The Cross Section of Inverse Beta Decay

Giulia Ricciardi, N. Vignaroli, and F. Vissani

1 Introduction

The interaction process

$$\bar{\nu} + p \rightarrow e^{+} + n \tag{1}$$

is known as inverse β decay (IBD). Its cross section was estimated for the first time by Bethe and Peierls in 1934 [1], using the Fermi theory, and gave an extremely small value ($\sigma \approx 10^{-44}$ cm^2 at $E(\bar{\nu}) = 2$ MeV), leading them to believe that the neutrino is an "undetectable particle". It is kind of ironic that in 1956 the inverse beta decay was exactly the process that allowed the first direct experimental evidence of

G. Ricciardi (✉)
Dipartimento di Fisica E. Pancini, Università di Napoli Federico II, Naples, Italy

INFN, Naples, Italy

MSA, Naples, Italy
e-mail: giulia.ricciardi2@unina.it; giulia.ricciardi@na.infn.it

N. Vignaroli
Dipartimento di Matematica e Fisica E. De Giorgi, Università del Salento, Lecce, Italy

INFN, Lecce, Italy
e-mail: natascia.vignaroli@unisalento.it

F. Vissani
INFN Laboratori Nazionali del Gran Sasso, L'Aquila, Italy
e-mail: francesco.vissani@lngs.infn.it

Y. Tayalati, M. Gouighri (eds.), *The First African Conference on High Energy Physics*, Springer Proceedings in Physics 425,
https://doi.org/10.1007/978-3-031-88933-2_15

neutrinos [2].[1] Since then, the IBD has always played a central role in neutrino physics, giving rise to a positron and a neutron, both potentially observable, and being in common use at energies below ∼20 MeV and its wide use in water- or hydrocarbon-based detectors, which are relatively cheap materials and rich in target protons. As a consequence, an accurate estimate of its cross section is essential for the accuracy of the results in several and different neutrino experiments. In this work, we discuss its accurate evaluation [4] and relevance in the current experimental framework.

2 The Cross Section Evaluation

We summarize the main steps needed to build the IBD cross section:[2]

1. write down the matrix element of the charged weak current between proton and neutron states, which in the most general form depends on six dimensionless Lorentz invariant form factors:

$$\mathcal{J}_\mu = \bar{u}_n\left(f_1\gamma_\mu + g_1\gamma_\mu\gamma_5 + i f_2\sigma_{\mu\nu}\frac{q^\nu}{2M} + g_2\frac{q_\mu}{M}\gamma_5 + f_3\frac{q_\mu}{M} + i g_3\sigma_{\mu\nu}\frac{q^\nu}{2M}\gamma_5\right)u_p; \tag{2}$$

2. calculate the tree level amplitude for the IBD, by using the lepton current and the W propagator. By squaring and averaging, one obtains the differential cross section

$$\frac{d\sigma}{dt} = \frac{G_F^2\cos^2\theta_C}{64\pi(s-m_p^2)^2}\,\overline{|\mathcal{M}^2|} \tag{3}$$

where G_F is the Fermi coupling, the parameters $s = (p_\nu + p_p)^2$, $t = (p_\nu - p_e)^2$ are Mandelstam variables and θ_C is the Cabibbo angle (that is linked to the $u - d$ element of the CKM matrix by the equality $\cos\theta_C = V_{\mathrm{ud}}$);
3. include the radiative corrections at leading order [5] using:

$$d\sigma(E_\nu, E_e) \to d\sigma(E_\nu, E_e)\left[1 + \frac{\alpha}{\pi}\left(6.00 + \frac{3}{2}\log\frac{m_p}{2E_e} + 1.2\left(\frac{m_e}{E_e}\right)^{1.5}\right)\right] \tag{4}$$

[1] For a brief historical survey see e.g. [3].

[2] For details see Ref. [4].

where α is the fine-structure constant. Next order corrections and other effects such as isospin breaking are estimated to be small and neglected in the low range of neutrino energies (below $\sim$10 MeV).

A new series of modern calculations was started by Vogel and Beacom in 1999 [6] and continued by Strumia and Vissani [7] in 2002. An updated and accurate assessment of the uncertainties was addressed in 2023 by Ricciardi, Vignaroli and Vissani in 2023 [4].

The estimation of theoretical uncertainties is crucial in experimental measurements aiming at high precision. In the cross section built as outlined before, the uncertainties are reflection of the uncertainties in the knowledge of the form factors and the Cabibbo angle.

The hadronic current $\mathcal{J}_\mu$ includes the vector and axial vector terms with form factors f_3 and g_3, that transform as second class currents under G-parity, according to a classification due to Weinberg [8]. G-parity is a symmetry for strong interactions, broken by mass differences. It turns out that second class currents can be safely neglected at the current level of precision [4].

The input parameters which determine the leading uncertainties in the cross section evaluation vary with the energy range; they are

- the Cabibbo angle and the axial coupling $g_1(0)$ (lowest energies);
- the axial radius r_A (higher energies).

The magnitude of the mixing element $V_{ud} = \cos\theta_C$ can be derived directly from the super-allowed $0^+ \to 0^+$ nuclear beta decays, which are pure vector transitions. The result is in tension with the one derived indirectly from the CKM unitarity, using the currently accepted values for $|V_{us}|$ and $|V_{ub}|$. That has been interpreted as a failure of unitarity of around 2σ and possibly more [9]. We believe it is reasonable to assume that this difference merely signals limits to the available interpretations and measurements, and we combine the data, enlarging all errors contributing to the result by the scale factor $S = \sqrt{\chi^2/(N-1)} = 2.0$ for a conservative estimation of the uncertainty, in agreement with the PDG suggestion [10]. We obtain $V_{\text{ud}} = 0.9743(3)$.

There are eight different measurements of the normalized axial coupling $\lambda = -g_1(0)/f_1(0)$ using decay polarized neutrons, with the latest measurement, published in 2019 by the Perkeo III collaboration [11], being much more precise than the others. In a conservative perspective, we include all results, but enlarging their error by a factor 2 to take into account discrepancies among older and newer results. That yields the value $\lambda = 1.2760(5)$, which is within 1σ from Ref. [11] and agrees with the global average.

The theoretical relation [12]

$$\frac{1}{\tau_{\text{n}}} = \frac{V_{\text{ud}}^2\,(1+3\lambda^2)}{4906.4 \pm 1.7\ \text{s}} \tag{5}$$

links both λ and V_{ud} to the average lifetime of the neutron τ_n. By propagating the errors, we find the prediction $\tau_{\mathrm{n}}(\mathrm{SM}) = 878.38 \pm 0.89$ s. Since the neutron decay lifetime τ_n is measured, Eq. (5) could help us to improve the inferences on the IBD cross section. There are two methods of measurement for the neutron decay lifetime. In experiments "in-bottle" or "storage", ultra-cold neutrons are trapped and, after a holding time T, they are released and the number $N(T)$ of surviving ultra-cold neutrons is counted. This is repeated for two or more different holding times, with T ranging from a few minutes up to some fraction of an hour. If no ultra-cold neutrons are lost, τ_n is obtained from an exponential fit to $N(T) \propto \exp(-T/\tau_n)$, and no absolute measurement is required. In "beam" experiments the neutron decay lifetime is directly calculated by counting decay products (protons and/or electrons) from a thermal neutron beam, while simultaneously measuring the neutron beam density.[3] There is a discrepancy among these results. In particular the value reported by the most precise beam experiment conducted at the NIST Center for Neutron Research [14, 15] is 8.9s (3.9σ) higher than the average storage value. This discrepancy between the two set of τ_n measurements has been widely discussed in literature, without reaching general consensus. Since the values of V_{ud} and λ are consistent with the values from the storage approach, and incompatible with the ones from the beam approach, we use only the former data set, assuming that the others are affected by a systematic deviation, which is not yet fully understood. The observed value of the neutron lifetime, together with the SM prediction, yields the relationship $V_{\mathrm{ud}} = 2.36323(75)/\sqrt{1+3\lambda^2}$.

In summary, by propagating the uncertainty factors we find that the cross section is known with $\delta\sigma_{\bar{\nu}_e p}/\sigma_{\bar{\nu}_e p} = 0.1\%$ for low values of electron anti-neutrino energies, which is four times better than [7].

At higher energies, the cross section becomes sensible to the momenta dependence of the form factors. This is generally expressed by phenomenological descriptions as the commonly used dipolar approximation. Since we are mostly concerned with low energy processes, whatever the expression of the full dependence is, we only need the first terms of its Taylor expansion. This is a rather general description and lessens the dependence on the phenomenological approximations, which in the most common cases are not even optimised for the energies we are discussing. Thus we set $g_1/g_1(0) = 1 + q^2 r_A^2/6$. The current value of the axial mass [16] $M_A = 1014 \pm 14$ MeV in the dipolar approximation implies $r_A^2 = 0.455 \pm 0.013$ fm^2; a determination that does not assume the double dipole has an error larger of about one order of magnitude $r_A^2 = 0.46 \pm 0.12$ fm^2.

By including the uncertainty on r_A, we find $\delta\sigma_{\bar{\nu}_e p}/\sigma_{\bar{\nu}_e p} = 1.1\%(E_\nu/50\,\mathrm{MeV})^2$ in the region above $\sim$10 MeV, a value which is 3 times larger than in [7].

[3] In the past years a third method for measuring the neutron life-time has been devised, which uses spacecraft-based neutron detectors to count the relative neutron flux as a function of altitude [13]. Its precision is not yet competitive.

While the cross section values are consistent with previous ones, the increase in its uncertainty range motivates attempts to improve the description of form factors at the energies we are considering.

3 Conclusions

The IBD plays an important role in present and future experiments studying neutrinos at low and intermediate energy ranges.

The detection of geo-neutrinos, whose energies extend up to about 2.5 MeV, is realised through IBD. In essentially all oscillation studies at reactors, electron antineutrinos are detected by interactions with free protons via IBD. Reactor neutrino energy spectra end at $\sim$10 MeV. Some of supernova experiments based on water Cherenkov detector are primarily sensitive to IBD interactions, revealing the Cherenkov light of the final-state positron. The inverse beta decay of supernova neutrinos can also be observed in scintillation detectors. Neutrino fluxes from supernovae go up to 50 MeV.

A reliable knowledge of the cross section of the IBD is based on a set of theoretical concepts and on measurements of the key parameters. We assessed the uncertainty on the cross section, which is relevant to experimental advances and increasingly large statistical samples. In order to improve the actual estimates, we need to address the reason of discrepancy in τ_n measurements and the unitarity issue in $|V_{ud}|$. This is particularly significant at the low energies which are relevant e.g. for reactor neutrinos. A priority, especially for supernova studies, is also to refine the description of the axial form factor in the 100 MeV range, thus decreasing the uncertainty due to r_A.

Acknowledgments This work was partially supported by INFN research initiative ENP and by the research grant number 2022E2J4RK "PANTHEON: Perspectives in Astroparticle and Neutrino THEory with Old and New messengers" under the program PRIN 2022 funded by the Italian Ministero dell'Università e della Ricerca (MUR) and by the European Union – Next Generation EU.

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