a distance of 57 cm. Stimuli were generated and presented with Matlab 7.6, using PsychToolbox routines (Brainard, 1997) running on a Macintosh laptop. Sounds were played by two loudspeakers (Trust SP-2420) flanking the computer screen. Speaker separation was around 80 cm and intensity 75 dB at the sound source. Haptic stimuli were delivered by a modified speaker resting on the index finger of the non-dominant hand (the left, for all the participants).

2.1. Experiment I: enumeration

2.1.1. Participants

Ten naive subjects (mean age: 26 ± 3) with normal or corrected-to-normal vision participated. Four subjects were tested in the visual attentional load and in one (of the two) auditory distractor paradigms (frequency discrimination). Four subjects (including three new) were tested on a different auditory attentional load task (time bisection). Finally, three new subjects performed the haptic load task (time bisection). All subjects performed the single task condition.

2.1.2. Stimuli and procedure

Each trial started with a fixation point (randomly displayed for a random interval from 200 to 2000 ms), followed by the simultaneous presentation of both distractors and numerosity task, both lasting 230 ms, followed immediately by a mask (600×600 pixels, randomly black or white) for 250 ms. The *numerosity stimulus* was a cloud of non-overlapping dots varying in number from 1 to 10, which subjects were required to enumerate. Dots were half-white and half-black so luminance was not a cue to number. Each dot was 0.4° in diameter, with position chosen at random within a matrix of 18° diameter. The visual distractor comprised four centrally positioned coloured squares, each subtending 3° of visual angle. The stimulus was classed as a target if a specific conjunction of colour and spatial arrangement was satisfied: two green squares along the right diagonal or two yellow squares along the left diagonal. Two separate auditory distractors were used: pitch discrimination and interval discrimination. For the pitch discrimination, three tones (each 30 ms) were played equi-spaced within 250 ms. Two reference stimuli had the same frequency, while the target to be detected (chosen at random) differed by ±40% Hz. Both the sign of the increase (increase or decrease) and the reference frequency (400–1000 Hz) were chosen randomly on each trial. For the auditory interval discrimination task, we performed *interval bisection* of three 1300 Hz, 10 ms tones. The first and the third were always played at 0 and 250 ms, the second at a variable interval (60, 80, 90, 110, 120 or 140 ms): subjects reported whether it was closer to the first or third tone. The haptic distractor task was like the auditory time bisection, with taps to the hand instead of tones. Taps were delivered by the coil of a small speaker resting on the hand, through which a 10 ms tone of 80 Hz was played. Like the auditory time bisection task, subjects determined whether the second tap was nearer in time to the first or third (same conditions as for audition). To prevent the use of auditory cues, subjects wore noise-reduction headphones that played white noise. In the single task condition, distractor stimuli were presented on all trials, but subjects were instructed to ignore them. These conditions were re-run separately for all distractor conditions (visual, auditory and tactile).

Numerosity responses were recorded only if the distractor task was correct. We measured 10 levels of numerosity (from 1 to 10) and five attentional conditions (visual, haptic, two different auditory dual-task and single-task), yielding a total of 5500 trials (equally divided between subjects and conditions). In separate sessions we measured enumerations where subjects were not required to do the distractor task (although the stimuli were always displayed).

In this experiment we also asked subjects to perform the auditory frequency-discrimination and visual conjunction task together, to verify that they did not interfere with each other (as others have previously reported).

2.2. Experiment II: numberline mapping

Three new naive subjects were recruited (mean age: 26 ± 2), with normal or corrected-to-normal vision, and who had not participated in the previous study participated in this one.

The general conditions (apparatus, etc.) were like the previous experiment, unless otherwise stated. Throughout each trial a "numberline" was displayed, a 25 cm line without markings, with sample dot-clouds representing the extremes: one dot on the left and 100 dots on the right (see Fig. 1). On subject initiation, both distractor and dot-cloud stimuli were presented for 230 ms, followed by a random-noise mask (described above) that remained on until the subject responded. In separate sessions we measured three different attentional conditions: single-task, visual distractor (described above) and auditory distractor (the frequency-discrimination task). As before, subjects responded first to the distractor task (when appropriate).

The numerosity stimulus was like the previous, a cloud of non-overlapping dots, half-white, half-black at 90% contrast, falling inside a circle of 8° diameter (sparing the central 1°). The numerosities were randomly selected from the set: 2, 3, 4, 5, 6, 18, 25, 42, 67, 71, 86 following Siegler and Opfer (2003). To discourage observers using strategies other than numerosity (such as texture density), on each trial we kept constant either the total covered area at 8° by varying individual dot size, or constant individual dot size of 0.4°, varying total area covered), Thus on average, neither dot size nor total covered area correlated with numerosity. Subjects clicked a mouse pointer on the position of the number line corresponding to the estimated numerosity. As before, numberline data were recorded only if the distractor task was correct.

Each block measured one of the three conditions (single and two dual-task), presenting 10 test stimuli of different numerosity presented in random order once. About five blocks were run for each condition, order randomized between observers.

2.2.1. Bayesian modelling

We modelled numberline mapping with the Bayesian model developed by Anobile, Cicchini, and Burr (2012), which assumes that subjects base their performance on a distribution that combines both their sensory estimates and an *apriori* hypothesis about the stimulus. Bayes' rule states that:

$$p(r|n) \propto p(n|r)p(r) \tag{1}$$

where r is the response and n is the numerosity of the stimulus. $p(n|r)$ is typically termed the likelihood, $p(r)$ the *prior* and $p(r|n)$ the *posterior*. We model likelihood with a gaussian distribution centred on the stimulus, with width given by Weber's law (Weber fraction times number). The *prior* is also modelled as a gaussian distribution centred on the mean of the stimulus range, with variable width (standard deviation). Bayes' Law states that the optimal

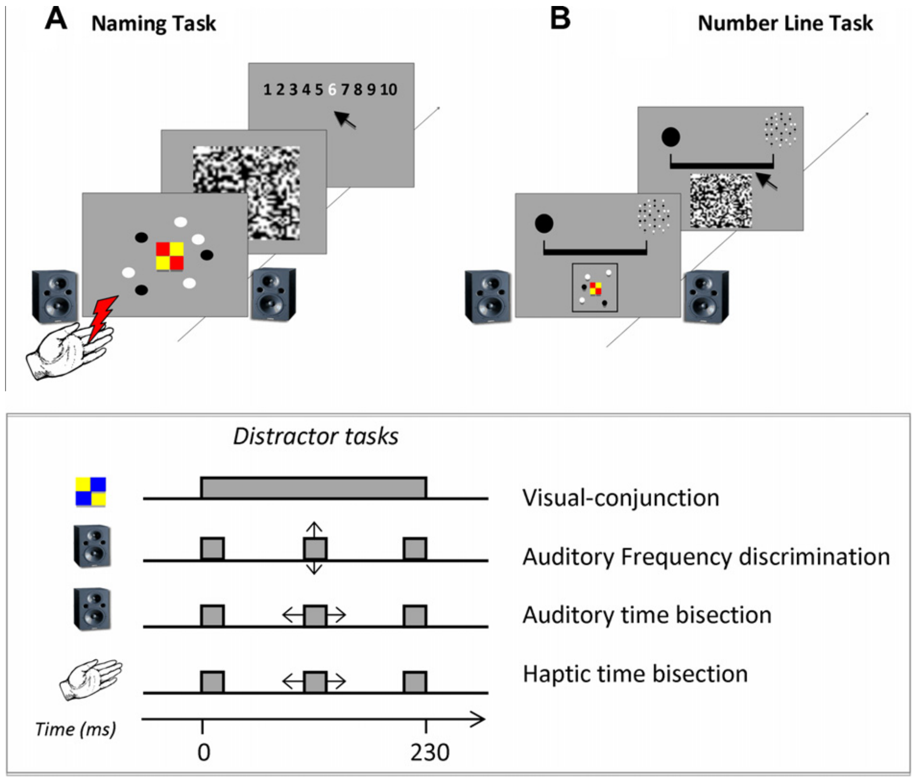

Fig. 1. Illustration of stimulus sequences for the two paradigms. (A) Naming task. Each trial starts with a fixation point (randomly displayed for 200–2000), followed by the numerosity stimulus (dot cloud), together with the distractor. Both last for 230 ms, immediately followed by a binary pixel mask (200 ms). Subjects responded first to the distractor task (described in the Section 2), then enumerated the numerosity. (B) Number line. At trial onset, observers viewed the number line, marked with a single dot to the left and 100 dots to the right. On key press, the test dot stimulus appears, together with the visual conjunction stimulus in the centre of the dot cloud. In the auditory dual-task condition three tones were also played.

combination of information is obtained point-wise multiplication of the two gaussian distributions:

$$\phi(r|n) \propto N(\mu_r, \sigma_r^2)N(\mu_P, \sigma_P^2) \tag{2}$$

where $\mathcal{N}$ indicates the gaussian function. The resulting distribution is itself gaussian whose centre is given by a weighted average of the centres of the likelihood and that of the prior:

$$\hat{r} = \mu_P \frac{\sigma_r^2}{\sigma_r^2 + \sigma_P^2} + \mu_r \frac{\sigma_P^2}{\sigma_r^2 + \sigma_P^2} \tag{3}$$

$$\begin{aligned} \mu_r &= n \\ \sigma_r &= wn \end{aligned} \tag{4}$$

where w is the *Weber fraction*, assumed constant. σ_r increases linearly with n, so the prior will have a weight proportional to n^2. For low numbers, the posterior distribution should be centred on the physical sensory number, while for higher numbers, the posterior estimates are attracted towards the prior (see Fig. 3A of Anobile, Cicchini, and Burr (2012)).

The final equation for the curves of Fig. 3 is obtained by substituting Eq. (4) into Eq. (3) and simplifying:

$$\hat{r} = \frac{\mu_P w^2 n^2 + \sigma_P^2 n}{w^2 n^2 + \sigma_P^2} \tag{5}$$

The shape of the function depends only on the position and width of the prior. By inspection it is obvious that as $\sigma_P^2 \to 0$, $\hat{r} \to \mu_P$ (total regression to the mean), and as $\sigma_P^2 \to \infty$, $\hat{r} \to n$ (veridical response). For intermediate values, the equation follows a Naka–Rushton-like rule, compressing towards the mean of the prior (μ_P).

3. Results

3.1. Experiment I: enumeration of low numerosities

As detailed above, we asked subjects to estimate the numerosity of dot-clouds, both when presented alone and with the various distractor tasks: visual conjunction detection, auditory frequency discrimination, auditory interval bisection task and tactile interval bisection task. Fig. 2 shows response distributions averaged over all subjects for two sample stimuli (3-dot and 6-dot), which we approximate by Gaussian distributions (on logarithmic abscissa). In the single task conditions the response distributions are narrow, particularly for the 3-dot stimulus (within the subitizing range). In the dual task conditions, the response distributions are broader. The effect of attention is clearly greater on the 3-dot than the 6-dot distribution, as they are so narrow without attentional load. With attentional load subjects begin to make errors in estimating the number of presented dots, deviating from veridicality by one or even more units. However, the mean remains virtually unchanged, around three.

We calculated separately for each subject the mean and standard deviation, to yield respectively estimates of accuracy and precision, which were then averaged and shown in Fig. 3. Fig. 3A plots precision as average Weber fraction (standard deviation divided by dot number) of the subjects for each attentional condition, as a function of dot number (excluding the extremes 1 and 10), for the various conditions. For the single-task condition, Weber fractions are near zero in the subitizing range, but rise to about 0.1 for numbers 5 and higher. This pattern changes completely under attentional load. When subjects were required to perform a concomitant dual-task – visual, auditory or tactile – precision was

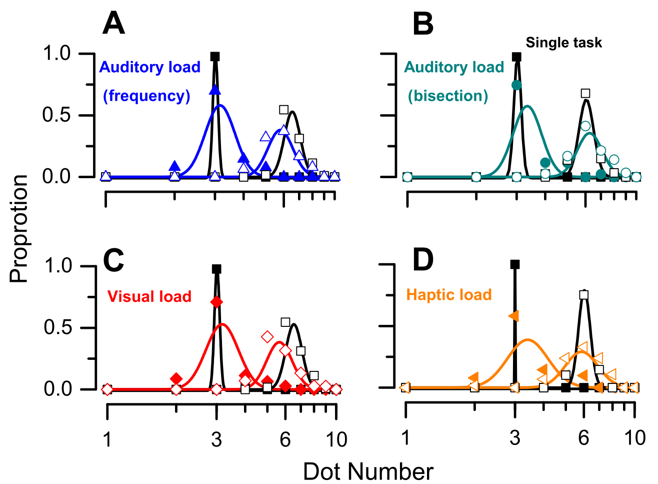

Fig. 2. Sample response distribution for enumerating 3 or 6 dot stimuli numbers in single task (black) and dual task conditions. Filled symbols show responses to 3-dot stimuli, hollow symbols to 6-dot stimuli. Several dual task conditions are shown: (A) Auditory frequency discrimination (blue). (B) Auditory time bisection (cyan). (C) Visual conjunction (red). (D) Haptic time bisection (orange). Best fitting log–gaussian curves are shown as continuous lines.

severely impaired in the subitizing range, with Weber fractions rising to 0.2 or higher. Precision was also impaired for the higher numbers, but by a lesser extent. This confirms the results of Burr, Turi, and Anobile (2010), and further shows that a distractor task in any modality, not just vision, impacts heavily on subitizing. In fact the worst performance was obtained with the tactile distractors. It is not clear why this is so, but perhaps the tactile task was, for some reason, more demanding.

Fig. 3B plots the average perceived numerosity, the mean responses for each numerosity, averaged over subjects. In general, these estimates were quite accurate (bias-free) in all conditions, following reasonably closely the actual target number (dashed diagonal). The only small deviation from veridicality was for the higher numbers (7–8–9), which tended to be slightly underestimated. This shows that the errors in the subitizing range were not simply due to some elements not being seen, as this would have lead to a systematic under-estimation of numerosity.

To be certain that the distractors tasks were performed appropriately during the dual-task conditions, we also measured in separate sessions the baseline performance of on the different distractor tasks. Performance on average does not change when these tasks were performed alone or within dual-task paradigm. Mean performances were 98%, 77%, 83% and 83% respectively for the visual colour-orientation conjunction, auditory frequency discrimination, auditory time bisection and haptic time bisection task when performed alone, compared with 97%, 75%, 80% and 81% when performed in the dual-task paradigm. The similar performance suggests that they made similar attentional demands on the subjects. As a final test of the independence of auditory and visual attention, we measured performance on the two distractor-stimuli – visual conjunction and auditory frequency discrimination – in the presence of the other. The methodology was exactly as before, except that subjects had to report on the conjunction task and the auditory-frequency task (and ignore the numerosity. Fig. 4 shows the results, for the auditory (A) and visual (B) tasks, measured alone and together with the task in the other modality. Clearly, doing two tasks in different modalities incurs little cost: performance, shown as percent correct responses, is little affected by the concomitant task.

3.2. Experiment II: mapping numbers onto space

Mapping onto the numberline is a standard task in number research. Subjects view a cloud of dots, estimate its numerosity and map that onto a line. Here we asked subjects to perform the task under dual-task conditions, with a visual or an auditory distractor. Fig. 5A–C shows numberline judgements for all three conditions (single-task, and visual and auditory frequency-discrimination distractors), averaged over all subjects. Without attentional load (A), the numberline is quite linear. With a concomitant visual conjunction task (B), the mapping shows a clear compressive non-linearity, as previous observed (Anobile, Cicchini, & Burr, 2012). However, the auditory distractor (C) had very little effect, leaving the mapping almost linear.

The curves are fits of the Bayesian model described in Anobile, Cicchini, and Burr (2012) and Section 2 (Eq. (5)). Best fits of the data were obtained with priors centred at 52 for single and auditory, and 40 for visual distractors: both near the mid-point of the stimulus range (2–86). If we assume a Weber fraction of 0.25 (agreeing with Ross (2003), and many other estimates), *prior* widths giving best fits are of 130, 34 and 10 for the single, auditory distractor and visual distractor respectively (the more narrow the prior, the greater the

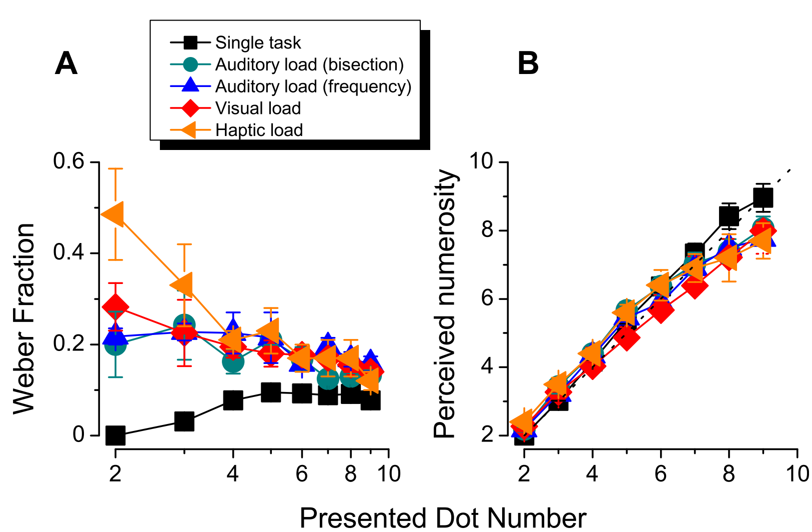

Fig. 3. Number enumeration. (A) Mean Weber fraction (standard deviation divided by physical number) as a function of target number, for the various distractor conditions. Attentional load strongly impairs precision in the subitizing range (4 and below), irrespective of the modality or type of distractor task. The effect at high numerosities was much less. (B) Attention had a little effect on average *accuracy*, with mean perceived numerosity nearly veridical over the range.

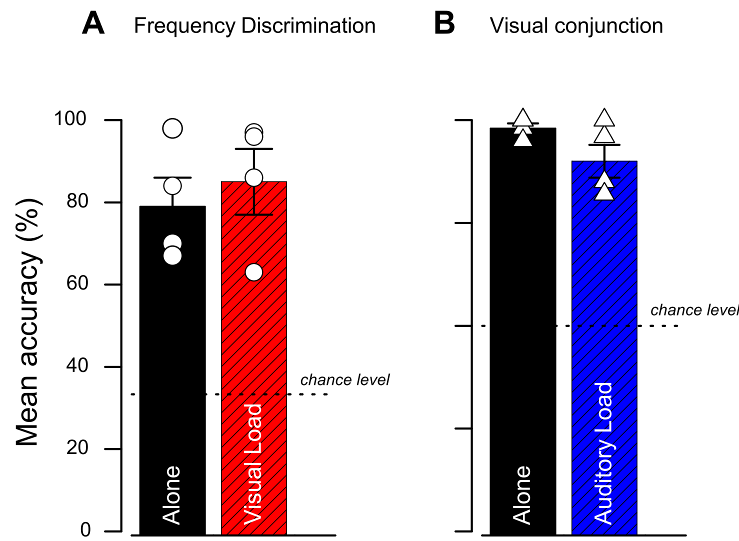

Fig. 4. (A) Average performance (shown as percent correct) for four subjects on the auditory distractor task (frequency discrimination), measured either alone (black) or in dual-task with the visual conjunction task (right-slanting red). The cross-modal distractor clearly did not affect performance. (B) The converse of (A): percent correct on the visual conjunction task measured either alone (black) and or in dual-task condition (right-slanting blue). Again, auditory attention had little effect on visual performance.

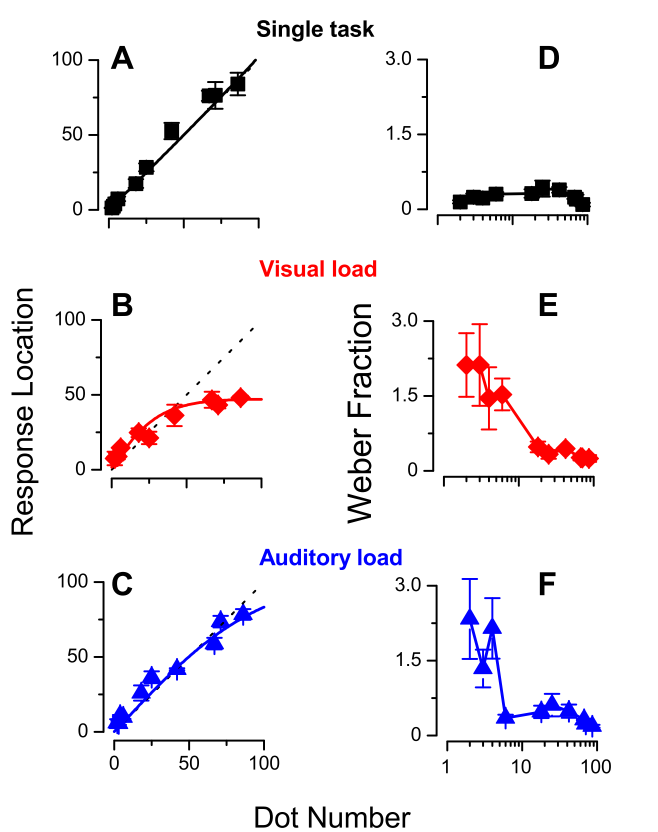

Fig. 5. Number line. (A–C) Mapped response (averaged across subjects), as a function of physical dot-number for different attentional load conditions: single task (A), visual conjunction (B) and auditory dual-task (C). The continuous curves are the fits of the Bayesian central tendency model, described in Section 2. (D–F) Mean Weber fraction as a function of numerosity (on logarithmic scale to display more clearly low numbers), again for single-task (D), visual (E) and auditory (F) distractors. Attentional load affects Weber fraction more for low (2–4) than high (6–98) numbers. Error bars represent ±1 s.e.m.

deviation from linearity). Assuming a higher or lower Weber would require the priors to be scaled commensurably.

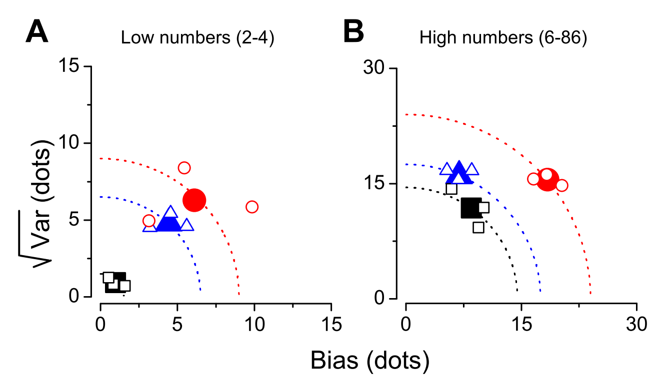

Fig. 6. Partitioning of the error of the numberline task into root-variance (average standard deviation of trials at a particular numerosity) and bias (average distance of the mean response from the physical numerosity), plotted separately for low numbers (2–4: panel A) and high numbers (6–86: panel B). Open symbols represent data of individual subjects, filled the average over subjects for each condition. Colour-coding as before: single – task black squares; visual – red diamonds; auditory – blue triangles. Total error is given by the distance of each symbol from the origin.

Fig. 5D–F plots the precision of the mapping, expressed as Weber Fraction (standard deviation normalized by dot number), with dot-number on a logarithmic abscissa (to bring out better the effects at low numbers). These results confirm those of Experiment I. Without attentional load (D), Weber fraction is low everywhere, including the subitizing range (slightly higher here than in Experiment I, presumably reflecting noise in positioning the pointer). However, with both visual and auditory distractors (E and F respectively), the Weber fraction increased considerably in the low number range, as in the previous experiment.

Following Jazayeri and Shadlen (2010), we partitioned error into two components: bias (inaccuracy) – the distance of the average mapping from the true value – and root-variance (imprecision) – the standard deviation of the individual trials. Fig. 6 shows the results of the numberline, partitioned in this way, separately for low (2–6) and for high numbers (18–86). This representation is revealing. For low numbers, the attentional demand increases both the bias and root-variance slightly more for vision modality compared with the single task condition. However, for high numbers only the visual attentional load increases the bias, the auditory distractors affecting only the root-variance (slightly). This is reflected in the non-linear mapping so clear in Fig. 5C, but not E.

4. Discussion

One of the main results of this study is to show that subitizing is affected by cross-attentional demands. While we confirmed previous work showing independent attentional resources for visual and auditory tasks for estimation of moderately high numerosities, *subitizing* of small quantities of visual items was strongly affected by concurrent attentionally demanding tasks in vision, audition (frequency discrimination or interval discrimination) or touch. For the distractor stimuli we used, all had similar effects, raising Weber fractions from virtually 0% to more than 30%.

This suggests that subitizing may be an *amodal* phenomenon, rather than strictly visual, an idea that finds support in some recent research showing that subitizing processes also operate in audition (Camos & Tillmann, 2008; Repp, 2007) and touch (Plaisier, Bergmann Tiest, & Kappers, 2009; Riggs et al., 2006). Estimating the numerosity of either visual or auditory stimuli causes increased activity of a right-lateralized fronto-parietal cortical network, independently of the modality of the stimuli (Piazza et al., 2006). All this suggests that subitizing may rely on supra-modal attentional resources. Estimation, however, was little affected by cross-modal attention, further evidence that it is an independent process.

It is not clear why subitizing is more affected by attention than estimation. One possibility is that it is a qualitatively different process, requiring more attentional resources than estimation. Indeed, it has been suggested that subitizing is directly linked to the capacity to individuate objects (Piazza et al., 2011). The cross-modal interference reported here tends to support this view, as all modalities may be contributing to object individuation. However, we cannot exclude other possibilities, such as there being some form of pre-normalization noise, highly dependent on attention, that would affect the low range of numbers more than the higher range. Further experimentation may be able to tease out these two possibilities.

A second goal of the study was to examine the effects of intra- and cross-modal attentional demand on mapping number onto space. Here we found that visual, but not auditory attentional load caused the mapping process to become strongly non-linear, with a logarithmic-like compression. Both auditory and visual distractors impaired the precision (Weber fraction) in the low numerosity range, agreeing with the previous result showing that cross-modal attentional load affects subitizing.

The compressive non-linearity we observed with visual attentional load is similar to the non-linearities observed with young children (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler & Opfer, 2003) children with dyscalculia (Ashkenazi & Henik, 2010; Geary et al., 2007, 2008) and adults without mathematical schooling (Dehaene et al., 2008). In all these cases, the mapping process has been described as "logarithmic". However, the fact that a logarithm describes the function does not necessarily imply that it reflects underlying logarithmic transformation. Anobile, Cicchini, and Burr (2012) have suggested that the compression may reflect a "central tendency of judgements", which has been studied for at least 100 years (Hollingworth, 1910) and recently revived in Bayesian terms (Jazayeri & Shadlen, 2010). In their version, the central tendency is a *Bayesian prior*, which combines with the sensory likelihood to produce a *posterior* biased towards the mean. Given that the likelihood is essentially the product of the Weber constant and dot number, and Weber fraction is fairly constant, the likelihood is much broader at the higher number range, and therefore more influenced by the prior. We modelled our numberline data with a simple Bayesian model that predicted both the compressive shape, and fitted the data well, accounting for about 95% of the variance.

What function does the *prior* serve? Jazayeri and Shadlen suggest that it serves to optimize performance, defined as the total error. Error can be partitioned into accuracy and precision, or bias and root-variance, as shown in Fig. 6. Total error is the Pythagorean sum of the two, the distance of the points from the origin. At low numerosities, both visual and auditory attentional loads affect performance, and they affect root-variance and bias in very similar amounts. As has been shown elsewhere (Cicchini et al., 2012; Jazayeri & Shadlen, 2010), increasing bias towards the mean optimizes performance, measured by total error. For high numerosities, however, the results were quite different. Visual attentional load caused a small increase in variance, but a large increase in bias, reflected in the compressive, non-linear mapping. Auditory attention had little effect on either bias or variance, agreeing with previous studies showing visual tasks to have separate attentional resources from audition.

In summary, this study examined how attentional tasks, either in the same and different sensory modalities, can affect numerosity perception. We show that enumerating numbers in the subitizing range is highly dependent on attentional resources, and these resources seem to be shared by the auditory and haptic systems. Attention also affects the higher range of numerosities, particularly when subjects are required to map number onto space. However, in this case, the attention-dependence seems to be specific for vision.

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