331 models and bilepton searches at the LHC

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Despite being remarkably predictive, the Standard Model leaves unanswered several important issues, which motivate an ongoing search for its extensions. One fashionable possibility are the so-called 331 models, where the electroweak gauge group is extended to $\mathrm{SU}_L(3)\times \mathrm{U}(1)$. We focus on a minimal extension which includes vectorlike quarks (VLQs) and new gauge bosons, performing a consistent analysis of the production at the LHC of a pair of doubly charged bileptons. We include for the first time all the relevant processes where VLQs contribute and, in particular, the associate production VLQ bilepton. Finally, we extract the bound on the bilepton mass, $m_Y > 1300$ GeV, from a reinterpretation of a recent ATLAS search for doubly charged Higgs bosons in multilepton final states.

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I. INTRODUCTION

The Standard Model (SM) based on the group $G_{\mathrm{SM}} = \mathrm{SU}(3)_c \times G_{\mathrm{ew}}$, where $G_{\mathrm{ew}} = \mathrm{SU}(2)_L \times \mathrm{U}(1)$, is a remarkably successful theory, but it has been proven to be incomplete and needs to be extended. Since the three gauge couplings, running with energy, are drawn nearer around 10^{15} GeV, an appealing possibility is that G_{SM} arises from the spontaneous breaking of a larger gauge group, characterized by only one structure constant which unifies strong and electroweak interactions. Most grand unified theories are based on the gauge groups SU(5) and SO(10) [1]. They predict proton decay, whose observation in itself provides a strong motivation. However, so far, experiments are just putting stronger and stronger lower limits on the proton lifetime.

A completely different approach is to consider minimal extensions of G_{SM} with not-simple gauge groups. There are many possibilities. The simplest one is to enlarge the electroweak sector to $G_{\mathrm{ew}} \times \mathrm{U}(1)'$ to add a nonstandard neutral gauge boson generally named Z'; see, for instance, Ref. [2]. The next case is inspired by the request to symmetrize the SM matter assignment, whose complete asymmetry between left- and right-handed fields is probably one of its less appealing features. In the so-called left-right models, G_{ew} is embedded into $\mathrm{SU}_L(2)\times \mathrm{SU}_R(2)\times \mathrm{U}(1)$ [3,4]. Another interesting possibility is to extend $\mathrm{SU}_L(2)$ with $\mathrm{SU}_L(N)$. In Refs. [5–8], G_{ew} is extended to $\mathrm{SU}_L(3)\times \mathrm{U}(1)$. These seminal works focus on the possibility to extend the Glashow-Iliopoulos-Maiani (GIM) mechanism [9] to more than four quarks and to suppress unwanted flavor-changing neutral currents. Then, in [10] has been proposed the first $\mathrm{SU}_L(3)\times \mathrm{U}_X(1)$ (where X is a generalized hypercharge; see below) model, where the three families of fermions are embedded into irreducible representations of the gauge group. The first complete model based on $\mathrm{SU}_L(3)\times \mathrm{U}_X(1)$ [hereafter named 331, where the first 3 refers to $\mathrm{SU}_c(3)$] has been proposed by Pisano and Pleitez [11] and Frampton [12].

In 331 models, the hypercharge operator $\hat{Y}$ is given by

$$\frac{\hat{Y}}{2} = \beta \hat{T}^8 + X\mathbb{1}, \tag{1}$$

where X is the quantum number associated with $\mathrm{U}_X(1)$ and β is a free parameter. Any 331 model is characterized, in general, by the pair (β, X), whose values are obviously related to the electric charges of the fields (fermions and bosons). By fixing the charges of the Standard Model fermions, the X charges are expressed as a function of the parameter β. Therefore, 331 models can be classified in terms of the value of β. The assumed correlation SM charges-β value does not naturally hold in cases where

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one considers additional multiplets including only beyond Standard Model (BSM) fermions, which then may have different X values given the same β (see, e.g., Ref. [13]).

In this work, we consider a minimal requirement that the new gauge bosons have at least integer charge values, although there are 331 models that discard this assumption; see, for instance, [14]. This implies that $|\beta| = n/\sqrt{3}$, where n is an odd integer number. On the other hand, by requiring that all gauge couplings are real, it follows that $|\beta| < 1.83$ (see also [15]). Therefore, models with $n > 3$ are excluded, and the only possible cases are $\beta = \pm 1/\sqrt{3}, \pm\sqrt{3}$.

Besides the specific value of the parameter β, 331 models are classified in terms of matter assignment into irreducible representation of $\mathrm{SU}_L(3)$, namely, triplets and antitriplets. It is well known that the anomaly cancellation implies that quark families cannot all be assigned to triplets or antitriplets. We derive the important relation

$$N_Q^{(3)} = (2N_F - N_\ell^{(3)})/3,$$

where $N_{Q,\ell}^{(3)}$ is the number of quark (lepton) families transforming as $\mathrm{SU}_L(3)$ triplet and N_F is the number of families. Both sides of the equality must be integers. This is true if the number of families N_F is a multiple of the color $N_c = 3$, as in minimal models where $N_\ell^{(\bar{3})} = 3$, $N_Q^{(3)} = 2$, and $N_Q^{(\bar{3})} = 1$. However, such proportionality is not strictly necessary, contrary to what is typically reported in the literature. An example is $N_F = 4$ with $N_\ell^{(3)} = N_\ell^{(\bar{3})} = 2$ that gives $N_Q^{(3)} = N_Q^{(\bar{3})} = 2$ (see also Ref. [16]).

Another constraint on 331 models is given by $\mathrm{SU}(3)_c$ asymptotic freedom. As in SM QCD, perturbative calculations limit the number of quark flavors to be equal to or less than 16. Since each family in 331 models has three flavors, that yields a constraint on the number of families: $N_F \leq 5$.

More details, including a general classification of 331 models and the passages needed to engineer a 331 model compatible with the SM, i.e., the choice of the scalar sector and the charge assignment of the matter fields, will be found in the Appendix A.

Even if from the group theory point of view extending $\mathrm{SU}_L(2)$ to $\mathrm{SU}_L(3)$ represents a small modification, from the phenomenological point of view it has important consequences.

(i) The matter assignment requires the introduction of extra fermions that can have exotic charges and can be vectorlike. In the minimal model we consider, there are two vectorlike quarks with charges $-4/3$ and $5/3$.
(ii) The scalar sector must be extended with extra Higgs fields. For example, in our case, the model contains three scalar $\mathrm{SU}_L(3)$ triplets.
(iii) The model has five more gauge bosons that can have exotic charges. In our case, the extra gauge bosons are Z', $W'^{\pm}$, and the doubly charged bilepton $Y^{\pm\pm}$.

The wealth of 331 models of new physics yields to opportunities for searching at colliders. We provide an inclusive study of the multilepton channel $pp \to \ell^\pm\ell^\pm\ell^\mp\ell^\mp + X$ at the LHC mediated by 331 new fields, in particular, by the pair production of bileptons $Y^{\pm\pm}$. The multilepton production process cross section is sizably modified by the presence of 331 new physics. This analysis tests a general version of 331 models with $\beta = \sqrt{3}$ and a minimal set of assumptions:

(i) all matter multiplets include SM fermions, excluding the possibility of fermion multiplets including only BSM fields;
(ii) all gauge bosons, including BSM ones, have integer charges;
(iii) all right-handed, $\mathrm{SU}(3)$ singlet, fermions have a left-handed counterpart.

The paper is organized as follows: In Sec. II, we present the main features of the model, and, in Sec. III, we discuss its phenomenology. The analysis of bilepton production at the LHC is reported in Sec. IV, and conclusions are given in Sec. V.

II. THE MODEL

A. Matter assignment

In this work, we focus on the minimal 331 model [17], where the first two generations of quarks transform as triplets under $\mathrm{SU}(3)_L$, whereas the third generation and all the leptons transform as antitriplets. In the appendixes, we discuss a general classification of 331 models and provide a detailed view of the one we consider. In the classification scheme presented in the appendixes, this model is indicated as $A[\sqrt{3}]$. Table I contains a summary of the matter content of the model. The fermions $D^{1,2}$ and T in the color triplet are exotic quarks with fractional electric charges $-4/3$ and $5/3$, respectively,[1] while the fermions ζ_L^a, $a = 1, 2, 3$, which are color singlets, are leptons with positive charge $+1$. All three BSM quarks can be assumed to carry the usual baryonic number for quarks, namely, $B = 1/3$. In the spirit of minimally extending the SM, we identify $\zeta_L^a = (\ell_R^a)^c$.

The three BSM fermions $D^{1,2}$ and T are vectorlike quarks (VLQs). If we consider the embedding of $\mathrm{SU}_L(2)$ into $\mathrm{SU}_L(3)$, we observe that the generators T^i ($i = 1, 2, 3$) do not act on the third component of $\mathrm{SU}_L(3)$ triplets and antitriplets. As a consequence, $D^{1,2}$ and T are singlets of the SM $\mathrm{SU}_L(2)$, and they do not couple to charged currents by the covariant derivative. Later on [see Eq. (13)], we explicitly show that their left- and right-handed components

[1]These exotic quarks do not have the same value charge of down-type quarks and top quark but maintain the same charge sign.

TABLE I. The fermion and scalar content of the $A[\sqrt{3}]$ model we are considering.

	$SU(3)_c$	$SU(3)_L$	$U(1)_X$		$SU(3)_c$	$SU(3)_L$	$U(1)_X$
$\begin{pmatrix} u_L^{1,2} \\ d_L^{1,2} \\ D_L^{1,2} \end{pmatrix}$	3	3	$-\frac{1}{3}$	u_R^a	3	1	$\frac{2}{3}$
				d_R^a	3	1	$-\frac{1}{3}$
				$D_R^{1,2}$	3	1	$-\frac{4}{3}$
$\begin{pmatrix} b_L \\ -t_L \\ T_L \end{pmatrix}$	3	$\bar{3}$	$\frac{2}{3}$	T_R	3	1	$\frac{5}{3}$
$\begin{pmatrix} \ell_L^a \\ -\nu_L^a \\ (\ell_R^a)^c \end{pmatrix}$	1	$\bar{3}$	0	χ	1	3	1
				ρ	1	3	0
				η	1	3	-1

behave the same way in the neutral weak interactions, resulting in a vector electroweak current, hence the epithet vectorlike.

Being singlets of the SM $SU_L(2)$, VLQs do not require coupling to $SU(2)_L$ scalar Higgs doublets to get mass. Hence, they are not excluded by current measurements of the production rate of the Higgs boson, which instead exclude the existence of a fourth generation of chiral quarks.

B. Gauge sector

In the first stage of spontaneous symmetry breaking (SSB) and with the choice $\beta = \sqrt{3}$, there emerge singly and doubly charged gauge bosons, $V_\mu^\pm$ (also named W'_μ) and $Y_\mu^{\pm\pm}$. The new charged gauge bosons are dubbed bileptons, because they couple at tree level to SM lepton pairs but not to SM quark pairs.[2] Bileptons occur in several models of common SM extensions, as grand unified theories, technicolor, and compositeness scenarios. We can assign them global quantum numbers, in contrast with the SM, where only fermions carry baryon or lepton numbers, not bosons.

One distinctive feature of our 331 model is that $Y_\mu^{\pm\pm}$ bileptons couple to BSM quarks. One can assign lepton number $L = -2$ to the gauge bosons V_μ^+ and Y_μ^{++} and $L = +2$ to V_μ^- and Y_μ^{--}. The 331 model does not conserve separate family lepton numbers, but the total lepton number is conserved if we assign $L = -2$ to the exotic quarks $D^{1,2}$ and $L = +2$ to the T quark. Then, the global symmetry $B - L$ of the SM maintains in the minimal model.

After both $SU(3)_L$ breaking, eight gauge bosons acquire a mass. We assume that spontaneous symmetry breaking occurs by triplets only, namely, the three Higgs fields ρ, η, and χ, defined in Eq. (B1). The charged gauge bosons are directly diagonal, while the neutral gauge boson matrix can be diagonalized as shown in Appendix B 2. Thus, we have

$$m_W^2 = \frac{g^2}{4}(v^2 + v'^2) \equiv \frac{g^2}{4} v_+^2, \tag{2}$$

$$m_V^2 = \frac{g^2}{4}(u^2 + v'^2) \simeq \frac{g^2}{4} u^2, \tag{3}$$

$$m_Y^2 = \frac{g^2}{4}(u^2 + v^2) \simeq \frac{g^2}{4} u^2, \tag{4}$$

$$m_A = 0, \tag{5}$$

$$m_Z^2 \simeq \frac{g^2}{4\cos^2\theta_W} v_{\rm SM}^2, \tag{6}$$

$$m_{Z'}^2 \simeq \frac{g^2 u^2 \cos^2\theta_W}{3[1 - (1+\beta^2)\sin^2\theta_W]}, \tag{7}$$

where v, v', and u are, respectively, ρ, η, and χ vacuum expectation values, A_μ is the SM photon field, and θ_W is the SM Weinberg angle (see the Appendix B for details). To correctly restore the SM, we require that $v_+^2 = v_{\rm SM}^2$, where $v_{\rm SM}$ is the Higgs boson vacuum expectation value and $u \gg v, v'$. These assumptions imply that $m_W \ll m_V \simeq m_Y$. Moreover, a useful relation between the masses of these bosons [that one can obtain by comparing Eqs. (B9) and (4)] holds:

$$\frac{m_{Z'}^2}{m_Y^2} \simeq \frac{4\cos^2\theta_W}{3[1 - (1+\beta^2)\sin^2\theta_W]}. \tag{8}$$

Fixing $\beta = \sqrt{3}$, we have[3]

$$\frac{m_{Z'}^2}{m_Y^2} \simeq \begin{cases} 13 & \text{for } \sin^2\theta_W(m_Z) \simeq 0.23, \\ 25 & \text{for } \sin^2\theta_W(1\ \text{TeV}) \simeq 0.24. \end{cases} \tag{9}$$

An important constraint for 331 models is obtained from the matching condition of the 331 coupling of the Z boson with fermions. By matching this coupling with the corresponding Standard Model coupling $g\sin^2\theta_W/\cos\theta_W$, one gets (see Appendix B for details)

$$\frac{g_X}{g} = \frac{\sqrt{6}\sin\theta_W}{\cos\theta_W \sin\theta_{331}} = \frac{\sqrt{6}\sin\theta_W}{\sqrt{1 - \sin^2\theta_W(1+\beta^2)}}. \tag{10}$$

We can see that the tree-level Eq. (10) exhibits a Landau pole when $\sin^2\theta_W = 1/(1+\beta^2)$. The energy scale of the Landau pole depends on the running of $\sin^2\theta_W$, which, in turn, depends on several aspects, such as radiative corrections, the matter content of the model, or the spontaneous symmetry-breaking conditions. In the SM framework, a Landau pole has been estimated, for the case $\beta = \sqrt{3}$,

[2] To avoid confusion with old literature, we remind that they have previously shared the name dilepton with events having two final state leptons.

[3] The running of the sinus is assumed in the SM framework; see, e.g., [18].

emerging at a scale of approximately 4 TeV [18]. Some recent analyses indicate how the Landau pole, and the nonperturbative regime which may appear even before the pole, could be pushed up to much higher energies [19,20].

C. Yukawa interactions

SM fermions get mass via the Yukawa coupling with the scalars. The gauge invariant Yukawa Lagrangian (for the $\beta = \sqrt{3}$ case) reads as

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} = &+\lambda^d_{i,a}\bar{Q}_i\rho d_{a,R} + \lambda^u_{i,a}\bar{Q}_i\eta u_{a,R} + \lambda^d_{3,a}\bar{Q}_3\eta^* d_{a,R} \\ &+ \lambda^u_{3,a}\bar{Q}_3\rho^* u_{a,R} + \lambda^J_{i,j}\bar{Q}_i\chi D_{j,R} + \lambda^J_{3,3}\bar{Q}_3\chi^* T_R \\ &+ \lambda^\ell_{a,b}\epsilon_{ijk}\bar{L}_{ai}L_{bj}\rho^*_k, \end{aligned} \tag{11}$$

where we have used the notations of the Appendix A and set $[Q^1, Q^2, Q^3] \equiv [3^1_Q, 3^2_Q, \bar{3}^3_Q]$ and $[L^1, L^2, L^3] \equiv [\bar{3}^1_\ell, \bar{3}^2_\ell, \bar{3}^3_\ell]$, with a, $b = 1, 2, 3$ and i, $j = 1, 2$. The first line and the first term in the second line corresponds to the masses of the Standard Model quarks, the last two terms in the second line to those of the VLQs, and the third pertains to the charged lepton masses. The Yukawa interactions involving quarks are 331-invariant independently from the β, whereas the ones accounting for the charged lepton masses [last line in Eq. (11)] are invariant only for $\beta = \sqrt{3}$ being $X[\rho] = 0$.

From the diagonalization of Eq. (11), we obtain the quark mass eigenstates u'_a and d'_a. The relation between left-handed and right-handed mass and interaction eigenstates reads as

$$\begin{pmatrix} u'_1 \\ u'_2 \\ u'_3 \end{pmatrix}_{L,R} = \mathcal{U}^{-1}_{L,R}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_{L,R}, \qquad \begin{pmatrix} d'_1 \\ d'_2 \\ d'_3 \end{pmatrix}_{L,R} = \mathcal{V}^{-1}_{L,R}\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}_{L,R}, \tag{12}$$

where $\mathcal{U}_{L,R}$ and $\mathcal{V}_{L,R}$ are four unitary matrices. The Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix V_{CKM} is recovered as the usual product $V_{\text{CKM}} \equiv \mathcal{U}^\dagger_L \cdot \mathcal{V}_L$.

We can safely take the new quarks D and T as mass eigenstates and omit their rotation matrices in the interactions. That is immediately evident for T, which, having charge $5/3$, cannot mix with the other quarks. In the case of D quarks, let us observe that in the interactions a possible rotation matrix is either canceled because of unitarity [see Eqs. (13) and (14)], or it appears together with $\mathcal{U}_L$ or $\mathcal{V}_L$ [see Eqs. (15) and (16)]. In the latter case, we can define new unitary $\mathcal{U}_L$ or $\mathcal{V}_L$ matrices as the product of the older $\mathcal{U}_L$ or $\mathcal{V}_L$ matrices, whose entries are unknown, and the D rotation matrix, similarly unknown.

D. Fermions and gauge boson new couplings

This section illustrates new interactions predicted by the model under investigation. We start from the interactions involving the SM gauge bosons and the new fermions. The $W^\pm_\mu$ bosons do not couple to VLQs, while the Z_μ boson does. The Lagrangian involving the Z_μ boson reads as

$$i\mathcal{L}^Z_F \supset +i\frac{g}{c_W}Z^\mu\left[\frac{4}{3}s^2_W\bar{D}^i_L\gamma_\mu D^i_L + \frac{4}{3}s^2_W\bar{D}^i_R\gamma_\mu D^i_R - \frac{5}{3}s^2_W\bar{T}_L\gamma_\mu T_L - \frac{5}{3}s^2_W\bar{T}_R\gamma_\mu T_R\right]. \tag{13}$$

Here, we report the terms involving the Z'_μ boson:

$$\begin{aligned} i\mathcal{L}^{Z'}_F = &\, i\frac{g}{2\sqrt{3}c_W\sqrt{1-4s^2_W}}\Big\{[1-4s^2_W](\bar{\nu}^a_L\gamma_\mu\nu^a_L + \bar{\ell}^a_L\gamma_\mu\ell^a_L) - [2+4s^2_W]\bar{\ell}^a_R\gamma_\mu\ell^a_R \\ &+ [-1+2s^2_W]\bar{u}'^a_L\gamma_\mu\delta_{ab}u'^b_L + 2c^2_W\bar{u}'^a_L\gamma_\mu u'^b_L\mathcal{U}^*_{L3a}\mathcal{U}_{L3b} + 4s^2_W\bar{u}'^a_R\gamma_\mu u'^b_R\delta_{ab} \\ &+ [-1+2s^2_W]\bar{d}'^a_L\gamma_\mu\delta_{ab}d'^b_L + 2c^2_W\bar{d}'^a_L\gamma_\mu d'^b_L\mathcal{V}^*_{L3a}\mathcal{V}_{L3b} - 2s^2_W d'^a_R\gamma_\mu d'^b_R\delta_{ab} \\ &+ 8s^2_W(\overline{D_L}^i\gamma_\mu D^j_L + \overline{D_R}^i\gamma_\mu D^j_R) + 10s^2_W(\overline{T_L}\gamma_\mu T_L + \overline{T_R}\gamma_\mu T_R)\Big\}. \end{aligned} \tag{14}$$

While in the SM the GIM mechanism guarantees flavor universality, Z'_μ interactions with quarks violate it, allowing processes like $e^+e^- \to \bar{u}c$ or $\bar{u}u \to \bar{u}c$.

The vertices involving V_μ read as

$$i\mathcal{L}^V_F = +i\frac{g}{\sqrt{2}}V^-_\mu\left[\bar{\nu}^a_L\gamma^\mu(\ell^a_R)^c + \bar{D}^i_L\gamma^\mu d'^a_L\mathcal{V}_{Lai} + \bar{u}'^a_L\gamma^\mu T_L\mathcal{U}^*_{L3a} + \text{H.c.}\right], \tag{15}$$

whereas the ones with the Y_μ boson are

$$i\mathcal{L}_F^Y = -i\frac{g}{\sqrt{2}}Y_\mu^{++}\left[\overline{(\ell_R^a)^c}\gamma^\mu\ell_L^a + \bar{u}_L^a\gamma^\mu D_L^i\mathcal{U}_{L\,ai}^* + \bar{T}_L\gamma^\mu d_L^a\mathcal{V}_{L3a} + \text{H.c.}\right]. \tag{16}$$

In the Appendix C, we show the Feynman diagrams of the couplings which are relevant to our phenomenological analysis.

We observe that the quark couplings to the new gauge bosons depend on the unknown entries of the rotation matrices $\mathcal{U}_L$ and $\mathcal{V}_L$, which do not cancel out in the interactions. For simplicity in our analysis, we fix as benchmark case $\mathcal{U}_L = \mathbf{1}$, i.e., the three by three unity matrix; from the definition of the CKM matrix $V_{\rm CKM}$, it follows that $\mathcal{V}_L = V_{\rm CKM}$. Under this assumption, D^i and T couple predominantly to the ith quark generation and to the third, respectively. For our phenomenological analysis, it is not worrisome having set $\mathcal{U}_L$ real, since a possible phase would not affect the diagrams in Fig. 3. It is possible to connect the entries of the rotation matrices to observables in kaon, B_d, and B_s systems, yielding absolute values that are generally of the same order of magnitude as ours [21].

III. PHENOMENOLOGY OF MINIMAL 331

This minimal 331 model is rich in new fields. In the fermion sector, we have two VLQs, and, in the gauge sector, we have one new neutral and two new charged bosons, namely,

$$(D^{1,2})^{-4/3}, \quad T^{5/3}, \quad Z'^0, \quad V^\pm, \quad Y^{\pm\pm}, \tag{17}$$

where we added an apex to indicate the charge of the particle.

Several phenomenological implications of the existence of these exotic particles have been investigated.

(i) At colliders, VLQs are typically identified by means of the decay channels VLQs $\to qW^\pm, qZ, qH$ [22–26]. In 331 models, these decay channels are suppressed and are replaced by the dominant ones VLQs $\to qV^\pm, qY^{\pm\pm}$. Therefore, the peculiar interactions of VLQs in 331 theories allow less conventional search strategies; see, for instance, Ref. [27], where the case of an exotic VLQ with charge $5/3$ has been considered and a lower bound on its mass of about 1.5 TeV is found by a recasting of the CMS analysis in Ref. [28]. An earlier study considering the decay of the VLQ with charge $5/3$ into a bilepton plus a b jet was presented in [29].

(ii) The phenomenology of Z' bosons in 331 has been studied quite in detail in the recent literature [30–33]. The Z' boson is produced through the Drell-Yan process and subsequently decays into pairs of SM particles or in exotic channels, like in pairs of VLQs. Note that these signatures are typical also of Z' bosons or, in general, of heavy resonances, which appear in other BSM theories, as those of composite Higgs models [34–37]. Therefore, it would be beneficial to identify, as we will do in this study, different signatures which are more characteristic of the 331 model. The dilepton channel gives significant constraints on the new vector, with bounds of the order of 4 TeV on the Z' mass, as extracted from the ATLAS and CMS searches, already with a luminosity of 139 fb^{-1} [31]. Note, however, that the Z' mass is expected to be significantly higher than those of V and Y [Eq. (9)]. While it will certainly be useful to continue in these searches for the Z', it is essential to explore new strategies for testing the 331 model that can be more distinctive of the underlying theory and, possibly, even more efficient.

(iii) Differently from the typical LHC phenomenology of W' resonances (such as sequential-SM W' [38], Kaluza-Klein W from extra dimension, or from composite Higgs theories [34]), which are dominantly produced via the Drell-Yan process,[4] in the case of the $V^\pm$ (W') from 331 models, other production channels must be considered. Indeed, their Drell-Yan production is precluded at tree level, given the absence of V couplings with pairs of SM fermions; see, e.g., Ref. [27]. Nevertheless, the $V^\pm$ could be produced by interactions with a VLQ and a SM quark. It could be, thus, interesting to examine less conventional channels, such as the one in Fig. 2 (bottom) in which the V is produced in association with a VLQ. We leave this analysis to a future investigation.

In our study, we propose signatures for 331 models which are also efficient as search channels for the exotic VLQs. In particular, based on the considerations above, we find particularly interesting to focus on the *inclusive* production of a pair of bilepton vector bosons $Y^{\pm\pm}$:

$$pp \to Y^{++}Y^{--}, \tag{18}$$

where each bilepton decays into a pair of same-sign charged leptons. Then, the process we look for is

$$pp \to \ell^+\ell^+\ell^-\ell^- + X \tag{19}$$

with a multilepton final state. We consider the *inclusive* production, allowing the presence of extra jets in the final state denoted here as X.

[4]The production via weak-boson fusion is typically subdominant, except when considering colliders at very high collision energies, such as FCC-hh [39].

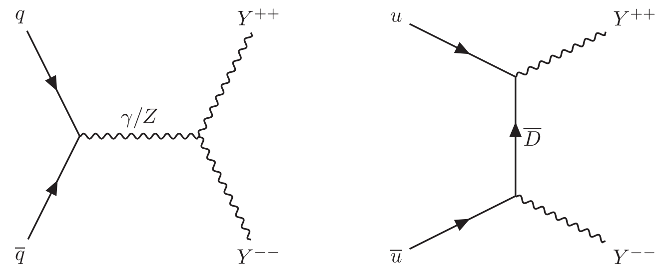

FIG. 1. Typical Feynman diagrams for $Y^{++}Y^{--}$ production: Drell-Yan-like processes (left) and t channel mediated by a vectorlike antiquark (right).

Exclusive production of bilepton pairs at colliders, with purely $Y^{++}Y^{--}$ in the final state, has been already studied; see, for instance, Refs. [17,40–42]. The typical diagram considered (at the parton level) are the Drell-Yan like processes (Fig. 1, left) or with the exchange of an exotic quark in the t channel (Fig. 1, right).

Another possibility [43] (not considered in the previous searches for exclusive bilepton pair production) is the production from gluon (g) interaction, associated to a VLQ, namely, $gu \to D^1 Y^{++}$, $gc \to D^2 Y^{++}$, and $gb \to TY^{--}$, as shown in Fig. 2. If

$$m_Y < m_D, \qquad m_T, \tag{20}$$

then the VLQs can decay in bileptons as $D^1 \to Y^{--}u$, $D^2 \to Y^{--}c$, and $T \to Y^{++}b$; see Fig. 3. The decay widths are reported in the Appendix C.

A contribution to the inclusive process (18) is, therefore, also given by

$$gq \to \mathrm{VLQ}Y \to Y^{++}Y^{--}q', \tag{21}$$

where the VLQ is either a D or a T fermion; q and q' are either an up, a charm, or a bottom SM quark. We remark that these processes are not included among the ones considered in Ref. [41], which focuses on the exclusive final state $Y^{++}Y^{--}jj$ with $j = q$, g at the parton level. Conservatively, in our study we will not include the D^2 contribution (i.e., we will consider either D^2 decoupled from the spectrum or $m_Y > m_{D^2}$ so that the $D^2 \to Yc$ decay is off shell and suppressed). This is a conservative choice, since an even larger VLQ contribution to the bilepton pair cross section is expected when D^2 can contribute. In agreement with this assumption, we indicate for simplicity $D^1 \equiv D$ hereafter and in Figs. 1 and 2.

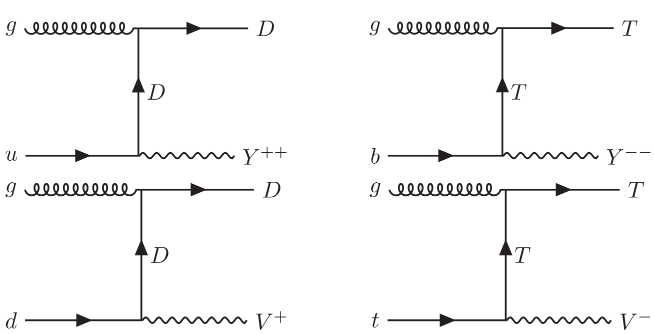

FIG. 2. Bilepton Y and V associated production with a VLQ.

To our knowledge, all previous studies have overlooked the contribution to the pair bilepton production given by the bilepton associated production with a VLQ (Fig. 2), which subsequently decays into a second bilepton; see Fig. 3. We find that instead this contribution is significant in a large portion of the model parameter space. We show in Fig. 4 the cross section for the VLQ-bilepton associated production, for the cases of D and T VLQ, as a function of the bilepton mass and for different VLQ mass values. Because of the smaller parton distribution function (PDF) for the initial b quark in the $T-Y$ associated production process, the latter has a significantly lower cross section compared to the $D-Y$ associated production. An analogous consideration applies to the case of the D^2-Y associated production, which depends on the c-quark PDF. Nevertheless, identifying these specific associated productions, possibly by applying a tailored analysis at a high luminosity, could give important confirmation about the flavor structure of the 331 model. In the case of the D VLQ, the associated production with a bilepton constitutes up to $\sim$20% of the total inclusive cross section for the bilepton pair production in a large portion of the parameter space, in particular, when m_Y approaches m_D (cf. Fig. 5). On top of their contribution to the inclusive bilepton pair production, these channels could be considered for a specific exclusive analysis. In this case, one could select the signal from the background by exploiting the peculiarities of the VLQ-Y associated production, namely, the presence in the final state of a heavy VLQ (whose invariant mass could be reconstructed), a jet and bileptons with hard p_T, and peculiar angular distributions. We leave this analysis for a future project.

Finally, another contribution to the inclusive bilepton pair production, which we have included in our study, comes from the QCD pair production of extra VLQs, which both decay into a bilepton plus a jet: $pp \to (\mathrm{VLQ} \to Yq)(\mathrm{VLQ} \to Yq) \to YYjj$, with $\mathrm{VLQ} = D$, T and $j = u$, b; see Fig. 6.

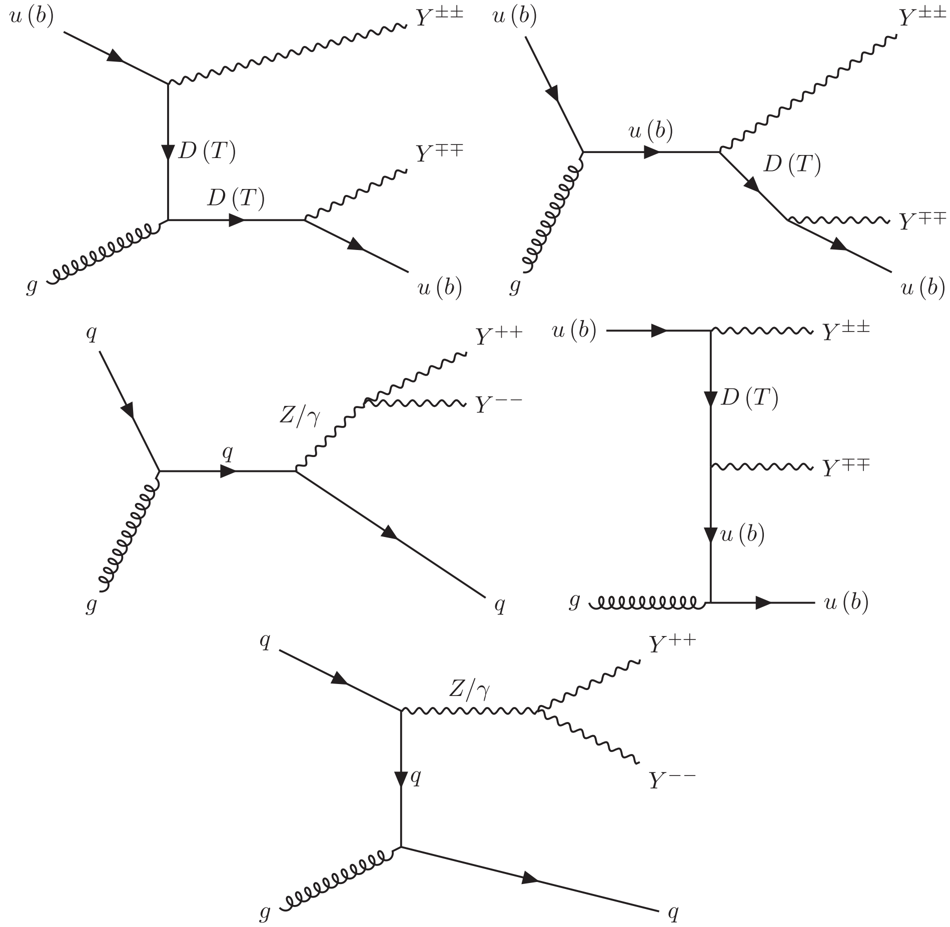

FIG. 3. Feynman diagrams for the dominant contributions to the process $gq \to Y^{++}Y^{--}j$ given by the VLQ-Y associated production considered in this study. The symbols g and q label a gluon and a SM quark, respectively; $j = q, g$ at the parton level.

IV. BILEPTON PAIR PRODUCTION AT THE LHC

Given the phenomenological considerations expressed in the previous section, we find particularly promising the search for bileptons at the LHC. We want to focus, in particular, on the inclusive process $pp \to \ell^\pm\ell^\pm\ell^\mp\ell^\mp$, where a pair of Y is produced on shell and then decay to same-sign leptons, allowing the presence of extra jets in the final state. We observe that $\mathrm{Br}(Y^{\pm\pm} \to \ell^\pm\ell^\pm) \simeq 1$ because this is the only decay channel at tree level with our mass choice. Our analysis consistently considers all possible contributions to the inclusive bilepton pair production and includes the processes whose diagrams are shown in Figs. 1, 3, and 6. For our analysis, we have

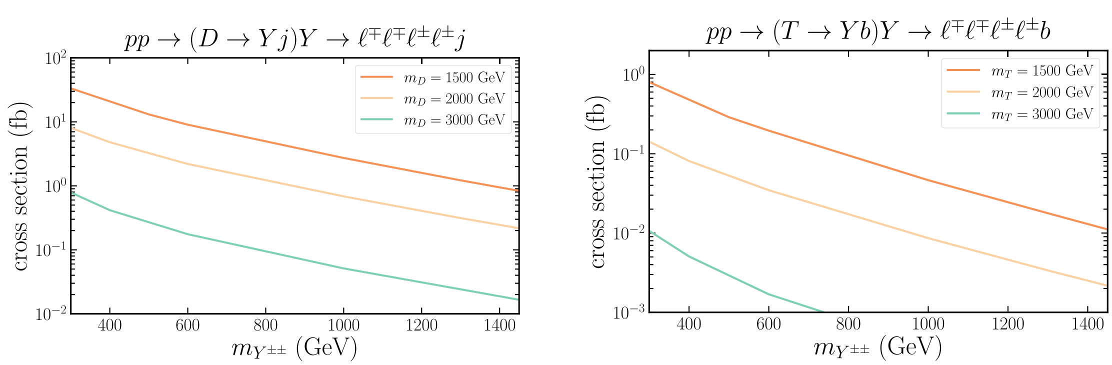

FIG. 4. Cross section for the associated production of a bilepton with a VLQ. We consider the subsequent decay of the VLQ into a bilepton plus a quark and the bilepton decay into electron or muon pairs.

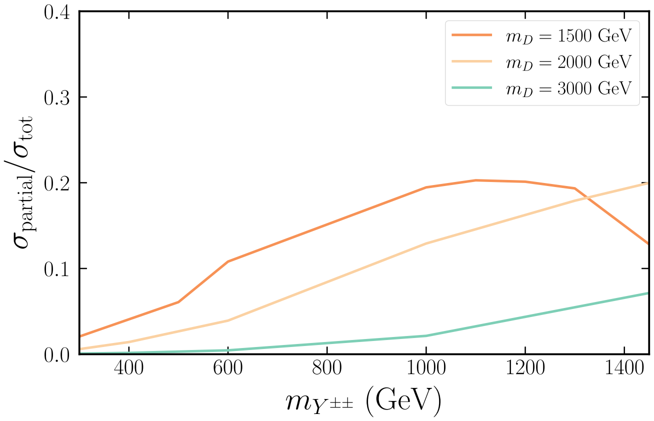

FIG. 5. Ratio of the partial cross section for the bilepton associated production with a D VLQ, which subsequently decays into a second bilepton, i.e., the $D - Y \to YYj$ associated production ($\sigma_{\rm partial}$) over the total $YY + X$ cross section ($\sigma_{\rm tot}$).

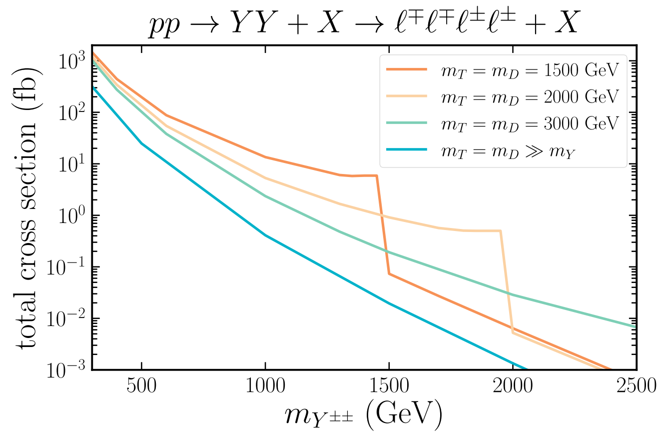

FIG. 7. Inclusive cross section for the process $pp \to \ell^{\pm}\ell^{\pm}\ell^{\mp}\ell^{\mp} + X$ as a function of the Y mass and for different values of the VLQ masses and in the case of decoupled VLQ.

implemented, by means of FEYNRULES [44], the 331 model we are considering ($A[\sqrt{3}]$) in a universal FEYNRULES output model [45], which we use for simulations and numeric calculations in MADGRAPH5 [46].

In Fig. 7, we provide the inclusive cross section for the process $pp \to \ell^{\pm}\ell^{\pm}\ell^{\mp}\ell^{\mp} + X$ as a function of the Y mass and for different values of the VLQ masses and in the case of decoupled VLQs. It is evident that processes involving VLQs which decay into bileptons constitute the dominant contribution to the bilepton pair inclusive cross section. Indeed, a large boost in the cross section, which reaches up to about 2 orders of magnitude near the kinematic threshold $m_{\rm VLQ} \simeq m_Y$, is observed when the VLQ $\to Yq$ decays are on shell. When these decays become off shell, the cross section reduces significantly, despite a residual contribution from VLQ exchanged in the t channel being still relevant.

A recent ATLAS search for doubly charged Higgs bosons, at the 13 TeV LHC with 139 fb^{-1}, focuses on multilepton final states with at least one same-sign lepton pair [47]. This analysis is, therefore, sensitive also to the bilepton pair production. We apply a recast of this analysis to extract a lower limit on the Y mass.

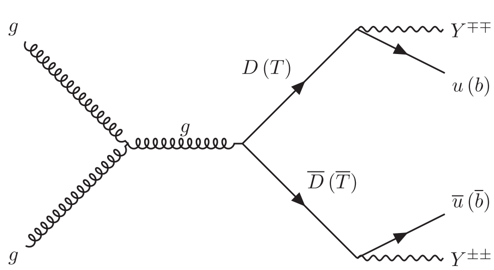

FIG. 6. QCD pair production of VLQs, leading to a $YYjj$ final state.

A. Reinterpretation of the ATLAS search for doubly charged Higgs bosons

We summarize in the following paragraphs the main selection criteria considered in the ATLAS analysis [47], also adopting their notation. The search considers a final state with two (SR2L), three (SR3L), or four (SR4L) leptons, either electrons or muons, passing triggering requirements. The leptons must satisfy the transverse momentum p_T and rapidity η requirements: $p_T > 40$ GeV and $|\eta_\mu| < 2.5$, $|\eta_e| < 2.47$ (for the electron, the region $1.37 < |\eta_e| < 1.52$ is vetoed). Electron candidates are discarded if their angular distance $\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2}$ from a jet (with $p_T > 20$ GeV and $|\eta| < 2.5$) satisfies $0.2 < \Delta R < 0.4$. If a muon and a jet featuring more than three tracks have a separation $\Delta R < 0.4$ and the muon's p_T is less than half of the jet's p_T, the muon is discarded. Signal regions are optimized to search for one same-charge lepton pair, in the case of two or three leptons, while four-lepton events are required to feature two same-charge pairs. The signal selection exploits, as the main variable to distinguish signal events from the background, the invariant mass of the two same-charge leptons with the highest p_T in the event, $m(\ell^{\pm}, \ell'^{\pm})_{\rm lead}$, and applies a cut:

$$m(\ell^{\pm}, \ell'^{\pm})_{\rm lead} \geq 300 \text{ GeV}. \tag{22}$$

Furthermore, for SR2L and SR3L, the vector sum of the two leading leptons' transverse momenta must exceed 300 GeV:

$$p_T(\ell^{\pm}, \ell'^{\pm})_{\rm lead} \geq 300 \text{ GeV} \quad (\text{SR2L, SR3L}), \tag{23}$$

while, for SR4L, the average invariant mass of the two same-charge lepton pairs must satisfy

TABLE II. Efficiencies of the inclusive bilepton pair production to the ATLAS selection [47] for several benchmark points of the minimal 331 model ($A[\sqrt{3}]$). We report the combined efficiencies for all three signal regions considered in the analysis, together with the specific efficiencies for the signal regions SR4L (upper values) and SR2L (lower values). The efficiencies for SR3L are similar to those obtained for SR4L.

m_Y (GeV)	300	600	1000	1300
$m_D = 1.5$ TeV	$0.14^{0.16}_{0.11}$	$0.37^{0.37}_{0.11}$	$0.36^{0.36}_{0.07}$	$0.33^{0.32}_{0.11}$
$m_D = 2.0$ TeV	$0.14^{0.13}_{0.10}$	$0.38^{0.37}_{0.08}$	$0.38^{0.37}_{0.07}$	$0.37^{0.36}_{0.14}$
$m_D = 3.0$ TeV	$0.13^{0.13}_{0.07}$	$0.38^{0.38}_{0.10}$	$0.38^{0.37}_{0.09}$	$0.39^{0.38}_{0.08}$
$m_D \gg m_Y$	$0.13^{0.13}_{0.06}$	$0.38^{0.38}_{0.09}$	$0.38^{0.37}_{0.09}$	$0.39^{0.38}_{0.08}$

$$\bar{m} = (m_{\ell^+\ell'^+} + m_{\ell^-\ell'^-})/2 \geq 300 \text{ GeV} \quad (\text{SR4L}). \quad (24)$$

On top of these cuts, for SR3L and SR4L, a Z-veto condition is required, excluding the presence of same-charge lepton pairs with invariant mass $71 \text{ GeV} < m(\ell, \ell') < 111.2$ GeV. For SR2L, the requirement on the separation $\Delta R(\ell^\pm, \ell'^\pm)_{\text{lead}} < 3.5$ is considered. Finally, events containing jets identified as originating from b quarks are vetoed.

We apply the above selection to the signal of inclusive bilepton pair production. Events are generated with MADGRAPH5 and then passed to PYTHIA8 [48] for showering and hadronization. Jets are clustered by using FASTJET [49] with an anti-kt algorithm with radius parameter $R = 0.4$. Conservatively, based on the details of lepton reconstruction procedures described in [47], we also consider the following identification efficiencies for an electron: 0.58, and for a muon: 0.95.

We obtain the overall signal efficiencies reported in Table II for several signal benchmark points (we also list the values for the separate efficiencies in the SR4L and SR2L regions). The b-veto applied in the ATLAS analysis completely suppresses the contribution from the VLQ T to the bilepton pair production.

For all of the benchmark points analyzed, except the lowest mass point, we find efficiencies for the bilepton pair process (ϵ_{YY}) which are comparable to those for doubly charged Higgs bosons (ϵ_{HH}) in the SR3L and in the SR4L regions. For the SR2L region, we find, in general, slightly lower efficiencies than those obtained for the charged Higgs signal. In order to correctly compare the cross section for our process $pp \to YY \to \ell^\pm\ell^\pm\ell^\mp\ell^\mp$ with the upper limit band on the multilepton cross section reported in Fig. 8 of the ATLAS search [47], we consider a scaling of our cross section values by a factor $\epsilon_{YY}/\epsilon_{HH}$. Conservatively, the scaling is not applied when this factor is greater than 1. We show the result of the comparison in Fig. 8. We find that in the full mass region covered by the ATLAS analysis, and for all of the considered values of the

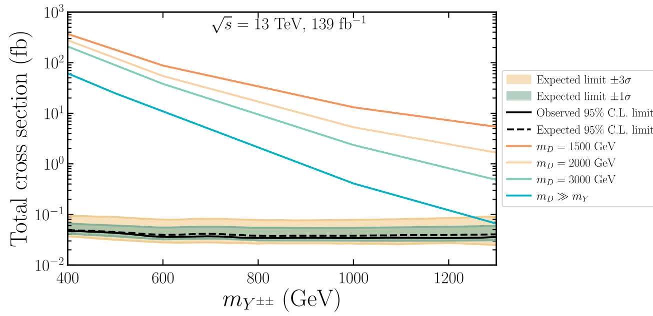

FIG. 8. Alongside our prediction for the multilepton cross section generated by the bilepton pair production in the minimal 331 model, we illustrate the results obtained by ATLAS in Ref. [47]. In particular, the solid and dashed lines represent, respectively, the observed and expected 90% C.L. upper limits on the multilepton cross section as a function of the bilepton mass (doubly charged Higgs in the ATLAS analysis). The shaded green and yellow bands represent the 1σ and 3σ uncertainty, respectively, around the combined expected limit.

mass of extra VLQs, the cross section for the process of inclusive bilepton pair production lies above the curve representing the 90% C.L. upper limit, with the lowest values obtained for the scenario of decoupled VLQs. We can therefore extract, independently of VLQ masses, the 90% C.L. bound on the bilepton mass:

$$m_Y > 1300 \text{ GeV}. \quad (25)$$

This value supersedes the one obtained in [42], $m_Y > 1200$ GeV, which was based on the results of a previous ATLAS search for doubly charged Higgs bosons in multilepton final states with 36.1 fb^{-1} at the 13 TeV LHC [50]. It is clear from Fig. 8 that the search, with the increasing of the luminosity expected at the High-Luminosity LHC phase, has a great potential to test even larger bilepton (and VLQs) mass values, especially when the intertwined contribution of VLQs is taken into account. We also emphasize that, considering the relation between the Z' and the bilepton masses in Eqs. (8) and (9), the bound we just derived on m_Y can be translated into an *indirect* lower limit on the Z' mass, which we find to be particularly significant:

$$m_{Z'} \gtrsim 6500 \text{ GeV}. \quad (26)$$

This limit, despite being indirect and subject to model-dependent assumptions related to the running of $\sin\theta_W$, is stronger than the one currently placed by direct searches for Z' at the LHC. Furthermore, the bound tests a region at significantly high Z' masses where direct searches may be inefficient, due to the large width of the resonance and the possible proximity to a nonperturbative regime of the theory.

V. CONCLUSIONS

We have considered a class of models, known as 331 models, characterized by the electroweak gauge group $SU_L(3) \times U(1)$. We have selected a minimal 331 model satisfying four general requests: (i) all matter multiplets include SM fermions—namely, there are not multiplets including only BSM fermions; (ii) all gauge bosons have integer charges; (iii) all right-handed fermions are $SU(3)_L$ singlet and possess a left-handed counterpart; and (iv) no gauge anomalies. These assumptions constrain the number of families: One possibility, although not the only one, is that it is a multiple of the color number. The model we have chosen has $\beta = \sqrt{3}$, the first two generations of quarks that transform as triplets under $SU_L(3)$, and the third generation of quark, together with the three lepton generations, that transform as antitriplets. In addition to the usual SM fields, this model accommodates some BSM fields, namely, three VLQs, $D^{1,2}$ and T, one neutral gauge boson Z'_μ, the charged boson $V^\pm_\mu$, and the doubly charged bilepton $Y^{\pm\pm}_\mu$. The intertwined phenomenology of bileptons and VLQs looks particularly promising for the search and characterization of 331 models at the LHC. In our analysis, we propose bilepton pair signatures for 331 models, which are also efficient as search channels for VLQs. In particular, we focus on the *inclusive* production of a pair of doubly charged bileptons, followed by the decay of each bilepton into a pair of same-sign charged leptons, that is, the process $pp \to Y^{++}Y^{--} \to \ell^+\ell^+\ell^-\ell^- + X$. The final state of this inclusive process has multileptons (with at least one same-sign lepton pair) and possibly extra jets.

In our analysis, we provide the values of the inclusive cross section for the process $pp \to \ell^+\ell^+\ell^-\ell^- + X$, as a function of the doubly charged bilepton mass. We include consistently the production of doubly charged bileptons from gluon interaction, involving VLQs. A highly massive VLQ can decay into a bilepton and a SM quark q, yielding a final state $Y^{++}Y^{--}q$ or $Y^{++}Y^{--}qq$, which contribute to the inclusive process under investigation. To the best of our knowledge, the production from gluon-quark interaction of a bilepton associated to a VLQ has not been taken into account in previous searches for inclusive or exclusive bilepton pair production. We find that instead this contribution is significant in a large portion of the model parameter space (up to ~20% of the total inclusive cross section for the bilepton pair production).

A recent ATLAS search for doubly charged Higgs bosons focuses on multilepton final states with at least one same-sign lepton pair [47]. We apply a recast of this analysis to extract a lower limit on the Y mass. We found, independently of the VLQ masses, a 90% C.L. lower limit on the mass of the doubly charged bilepton:

$$m_Y > 1.3 \text{ TeV}. \tag{27}$$

This value supersedes the previous one of $m_Y > 1.2$ TeV [42], based on 2017 ATLAS results.

We conclude by remarking that the channel we have investigated remains relevant with the increase of luminosity expected during the High-Luminosity LHC phase, due to its potential to test quite large bilepton (and VLQ) mass values.

ACKNOWLEDGMENTS

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APPENDIX A: BASIC FEATURES OF 331 MODELS

In this appendix, we present in more detail the general features of the 331 models. We are using the same convention of Ref. [21]. We do not discuss $SU(3)_C$ which does not change with respect to the SM. The generators of the $SU(3)_L$ gauge group are indicated with $\hat{T}^1, \ldots, \hat{T}^8$. The generators of the fundamental representation of $SU(3)_L$ are $T^a = \lambda^a/2$, where λ^i are the eight Gell-Mann matrices. For the $\bar{3}$ representation, the generators are $-(T^a)^T$. The normalization of the generators is $\mathrm{Tr}[T^i T^j] = \delta^{ij}/2$, and $\mathbb{1} = \mathrm{diag}(1, 1, 1)$ is the identity matrix in the fundamental space. It is common to define the $U(1)_X$ generator as $T^9 = \mathbb{1}/\sqrt{6}$. With this normalization $\mathrm{Tr}[T^9 T^9] = 1/2$, similarly to the one of $SU(3)_L$ generators.

In analogy with the SM, the electric charge is defined as the diagonal operator

$$\hat{Q} = \hat{T}^3 + \frac{\hat{Y}}{2}, \tag{A1}$$

where we identify the hypercharge operator $\hat{Y}$ as in Eq. (1):

$$\frac{\hat{Y}}{2} = \beta \hat{T}^8 + X\mathbb{1}. \tag{A2}$$

In the SM, left-handed fermions are assigned to doublets of $SU_L(2)$. Similarly, in 331 models, one assigns left-handed fields to fundamental irreducible representations (3 or $\bar{3}$) of $SU_L(3)$. We indicate them generically with $3_{Q,\ell}$ and $\bar{3}_{Q,\ell}$, distinguishing between quark triplets and antitriplets (3_Q and $\bar{3}_Q$) and lepton ones (3_ℓ and $\bar{3}_\ell$). Thus, for a generic SM quark doublet (u_L, d_L) we set

$$3_Q = \begin{pmatrix} u_L \\ d_L \\ \psi_L \end{pmatrix}_i, \qquad \bar{3}_Q = \begin{pmatrix} d_L \\ -u_L \\ \Psi_L \end{pmatrix}_j. \tag{A3}$$

In a similar way for a generic SM lepton doublet (ν_L, ℓ_L), we set

$$3_\ell = \begin{pmatrix} \nu_L \\ \ell_L \\ \xi_L \end{pmatrix}_k, \qquad \bar{3}_\ell = \begin{pmatrix} \ell_L \\ -\nu_L \\ \zeta_L \end{pmatrix}_r. \tag{A4}$$

The field assigned to the third components of these triplets are the left-handed component of a field beyond the SM; namely, we have introduced new fields ψ_i, Ψ_j, ξ_k, and ζ_r. The indexes i, j, k, and r refer to flavor.

The action of the charge operator $\hat{\mathcal{Q}}$ on 3 and $\bar{3}$ representations is given by the eigenvalues of the eigenstates $|3_{Q,\ell}\rangle$ and $|\bar{3}_{Q,\ell}\rangle$ that we indicate, respectively, as $\mathcal{Q}[3_{Q,\ell}]$ and $\mathcal{Q}[\bar{3}_{Q,\ell}]$. We have

$$\mathcal{Q}[3_{Q,\ell}(\bar{3}_{Q,\ell})] = \begin{pmatrix} \pm\frac{1}{2} \pm \frac{\beta}{2\sqrt{3}} + X[3_{Q,\ell}(\bar{3}_{Q,\ell})] & 0 & 0 \\ 0 & \mp\frac{1}{2} \pm \frac{\beta}{2\sqrt{3}} + X[3_{Q,\ell}(\bar{3}_{Q,\ell})] & 0 \\ 0 & 0 & \mp\frac{\beta}{\sqrt{3}} + X[3_{Q,\ell}(\bar{3}_{Q,\ell})] \end{pmatrix}, \tag{A5}$$

The up (down) signs correspond to the 3 ($\bar{3}$) representations. The X values for $3_{Q,\ell}$ and $\bar{3}_{Q,\ell}$ states are indicated, respectively, as $X[3_{Q,\ell}]$ and $X[\bar{3}_{Q,\ell}]$. We will use a similar notation $X[f]$ and $X[\bar{f}]$ if, instead of referring to all fields in an entire multiplet, we refer to the specific field f.

The charges of the first two components of (A3) and (A4) must match the charges of the corresponding SM fields. This matching fixes uniquely the X values for the fermions as functions of β. Let us consider the case of the quark triplet 3_Q. If we match the up quark charge $1/2 + \beta/(2\sqrt{3}) + X[3_Q] = 2/3$, we obtain $X[3_Q] = 1/6 - \beta/(2\sqrt{3})$. The correct down quark charge follows automatically, and it does not represent an independent constraint. In the same way, we get $X[3_\ell] = -1/2 - \beta/(2\sqrt{3})$ in the lepton sector.

Let us now consider the entries of the matrix (A5) for the antitriplets. By repeating the same reasoning for the quark assignment in (A3), we get $X[\bar{3}_Q] = 1/6 + \beta/(2\sqrt{3})$, whereas for the lepton one in (A4) we obtain $X[\bar{3}_\ell] = -1/2 + \beta/(2\sqrt{3})$.[5] Negative values for β can be related to the positive ones by taking the complex conjugate in the covariant derivative of each model, which, in turn, is equivalent to replacing 3 with $\bar{3}$ in the fermion content of each particular model.

The nonstandard fields ψ_i, Ψ_j, ξ_k, and ζ_r have the same X values as the other members of the respective (anti)triplet because of gauge invariance. Thus, using Eq. (A5), we deduce their charges: $\mathcal{Q}[\psi_L] = 1/6 - \sqrt{3}/2\beta$, $\mathcal{Q}[\Psi_L] = 1/6 + \sqrt{3}/2\beta$, $\mathcal{Q}[\xi_L] = -1/2 - \sqrt{3}/2\beta$, and $\mathcal{Q}[\zeta_L] = -1/2 + \sqrt{3}/2\beta$.

As far as right-handed fields are concerned, we assume, in analogy to the SM, that they are assigned to singlet representations of $SU_L(3)$. Since the action of the charge operator $\hat{\mathcal{Q}}$ on the singlet is given by $\hat{\mathcal{Q}}|1\rangle = X|1\rangle$, the X value of a right-handed field coincides with its charge. In our notation, given a left-handed fermion field f_L, belonging to a triplet or antitriplet $SU_L(3)$ representation, we indicate with f_R the corresponding right-handed field in the $SU_L(3)$ singlet representation. Let us observe that the presence of right-handed quark singlets is demanded for the model to be $SU(3)_C$ vectorlike. Table III summarizes the quantum numbers of the 331 fermions.

Let us now us consider the 331 gauge bosons. Differently from the standard models, we have eight W^a_μ bosons for $SU_L(3)$ and one $B_{X\mu}$ boson for $U_X(1)$. The covariant derivatives acting on field representations are

TABLE III. Assigned representations and X values of matter fields in terms of the parameter β for a generic 331 model. The subscript i is a generic generation index.

	$SU_c(3)$	$SU_L(3)$	$U_X(1)$		$SU_c(3)$	$SU_L(3)$	$U_X(1)$
$\begin{pmatrix} u_L \\ d_L \\ \psi_L \end{pmatrix}_i$	3	3	$\frac{1}{6} - \frac{\beta}{2\sqrt{3}}$	$\begin{pmatrix} \nu_L \\ \ell_L \\ \xi_L \end{pmatrix}_i$	1	3	$-\frac{1}{2} - \frac{\beta}{2\sqrt{3}}$
$\begin{pmatrix} d_L \\ -u_L \\ \Psi_L \end{pmatrix}_i$	3	$\bar{3}$	$\frac{1}{6} + \frac{\beta}{2\sqrt{3}}$	$\begin{pmatrix} \ell_L \\ -\nu_L \\ \zeta_L \end{pmatrix}_i$	1	$\bar{3}$	$-\frac{1}{2} + \frac{\beta}{2\sqrt{3}}$
u_{R_i}	3	1	$\frac{2}{3}$				
d_{R_i}	3	1	$-\frac{1}{3}$	ℓ_{R_i}	1	1	-1
ψ_{R_i}	3	1	$\frac{1}{6} - \frac{\sqrt{3}\beta}{2}$	ξ_{R_i}	1	1	$-\frac{1}{2} - \frac{\sqrt{3}\beta}{2}$
Ψ_{R_i}	3	1	$\frac{1}{6} + \frac{\sqrt{3}\beta}{2}$	ζ_{R_i}	1	1	$-\frac{1}{2} + \frac{\sqrt{3}\beta}{2}$

[5] We are using the same convention of Ref. [21], opposite to the one in Ref. [16].

(i) triplet: $D_\mu = \partial_\mu - igW^a_\mu T^a - ig_X X B_{X\mu} T^9$;
(ii) antitriplet: $D_\mu = \partial_\mu + igW^a_\mu (T^a)^T - ig_X X B_{X\mu} T^9$;
(iii) singlet: $D_\mu = \partial_\mu - ig_X X B_{X\mu} T^9$;

where the g and g_X are gauge couplings and a sum over a is understood.

We arrange the $\mathrm{SU}(3)_L$ gauge bosons W^a_μ as

$$W_\mu = W^a_\mu T^a = \frac{1}{2}\begin{pmatrix} W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu & \sqrt{2}W^+_\mu & \sqrt{2}Y^{Q_Y}_\mu \\ \sqrt{2}W^-_\mu & -W^3_\mu + \frac{1}{\sqrt{3}}W^8_\mu & \sqrt{2}V^{Q_V}_\mu \\ \sqrt{2}Y^{-Q_Y}_\mu & \sqrt{2}V^{-Q_V}_\mu & -\frac{2}{\sqrt{3}}W^8_\mu \end{pmatrix}, \tag{A6}$$

where

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu),$$
$$V^{\pm Q_V}_\mu = \frac{1}{\sqrt{2}}(W^6_\mu \mp iW^7_\mu),$$
$$Y^{\pm Q_Y}_\mu = \frac{1}{\sqrt{2}}(W^4_\mu \mp iW^5_\mu). \tag{A7}$$

The action of the charge operator $\hat{\mathcal{Q}}$ on gauge bosons is given by the commutator $[\hat{\mathcal{Q}}, W_\mu]$, and it can be represented by the matrix

$$\mathcal{Q}^{\mathrm{GB}} = \begin{pmatrix} 0 & 1 & \frac{1}{2} + \beta\frac{\sqrt{3}}{2} \\ -1 & 0 & -\frac{1}{2} + \beta\frac{\sqrt{3}}{2} \\ -\frac{1}{2} - \beta\frac{\sqrt{3}}{2} & \frac{1}{2} - \beta\frac{\sqrt{3}}{2} & 0 \end{pmatrix}, \tag{A8}$$

where each entry represents the charge of the gauge boson in the corresponding entry of the matrix (A6). From Eq. (A8), we see that there are

(i) three neutral gauge bosons, W^3_μ, W^8_μ, and $B_{X\mu}$—after the spontaneous symmetry breakings, the neutral gauge bosons mix, originating three mass eigenstates, the SM γ and Z^0 plus an additional neutral BSM gauge boson, generally indicated with Z';
(ii) two bosons with unit charges, corresponding to $W^\pm$;
(iii) four BSM bosons $Y^{\pm Q_Y}_\mu$, $V^{\pm Q_V}_\mu$, with charges $\pm\mathcal{Q}_Y = \pm\frac{1}{2} \pm \beta\frac{\sqrt{3}}{2}$ and $\pm\mathcal{Q}_V = \mp\frac{1}{2} \pm \beta\frac{\sqrt{3}}{2}$, that depend on the value of β.

It is useful at this point to summarize the electric charges of BSM fermions and BSM bosons:

Field	ψ	Ψ	ξ	ζ	Z'	V	Y
$\mathcal{Q}$	$\frac{1}{6} - \frac{\sqrt{3}}{2}\beta$	$\frac{1}{6} + \frac{\sqrt{3}}{2}\beta$	$-\frac{1}{2} - \frac{\sqrt{3}}{2}\beta$	$-\frac{1}{2} + \frac{\sqrt{3}}{2}\beta$	0	$\mp\frac{1}{2} \pm \frac{\sqrt{3}}{2}\beta$	$\pm\frac{1}{2} \pm \frac{\sqrt{3}}{2}\beta$

Depending on the choice of the parameter β, the electric charges of the new fields can be exotic. By exotic, we mean charges different from $\pm 1/3$ and $\pm 2/3$ for extra quarks and different from ± 1 for extra leptons and gauge bosons.

We consider as a minimal requirement that the new gauge bosons have at least integer charge values, even if there are 331 models that discard this assumption; see, for instance, [14].

By imposing that the new gauge bosons $Y^{\pm Q_Y}_\mu$ and $V^{\pm Q_V}_\mu$ have integer charge, it follows that the allowed values for the parameter β are

$$\beta = \pm\frac{2n+1}{\sqrt{3}} \equiv \frac{r}{\sqrt{3}}, \tag{A9}$$

where r is an odd integer number (positive or negative). A few examples of X and $\hat{\mathcal{Q}}$ charges for new fields are reported in Table IV, corresponding to $r = \pm 1, \pm 3, \pm 5$. We immediately see that only the choice $r = \pm 1$ guarantees no exotic charges. It is necessary to warn that another constraint ($|r| \leq 3$) comes from spontaneous symmetry breaking, as we will see in Appendix II B. In Table IV, we report the case $\beta = \pm 5/\sqrt{3}$ only to remark that in the minimal assumptions made up to now there is nothing that prevents $|r|$ from having values larger than 3. From Table IV, it is also manifest that the exchange $r \leftrightarrow -r$ is equivalent to trade triplets for antitriplets and vice versa.

The 331 models are fully characterized by the choice of the electric charge embedding, namely, the pair (β, X), of its fermionic content and multiplet assignments, and of the scalar sector. There are two major quantum field theory constraints: the absence of gauge anomalies and the possibility of QCD asymptotic freedom.

The fermionic structure is restricted by requiring the cancellation of the triangular anomalies. Both $\mathrm{SU}(3)_L$ and $\mathrm{U}(1)_X$ are anomalous, contrary to the SM, where only $\mathrm{U}(1)_Y$ is. In the SM, gauge anomalies are canceled out for every fermion family. This property is lost in 331 models, and correlation between families is needed. Reference [16] discusses in detail this topic for an arbitrary value of β and

TABLE IV. We list the X values and the charges for the third components of quark triplet 3_Q, lepton triplet 3_ℓ, quark antitriplet $\bar{3}_Q$, lepton antitriplet $\bar{3}_\ell$, and the absolute values of the charges of BSM gauge bosons for sample values of r ($\beta = r/\sqrt{3}$).

r	+1	−1	+3	−3	+5	−5
$X[\psi]$	0	1/3	−1/3	2/3	−2/3	1
$X[\Psi]$	1/3	0	2/3	−1/3	1	−2/3
$X[\xi]$	−2/3	−1/3	−1	0	−4/3	1/3
$X(\zeta)$	−1/3	−2/3	0	−1	1/3	−4/3
$\mathcal{Q}[\psi]$	−1/3	2/3	−4/3	5/3	−7/3	8/3
$\mathcal{Q}[\Psi]$	2/3	−1/3	5/3	−4/3	8/3	−7/3
$\mathcal{Q}[\xi]$	−1	0	−2	1	−3	2
$\mathcal{Q}[\zeta]$	0	−1	1	−2	2	−3
$\lvert\mathcal{Q}[V]\rvert$	0	1	1	2	2	3
$\lvert\mathcal{Q}[Y]\rvert$	1	0	2	1	3	2

number of generations. Below, we outline anomaly cancellation calculations assuming $N^{(3)}_{Q(\ell)}$ and $N^{(\bar{3})}_{Q(\ell)}$ quark (lepton) generations transforming as triplets and antitriplets with the structure outlined above. We underline that, when we refer to lepton generations, we mean generations of active leptons, whose left-handed parts are embedded in triplets or antitriplets. We define the total number of quark (lepton) generations as

$$N^F_{Q(\ell)} = N^{(3)}_{Q(\ell)} + N^{(\bar{3})}_{Q(\ell)}. \tag{A10}$$

The anomaly cancellations with their implications read as follows.

(A1) $[\mathrm{SU}(3)_L]^3$.—

$$N_c N_Q^{(3)} - N_c N_Q^{(\bar{3})} + N_\ell^{(3)} - N_\ell^{(\bar{3})} = 0$$
$$\Rightarrow N_Q^{(3)} = \frac{N_c N_Q^F + N_\ell^F}{2N_c} - \frac{N_\ell^{(3)}}{N_c}.$$

(A2) $[\mathrm{SU}(3)_L]^2 \otimes \mathrm{U}(1)_X$.—

$$3N_c\left[N_Q^{(3)} X[3_Q] + N_Q^{(\bar{3})} X[\bar{3}_Q]\right] + 3N_\ell^{(3)} X[3_\ell] + 3N_\ell^{(\bar{3})} X[\bar{3}_\ell] = 0$$
$$\Rightarrow N_Q^F = \frac{3}{N_c} N_\ell^F$$

[assuming (A1)].

(A3) $[\mathrm{U}(1)_X]^3$.—

$$N_c\left[N_Q^{(3)}(3X[3_Q]^3 - X[\psi_R]^3) + N_Q^{(\bar{3})}(3X[\bar{3}_Q]^3 - X[\Psi_R]^3) - N_Q^F(X[u_R]^3 + X[d_R]^3)\right]$$
$$+ N_\ell^{(3)}(3X[3_\ell]^3 - X[\xi_R]^3) + N_\ell^{(\bar{3})}(3X[\bar{3}_\ell]^3 - X[\zeta_R]^3) - N_\ell^F X[\ell_R]^3 = 0$$
$$\Rightarrow \text{no condition (always true)}.$$

(A4) $[\mathrm{SU}(3)_c]^2 \otimes \mathrm{U}(1)_X$.—

$$N_c\left[N_Q^{(3)}(3X[3_Q] - X[\psi_R]) + N_Q^{(\bar{3})}(3X[\bar{3}_Q] - X[\Psi_R]) - N_Q^F(X[u_R] + X[d_R])\right] = 0$$
$$\Rightarrow \text{no condition (always true)}.$$

(A5) $[\mathrm{Grav}]^2 \otimes \mathrm{U}(1)_X$.—

$$N_c\left[N_Q^{(3)}(3X[3_Q] - X[\psi_R]) + N_Q^{(\bar{3})}(3X[\bar{3}_Q] - X[\Psi_R]) - N_Q^F(X[u_R] + X[d_R])\right]$$
$$+ N_\ell^{(3)}(3X[3_\ell] - X[\xi_R]) + N_\ell^{(\bar{3})}(3X[\bar{3}_\ell] - X[\zeta_R]) - N_\ell^F X[\ell_R] = 0$$
$$\Rightarrow \text{no condition (always true)}.$$

N_c refers to the color $\mathrm{SU}_3(N_c)$, always fixed to $N_c = 3$. The minus signs in the sum are due to the different fields chirality. We remark that these relations hold even under the common assumption $\zeta^a_L = (\ell^a_R)^c$.

Before drawing conclusions from the anomaly cancellation, let us summarize the assumptions we have done on an otherwise completely general 331 model.

(i) All matter multiplets include SM fermions, excluding the possibility of fermion multiplets including only BSM fields;
(ii) all gauge bosons, including BSM ones, have integer charges;
(iii) all right-handed, SU(3) singlet, fermions have a left-handed counterpart.

We underline that we could have assumed the presence of additional sterile right-handed or left-handed neutrinos in an arbitrary number without changing the above (A1)–(A5) conditions. Nor would it have changed the definition of $N^F_{Q(\ell)}$, which refers to weak interacting fermions.

We found from relation (A2)

$$N^F_\ell = N^F_Q \equiv N_F; \tag{A11}$$

namely, the number of quark and charged lepton families must be the same. Combining (A1) with (A2), it follows that

$$N^{(3)}_Q = \frac{1}{3}(2N_F - N^{(3)}_\ell). \tag{A12}$$

Both sides of the equality must be integers. This is true if N_F is a multiple of the color $N_c = 3$, as in minimal models where $N^{(\bar{3})}_\ell = 0$. However, such proportionality is not strictly necessary. An example is $N_F = 4$ with $N^{(3)}_\ell = N^{(\bar{3})}_\ell = 2$ that gives $N^{(3)}_Q = N^{(\bar{3})}_Q = 2$ (see also Ref. [16]).

Another constraint is given by $\mathrm{SU}(3)_c$ asymptotic freedom. As in SM QCD, perturbative calculations limit the number of quark flavors to be equal to or less than 16. Since each family in 331 models has three flavors, that yields a constraint on the number of families: $N_F \leq 5$.

In summary, under the minimal assumptions discussed above, we can classify 331 models by means of $\beta = r/\sqrt{3}$, where $r = \pm 1, \pm 3$ (see, for instance, [51]), the family number $N_F \leq 5$, and the number of triplets or antitriplets for either quarks or leptons.

Fixing one of the four possible values for β, and assuming $N_c = N_F = 3$, we have two possible choices, namely, $N^3_Q = 2,\ 1$, that we name case $A[\beta]$ and $B[\beta]$, respectively. In Table V, we report the numbers of quark and leptons in triplets and antitriplets according to the relations (A10)–(A12). Note that this classification works only when triplets and antitriplets are used, but it is also possible to assign matter fields to different representations like sextets; see, for instance, Ref. [15].

Models $A[\beta]$ and $B[-\beta]$ are related by the simultaneous exchange of fermion triplets → antitriplets and $\beta \to -\beta$. It implies that matter and gauge fields possess the same set of charges in both models.[6] Several of these models have been discussed in the literature: $A[1/\sqrt{3}]$ [21,53,54], $A[-1/\sqrt{3}]$ [31,55–57], and $A[\sqrt{3}]$ [10–12,17,58–61]. The case $A[\sqrt{3}]$ is known as the minimal 331 model and will be considered in this work.

We remark that, in order to fully specify a model, fixing the matter content as above is not enough, but it has to be complemented by the assignment of the Higgs structures and vacuum alignments; see below.

[6]A complete symmetry between the two models requires additional conditions in the Higgs sector [52].

TABLE V. Number of triplets and antitriplets in the lepton and quark sectors in cases $A[\beta]$ and $B[\beta]$ (see the text).

	$N^{(3)}_Q$	$N^{(\bar{3})}_Q$	$N^{(3)}_\ell$	$N^{(\bar{3})}_\ell$
$A[\beta]$	2	1	0	3
$B[\beta]$	1	2	3	0

APPENDIX B: THE MODEL

1. The scalar sector

As mentioned in the introduction, the SSB pattern of 331 models occurs in two steps: The first one allows one to recover the SM group structure, and the second one is identifiable as the electroweak SSB. The 331 models are intrinsically multi-Higgs models, which, as in the SM, are $\mathrm{SU}(3)_C$ singlets acquiring nonvanishing vacuum expectation values after SSB. They are responsible of mass terms for gauge bosons and fermions coupling to them. The requirement of gauge invariance of the Lagrangian term coupling one Higgs and two fermions constrains the Higgs representations. Namely, since the fermions ψ transform either as a 3 or as a $\bar{3}$ under $\mathrm{SU}(3)_L$, we have only the following possibilities [62] for the Higgs field Φ:

(i) $\bar{\psi}_L \psi_R \Phi$, with $\Phi \sim 3$;
(ii) $\bar{\psi}_L (\psi_L)^c \Phi$, with $\Phi \sim 6$;
(iii) $\bar{\psi}_R (\psi_R)^c \Phi$, with $\Phi \sim 1$;
(iv) $\bar{\psi}_L (\psi'_L)^c \Phi$, with $\Phi \sim \bar{3}$.

The last possibility is allowed only if ψ'_L are completely exotic triplets with charged particles which are right-handed charge-conjugated components [13] and otherwise is zero. Singlets scalars are also scalars under $\mathrm{U}(1)_X$ because of electromagnetic invariance. Thus, after the two steps of SSB, their vacuum expectation value will never give rise to a mass term for the gauge bosons or the charged fermions. Sextets are needed in some models, for instance, to give Majorana masses to neutrinos as in Ref. [58] or to allow a doubly charged scalar to decay into same-sign leptons [17]. Both singlets and sextets are not needed for the model and the phenomenology we are investigating, so we limit to the simpler, and widely used in the literature, assumption of SSB by triplets only. The X values of the triplets, and, therefore, also the charges of their components, are constrained by the requirement that the above Higgs-fermion coupling terms in the Lagrangian are also invariant under $\mathrm{U}(1)_X$.

The Higgs triplets are

$$\chi = \begin{pmatrix} \chi^{++} \\ \chi^{+} \\ \chi^{0} \end{pmatrix}, \qquad \rho = \begin{pmatrix} \rho^{+} \\ \rho^{0} \\ \rho^{-} \end{pmatrix}, \qquad \eta = \begin{pmatrix} \eta^{0} \\ \eta^{-} \\ \eta^{--} \end{pmatrix}. \tag{B1}$$

Their X values are listed in Table I. The neutral components acquire a vacuum expectation value, namely, $\langle\chi^0\rangle \equiv u$, $\langle\rho^0\rangle \equiv v$, and $\langle\eta^0\rangle \equiv v'$. In the first SSB, triggered by the triplet χ, there are five gauge fields that acquire mass. In the second SSB, which requires the pair of triplets ρ and η, three gauge fields acquire mass, leaving massless the field associated to the unbroken charge generator, that is, the photon.

2. The gauge sector

Boson masses stem from the expansion of the covariant derivatives of the scalar sector:

$$\mathcal{L}_S = (D_\mu\chi)(D^\mu\chi)^\dagger + (D_\mu\rho)(D^\mu\rho)^\dagger + (D_\mu\eta)(D^\mu\eta)^\dagger. \quad \text{(B2)}$$

The neutral gauge bosons matrix is not diagonal. In the base $W^3_\mu, W^8_\mu, B_{X\mu}$, it is given by

$$M_0 = \frac{1}{4}\begin{pmatrix} g^2 v_+^2 & \frac{g^2}{\sqrt{3}} v_-^2 & -\frac{gg_X}{3\sqrt{2}}(v_+^2\sqrt{3} - v_-^2\beta) \\ \frac{g^2}{\sqrt{3}} v_-^2 & \frac{g^2}{3}(4u^2 + v_+^2) & -\frac{gg_X}{9\sqrt{2}}(u^2 4\sqrt{3}\beta + v_+^2\sqrt{3}\beta + 3v_-^2) \\ -\frac{gg_X}{3\sqrt{2}}(v_+^2\sqrt{3} - v_-^2\beta) & -\frac{gg_X}{9\sqrt{2}}(u^2 4\sqrt{3}\beta + v_+^2\sqrt{3}\beta + 3v_-^2) & \frac{g_X^2}{18}(4u^2\beta^2 + v_+^2(3+\beta^2) + v_-^2 2\sqrt{3}) \end{pmatrix}, \quad \text{(B3)}$$

where $v_-^2 = v'^2 - v^2$.

Since $u^2 \gg v_+^2, v_-^2$ we can diagonalize this matrix in two steps. In the first one, $B_{X\mu}$ and W^8_μ mix, giving rise to B'_μ and Z'_μ as

$$\begin{pmatrix} W^8_\mu \\ B_{X\mu} \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_{331} & -\sin\theta_{331} \\ \sin\theta_{331} & \cos\theta_{331} \end{pmatrix}\begin{pmatrix} B'_\mu \\ Z'_\mu \end{pmatrix}. \quad \text{(B4)}$$

With the above definition of the mixing angle, by diagonalizing M_0 we obtain

$$\sin\theta_{331} \simeq \frac{g}{\sqrt{g^2 + g_X^2\beta^2/6}}. \quad \text{(B5)}$$

Then, at the second step, B'_μ and Z'_μ mix, both with W^3_μ, but the mixing between Z'_μ and W^3_μ is small compared to the one between B'_μ and W^3_μ; therefore, the first one can be neglected, and we have

$$\begin{pmatrix} W^3_\mu \\ B'_{X\mu} \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \quad \text{(B6)}$$

where A_μ is the Standard Model photon field. The corresponding boson masses are

$$m_A = 0, \quad \text{(B7)}$$

$$m_Z^2 \simeq \frac{g^2}{4\cos^2\theta_W} v_{\rm SM}^2, \quad \text{(B8)}$$

$$m_{Z'}^2 \simeq \frac{g^2 u^2 \cos^2\theta_W}{3[1 - (1+\beta^2)\sin^2\theta_W]}. \quad \text{(B9)}$$

We note that $m_W^2/m_Z^2 = \cos^2\theta_W$ and, therefore, we can identify θ_W with the Weinberg angle.

An important constraint for 331 models is obtained by imposing a matching condition for the coupling of the Z boson with fermions. In particular, considering the right-handed leptons ℓ_R with X charge $X[\ell_R] = -1$, the covariant derivative is given by

$$D_\mu\ell_R = \partial_\mu\ell_R + ig_X\frac{B_{X_\mu}}{\sqrt{6}}\ell_R. \quad \text{(B10)}$$

From Eqs. (B4) and (B6), we have that

$$B_{X_\mu} = \cos\theta_{331} Z'_\mu - \sin\theta_{331}\sin\theta_W Z_\mu + \sin\theta_{331}\cos\theta_W A_\mu. \quad \text{(B11)}$$

Then, by singling out the coupling with the Z boson and by matching with the corresponding Standard Model coupling $g\sin^2\theta_W/\cos\theta_W$, we get

$$\frac{g_X}{g} = \frac{\sqrt{6}\sin\theta_W}{\cos\theta_W\sin\theta_{331}} = \frac{\sqrt{6}\sin\theta_W}{\sqrt{1 - \sin^2\theta_W(1+\beta^2)}}, \quad \text{(B12)}$$

where we have used Eq. (B5). Beyond tree level, the precise definition of $\sin^2\theta_W$ is renormalization scheme and energy dependent. Extending the tree-level formula $m_W^2/m_Z^2 = \cos^2\theta_W$ to all orders of perturbation theory results in the on-shell value of $\sin^2\theta_W$ [63]. Another widely used convention is the modified minimal subtraction ($\overline{\rm MS}$) scheme. We can see that the tree-level Eq. (B12) exhibits a Landau pole when $\sin^2\theta_W = 1/(1+\beta^2)$. As in other non-asymptotically-free theories, at the Landau pole the coupling constant g_X becomes infinite. The energy scale of the Landau pole depends on the running of $\sin^2\theta_W$, which, in turn, depends on the matter content of the model. Besides, in the on-shell scheme, radiative corrections for $\sin^2\theta_W$ include m_t^2-dependent terms which can cause large corrections in higher orders. $\sin^2\theta_W$ depends not only on the gauge couplings, but also on the spontaneous symmetry breaking. In the SM framework, using the $\overline{\mathrm{MS}}$ scheme, it has been estimated that values of $\sin^2\theta_W$ around the upper limit of the constraint $\sin^2\theta_W < 0.25$ do not arise before ≈ 4 TeV [18]. There are one-loop analyses that estimate that at large values of the coupling, that is, even before the Landau pole is reached, the model may lose its perturbative character [19,64–66] (see also the discussion in [67]). At the same time, some of these analyses indicate how to push the nonperturbative regime, and the Landau pole, up to much higher scales, including, for example, heavy resonances as active degrees of freedom in the running [19,20]. We stress anyway that the typical energy scale of bilepton-pair production we consider is of the order of $\approx 2m_Y \approx 2$ TeV. Given the considerations above, we assume that no significative barrier is posed by the Landau pole to the perturbative analysis carried out in the present study.

We also note that the left-hand side of Eq. (B12) is real if (see also [15])

$$|\beta| < \frac{\cos\theta_W}{\sin\theta_W} \simeq 1.83. \tag{B13}$$

This upper limit of $|\beta|$ is extremely important for the model building. As shown above, the only allowed values for $|\beta|$ are $r/\sqrt{3}$ with r integer. The constraint coming from Eq. (B13) yields $n \leq 3$, and, therefore, models with $n > 3$ are not allowed in this minimal framework.

APPENDIX C: DECAY RATES AND VERTICES

Here, we report the decay widths of the VLQs introduced in the model:

$$\Gamma(T \to Y^{++} d^a) = \frac{g^2|\mathcal{V}_{L3a}|^2}{64\pi} \frac{m_T^6 - 3m_T^2 m_Y^4 + 2m_Y^6}{m_T^3 m_Y^2}, \tag{C1}$$

$$\Gamma(D^i \to Y^{--} u^a) = \frac{g^2|\mathcal{U}_{Lia}|^2}{64\pi} \frac{m_D^6 - 3m_D^2 m_Y^4 + 2m_Y^6}{m_D^3 m_Y^2}, \tag{C2}$$

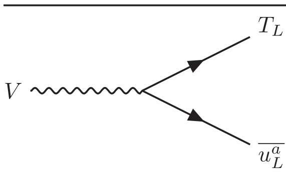
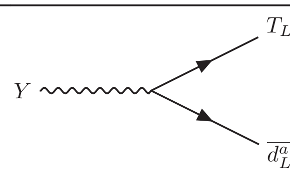
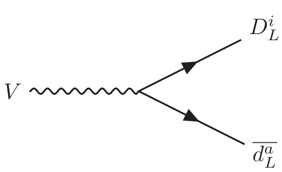
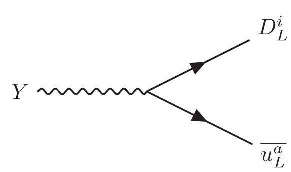
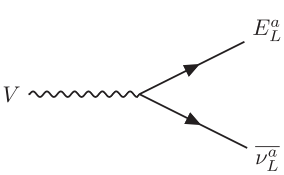
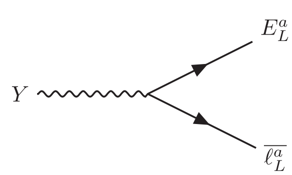
$$\Gamma(T \to V^{+} u^a) = \frac{g^2|\mathcal{U}_{L3a}|^2}{64\pi} \frac{m_T^6 - 3m_T^2 m_V^4 + 2m_V^6}{m_T^3 m_V^2}, \tag{C3}$$

$$\Gamma(D^i \to V^{-} d^a) = \frac{g^2|\mathcal{V}_{Lia}|^2}{64\pi} \frac{m_D^6 - 3m_D^2 m_V^4 + 2m_V^6}{m_D^3 m_V^2}, \tag{C4}$$

where m_X stays for the mass of the X field. The bilepton Y decay width is

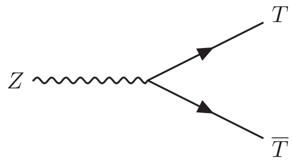
$$\Gamma(Y \to \ell^\pm \ell^\pm) = \frac{g^2}{12\pi} m_Y. \tag{C5}$$

In the following tables, we give the relevant vertices of the model.

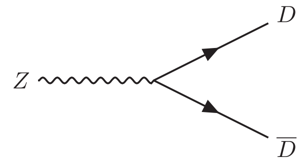
Gauge boson-fermion interactions			
Vertex	Coupling	Vertex	Coupling
V boson		Y boson	
	$\frac{g}{\sqrt{2}}\mathcal{U}_{L3a}$		$-\frac{g}{\sqrt{2}}\mathcal{V}_{L3a}$
	$\frac{g}{\sqrt{2}}\mathcal{V}_{Lia}$		$\frac{g}{\sqrt{2}}\mathcal{U}_{Lia}$
	$\frac{g}{\sqrt{2}}$		$-\frac{g}{\sqrt{2}}$

(Continued)

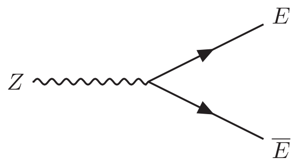
Z boson



$$\frac{g}{c_w}\left(\frac{1}{6}+\frac{\sqrt{3}}{2}\beta\right)s_w^2$$

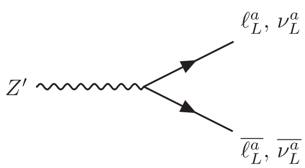


$$\frac{g}{c_w}\left(-\frac{1}{6}+\frac{\sqrt{3}}{2}\beta\right)s_w^2$$

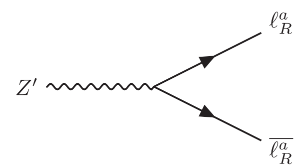


$$\frac{g}{2c_w}(1-\sqrt{3}\beta)s_w^2$$

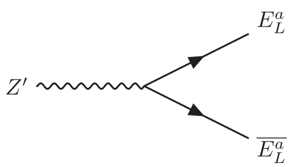
Z' boson



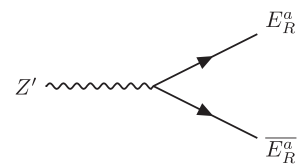
$$\frac{g[1-(1+\sqrt{3}\beta)s_w^2]}{2\sqrt{3}c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



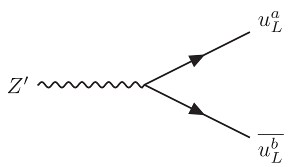
$$-\frac{g\beta s_w^2}{c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



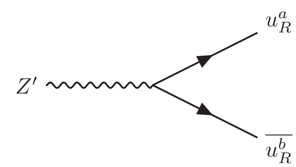
$$-\frac{g[2-(2-\sqrt{3}\beta(1-\sqrt{3}\beta))s_w^2]}{2\sqrt{3}c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



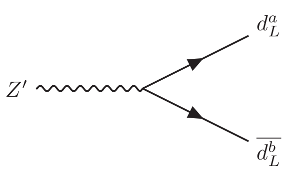
$$-\frac{g(1-\sqrt{3}\beta)\beta s_w^2}{2c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



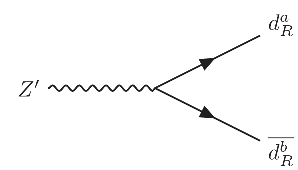
$$\frac{g[[-1+(\beta/\sqrt{3})s_w^2]\delta_{ab}+2c_w^2\mathcal{U}_{L3a}^*\mathcal{U}_{L3b}]}{2\sqrt{3}c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



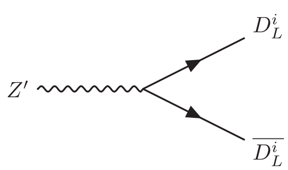
$$\frac{2g\beta s_w^2\delta_{ab}}{3c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



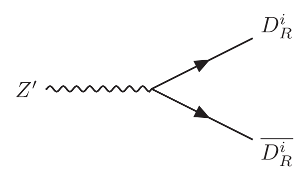
$$\frac{g[[-1+(\beta/\sqrt{3})s_w^2]\delta_{ab}+2c_w^2\mathcal{V}_{L3a}^*\mathcal{V}_{L3b}]}{2\sqrt{3}c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



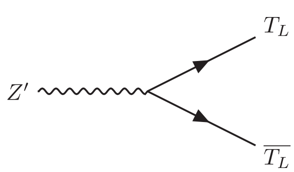
$$-\frac{g\beta s_w^2\delta_{ab}}{3c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



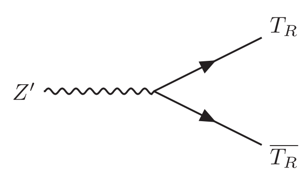
$$\frac{g[2+(-2\sqrt{3}+\beta(1-3\sqrt{3}\beta))s_w^2/\sqrt{3}]}{2\sqrt{3}c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



$$\frac{g\beta(1-3\sqrt{3}\beta)s_w^2}{6c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



$$\frac{g[-2+(2\sqrt{3}+\beta(1-3\sqrt{3}\beta))s_w^2/\sqrt{3}]}{2\sqrt{3}c_w\sqrt{1-(1+\beta^2)s_w^2}}$$



$$\frac{g\beta(1+3\sqrt{3}\beta)s_w^2}{6c_w\sqrt{1-(1+\beta^2)s_w^2}}$$

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