



Combining autonomous delivery robots and traditional vehicles with public transportation infrastructure in last-mile distribution[☆]

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ABSTRACT

The recent boom of e-commerce and the resulting concern about emissions increase are promoting the development of alternative paradigms for last-mile delivery, involving greener vehicles, such as air drones and autonomous delivery robots, which are also less sensitive to traffic congestion. In this paper, we study a last-mile distribution problem in which a heterogeneous fleet composed of both traditional vehicles and autonomous delivery robots is used to service a large number of customer requests in an urban area. The delivery robots use stations of the public transportation network as their depots. The problem amounts to building both van and delivery-robot routes in such a way as to minimize the distribution cost, meeting the constraints imposed by the different ranges and capacities of both vehicles. At the same time, another decision involves choosing at which stations of the public transportation network the delivery robots are based. For this purpose, we develop two variants of a matheuristic scheme based on the solution of a hybrid set-partitioning and location-based model. We perform thorough computational experiments on instances inspired by the problem of distributing drugs to pharmacies in the urban area of Rome (Italy) and show that, when the instance size grows, the proposed solution approach allows us to obtain significant cost reductions compared to both a straightforward solution of a mathematical model and a classical metaheuristic scheme. In addition, we show that combining autonomous delivery robots with traditional vehicles allows us to achieve important cost and emission reductions with respect to a traditional delivery scheme.

1. Introduction

In recent years, the e-commerce sector has been experiencing a huge rise in its volumes, with e-commerce sales which are expected to reach approximately 8 trillion dollars by 2026 worldwide (Statista, 2022). This is obviously causing an exponential increase of parcels to be delivered, accelerating the need for a more sustainable last-mile logistics. For instance, in the 2030 Agenda (United Nations, 2022), the debate surrounding the Sustainable Development Goals (SDGs) has stimulated various green logistics initiatives with the aim to enhance energy efficiency, as well as to reduce air pollutant emissions and the associated health impact. These factors have motivated the study and gradual introduction of innovative distribution paradigms based on new types of vehicles, such as Unmanned Aerial Vehicles (UAV, typically referred to as aerial drones, or simply drones) and Autonomous Delivery Robots (ADR, also referred to as ground drones).

UAVs and ADRs are used to deliver several types of products, such as medicines (or even organs for transplantation), food, and small parcels. They have various advantages compared to traditional delivery vehicles. First, they can contribute to significantly reducing parcel distribution times, especially in highly-congested urban areas or in limited traffic zones (where traditional vehicles might not be allowed to enter). Second, they are electrically-powered and just need a supervisor (who can typically manage more than a single vehicle at a time) for loading/unloading the parcels and for battery replacement. As a consequence, UAVs and ADRs have reduced operating costs compared to traditional delivery vehicles, which are subject to both fuel and driver costs. Third, such innovative vehicles have negligible emissions compared to traditional vans. Nevertheless, there are still some technological and regulation issues to overcome. Indeed, both UAVs and ADRs have limited capacity and autonomy. Furthermore, in

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(a) Sidewalk-ADR by Amazon



(b) Sidewalk-ADR by Startship



(c) Road-ADR by Nuru



(d) Robovan by Mercedes and Starship

Fig. 1. Examples of autonomous delivery robots.

many urban scenarios, especially in Europe, this type of service is not yet fully regulated, although it is expected to be so in the near future.

Kunze (2016) reports a comprehensive list of the main differences between UAVs and ADRs, as well as of their potential applications in a city-logistics context. In general, aerial vehicles can travel faster than ground vehicles, but they can typically transport one parcel at a time, whereas ADRs might have several compartments to deliver freight to different customers within a single trip. Moreover, ADRs have significant advantages in terms of energy consumption, given that they consume approximately three to 10 times less energy than UAVs (Kirschstein, 2020). Another important difference is that UAVs need a safe area to land, which can be very difficult to find in densely populated urban areas. In addition, ADRs are also advantageous from the point of view of noise pollution, given that they operate quietly, which is crucial for maintaining the quality of life in urban areas (Yuen et al., 2022). For all these reasons, UAVs are most suited for deliveries in rural areas, whereas ADRs are more advantageous for urban last-mile delivery. In fact, ADRs primarily operate on sidewalks and pedestrian pathways, bypassing traffic bottlenecks and ensuring more reliable delivery times, especially during peak hours (Mourad et al., 2021). The potential of ADRs is explored in Lemardelé et al. (2021), with the authors offering some insights for designing ADR-based delivery systems for different cities.

Concerning the distribution paradigm, deliveries can be made by ADRs alone or by them together with vans. In this latter case, it is worth mentioning the ‘Robovan’ partnership between Mercedes-Benz (providing the vans) and Starship Technologies (providing the ADRs), in which a van can host up to eight delivery robots that are transported to pre-determined drop-off locations, where they are unloaded to start their autonomous trips to complete deliveries (Crowe, 2016). Fig. 1 shows different types of ADRs, as well as the ‘Robovan’ concept. Another delivery scheme that has started to be studied recently is exploiting the Public Transportation (PT) system to take advantage of the possible synergies between freight and passenger flows in urban areas (freight-on-transit problems, see, e.g., Bosse et al., 2023; Delle Donne et al., 2023). In the context of ADRs, examples include integrated systems in which freight requests are delivered using a fleet of delivery robots that can use a public transportation service as part of their trips (De Maio et al., 2024; Mourad et al., 2021).

In this article, we deal with a problem in which last mile-distribution is carried out by traditional vans (located at one or more depots) along with a fleet of ADRs starting their trips from stations of the public transportation (PT) network (assumed to be the ADRs depots), where we assume that the freight to deliver arrives by transiting on the PT lines before the start of the working day. The problem entails assigning customers to either traditional vehicles or ADRs, as well as building both van and robot routes in such a way as to minimize the overall costs while satisfying the constraints imposed by the different ranges and capacities of both vehicles. At the same time, we must decide at which PT stations the ADRs are based. For this purpose, we develop two variants of a matheuristic scheme - based on a generalization of the approach by Bramel and Simchi-Levi (1995) - in which van and delivery robot routes are jointly designed. Our approach is tested on a set of realistic instances based on the urban area of Rome (Italy). In the following, we refer to this decision-making problem as the Location-Routing Problem with Vans and autonomous delivery Robots (LRPVR).

The remainder of the paper is organized as follows. In Section 2, we provide an overview of the literature related to our problem, which is formally described in Section 3. Then, in Section 4 we detail our solution approach, and in Section 5 we report the results of the computational experiments, followed by conclusions in Section 6.

2. Literature review

The proposed system combines ADRs and traditional delivery vehicles, resulting in a hybrid transport network integrating two components: routes of traditional vehicles originating from a central depot, and ADR routes starting from stations of the PT network. Both types of routes have the goal of servicing customers. The problem we study is related to several variants of the Vehicle Routing Problem (VRP), mainly those addressing heterogeneous fleets and hybrid delivery systems.

A first example of such problems is the Two-Echelon VRP (2E-VRP, see Gonzalez-Feliu (2011) and Sluijk et al. (2023) for systematic reviews), in which in the first echelon larger vehicles transport goods to intermediate locations. Then, in the second echelon, smaller vehicles perform the final deliveries to customers (Gonzalez-Feliu, 2013). For instance, Hemmelmayr et al. (2012) develop new neighborhood

search operators, used in a hierarchical scheme reflecting the multi-level nature of the problem, whereas [Lehmann and Winkenbach \(2024\)](#) introduce a variant of the 2E-VRP - combining features such as time windows, mixed pickup and delivery demand, or vehicle range constraints - solved by a matheuristic. However, even if we also have larger and smaller vehicles (the vans and the ADRs, respectively), our problem is not exactly a 2E-VRP, because we neglect how the goods arrive at the depots, and focus only on the last leg of the distribution process.

Our problem is also related to the literature on fleet size and mix VRPs ([Hoff et al., 2010](#)) and VRPs with heterogeneous fleet ([Baldacci et al., 2008](#)), in which the goal is to determine the routing and the composition of a mixed fleet, as well as on multi-depot VRPs ([Rodrigues Pereira Ramos et al., 2020](#)), for which the goal is to minimize the total travel cost, with the vehicles based at different depots. For instance, [Mancini \(2016\)](#) introduces an Adaptive Large Neighborhood Search (ALNS) metaheuristic to tackle a variant of the problem involving multiple periods and varying levels of incompatibility between customers, vehicles, and days. More recently, the increased interest for autonomous delivery vehicles, has stimulated the development of contributions employing ADRs as one element of the mixed fleet. A recent review of delivery problems involving ADRs can be found in [Srinivas et al. \(2022\)](#). Among the most relevant contributions, [Boysen et al. \(2018\)](#) explore a scenario involving a truck combined with several ADRs. After being loaded with the parcels to deliver at a central depot, the truck moves toward the city center, from which the ADRs start their trips. Each ADR serves a single customer and, after completing its task, it returns to a predetermined depot. To solve this problem, the authors propose an iterated local search approach. [Chen et al. \(2021\)](#) propose a VRP with time windows and delivery robots. They devise a two-stage matheuristic based on clustering techniques to handle instances of medium size. A variant of this problem is proposed in [Alfandari et al. \(2022\)](#), in which the objective is to minimize a tardiness measure computed with respect to the customers' delivery deadlines. [Yu et al. \(2022\)](#) suggest a two-echelon pickup and delivery scheme involving a van and a single robot. The van visits some parking nodes to drop off and/or pick up its robot, replenish and/or swap its battery if necessary. The authors propose a mixed-integer model, as well as an ALNS algorithm to solve larger instances. Then, [Yu et al. \(2024\)](#) study a variant of this problem in which, when the robots are on the vans, they can be recharged, thereby increasing the efficiency of the distribution system. Again, an ALNS algorithm is presented. Recently, [Kloster et al. \(2023\)](#) study a multiple Traveling Salesman Problem in which several trucks start their trips from a depot and can supply some packet stations, from which robots can then be launched to serve customers. To minimize the makespan, the authors propose two variants of a decomposition-based matheuristic, as well as an algorithm based on iterated local search. [Heimfarth et al. \(2022\)](#) generalize the above concepts and propose a mixed truck and robot delivery concept, in which the truck not only transports the robots to drop-off locations, but is also used for delivering goods to customers. This problem is solved by using a General Variable Neighborhood Search. Subsequently, [Ostermeier et al. \(2023\)](#) extend the previous problem to consider multiple trucks and develop a tailored heuristic solution based on Set Improvement Neighborhood Search. We observe that, in all the previously reported contributions, the ADRs are either launched from the trucks, or have capacity for a single shipment, or both. On the other hand, in our work we assume that robots have capacity for multiple deliveries during a tour and are based at their own depots, from which they depart and where they must return. In particular, this latter feature makes the problem more applicable in practice, because it eliminates the synchronization issues between trucks and robots.

Another research stream to which our problem is also related concerns Location-Routing Problems (LRPs, see [Mara et al., 2021](#)), since it includes the selection of the PT stations where the ADRs are based. Combining the LRP with the multi-depot and fleet size and mix issues

results in a multi-depot fleet size and mix LRP. In this light, the contribution closest to our paper is [Koç et al. \(2016\)](#), in which formulations and a heuristic are presented in presence of time windows. However, our paper differs from [Koç et al. \(2016\)](#) for two main aspects: (i) only ADRs may be based at the stations of the PT system; (ii) range constraints affect the design of the ADR routes.

Concerning the usage of matheuristic algorithms for related problems, many contributions have been proposed in the context of distribution logistics, not only in the case of last-mile delivery. However, none of such contributions directly tackles the same problem we are studying. To give a first example, [Villegas et al. \(2013\)](#) propose a two-phase matheuristic for a truck and trailer routing problem, in which the routes obtained in the first phase are used as columns in a set-partitioning formulation of the problem. Then, [Wang et al. \(2023\)](#) study a joint optimization problem of parcel allocation and crowd routing, and devise a data-driven column generation algorithm to solve the problem based on a set-partitioning formulation, as well as a rolling-horizon approach. [Mancini and Gansterer \(2022\)](#) tackle the VRP with occasional drivers and orders bundle, where, instead of assigning one customer per driver, drivers are assigned bundles of customers. The solution approach is a mathematical model for solving small instances, and a matheuristic based on Large Neighborhood Search (LNS) for larger instances. In [Mancini and Gansterer \(2021\)](#), the same authors propose a last-mile delivery problem combining the advantages of attended home delivery and usage of shared delivery locations, solved with a LNS-based matheuristic and an iterated local search procedure. In the context of dynamic VRPs, [He et al. \(2020\)](#) face a VRP with dynamic customers considering dual service (including home delivery and customer pickup) in last-mile delivery, and design a two-phase matheuristic including construction-improvement heuristics to solve the deterministic model and dynamic programming to include dynamic customers into the routes. More recently, [Pina-Pardo et al. \(2024\)](#) study a VRP where orders become available at the depot over time and can be collected by trucks via depot returns or resupplied by air drones en route. The authors propose a matheuristic approach, based on a tabu search scheme, that provides good solutions for both the truck-and-drone and the truck-only versions of the problem. Finally, in a first-mile context, [He et al. \(2024\)](#) study a ridesharing problem with public and private autonomous vehicles, for which they propose a branch-and-price procedure and a tailored column-generation matheuristic algorithm.

2.1. Contributions to the literature

The contributions of this paper to the existing literature are fourfold. First, we propose a combined set-partitioning and two-index vehicle flow formulation to model a last-mile location-routing problem using a heterogeneous fleet of traditional vehicles and autonomous delivery robots, with the latter starting their trips from stations of the public transportation network. We observe that, incorporating location decisions about the depots of one component of the fleet, namely the ADRs, makes our problem different from the last-mile vehicle routing problem with heterogeneous fleet. Second, we design a two-step matheuristic scheme, in which we first solve a hybrid set-partitioning and location-based model where van routes are approximated as a concentrator location problem, whose solution is then transformed into a solution to the routing problem in the second step. As a third contribution, we perform a comprehensive computational study on a set of realistic large-scale instances with up to 500 customers, located in the urban area of Rome (Italy). Finally, we derive additional insights showing the cost and emission advantages of using the proposed delivery infrastructure compared to a classical delivery scheme.

3. Problem definition

In this section, we present a formal definition of the LRPVR. We first give a problem narrative and then present a mathematical formulation.

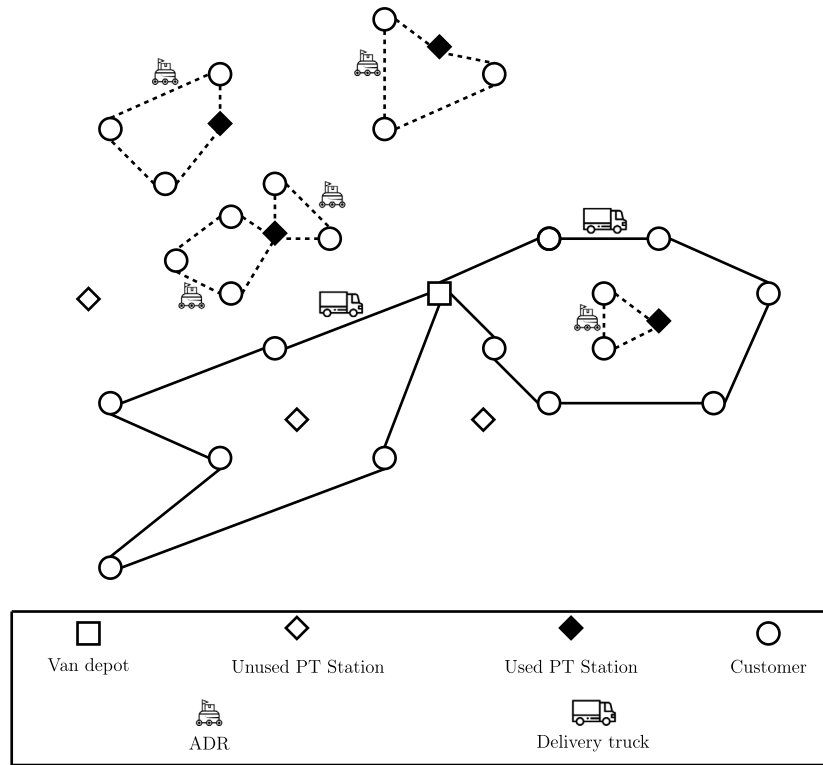


Fig. 2. Scheme of the proposed network.

3.1. Problem narrative

We consider a transportation service provider which uses a heterogeneous fleet of traditional vehicles (e.g., vans) and autonomous delivery robots to service a set of last-mile delivery requests. Such a scheme could be effectively applied across various sectors, including e-commerce, grocery delivery, and medical supplies distribution. Traditional vehicles are located at a central depot, whereas the starting/ending points of ADR routes must be determined among a set of potential locations, represented by stations of the PT network. We assume that the freight designated to be delivered by ADRs arrives at the stations by transiting on the PT lines. Since the focus of our work is on the last-mile delivery phase, we do not take into account how the goods arrive at the PT stations or at the van depot. In Fig. 2 we report a scheme of the proposed delivery infrastructure, along with a possible feasible solution. Here, the vans move from a central depot (the white square) to serve a subset of the customers. The remaining customers are serviced by ADRs, which start their routes from some PT stations (the black diamonds). The other PT stations not acting as ADR depots (the empty diamonds) remain unused. In the figure, the dashed lines represent the arcs traveled by ADRs, whereas the continuous lines are the paths followed by traditional vans.

Vans and delivery robots differ in capacity and autonomy. In particular, ADRs have a lower capacity and autonomy than vans but have the advantage of not being influenced by traffic, given that they only travel on sidewalks. Routes are feasible if the capacity and autonomy thresholds of vehicles are not exceeded.

To solve this problem, the provider must make several distinct, yet intertwined decisions: (i) determine which PT stations to use as depots of the ADRs; (ii) determine which delivery will be carried out by a van and which will be performed by a delivery robot; (iii) route both types of vehicles accordingly.

3.2. Mathematical modeling

In the following, we present a mathematical program that models the LRPVR, preceded by a description of the rationale behind the modeling choices and some general notation.

A solution to our problem partitions the set of customers into a set of ADR routes and a set of van routes. Concerning ADR routes, their peculiarity is that each one contains very few customers, because of the limited capacity and range of such vehicles (e.g., in our experiments the capacity and range of an ADR are set to three compartments and 16 km, respectively). Thus, a natural modeling choice leads to a set-partitioning model (Balinski & Quandt, 1964) in which all the feasible routes (satisfying the capacity and maximum distance constraints) are enumerated. Each route of an ADR starts from one of the PT stations (to be identified), visits the assigned customers, and terminates at the same PT station. With respect to van routes, we employ a classical two-index vehicle flow formulation in which the subtour elimination constraints are modeled in a Miller-Tucker-Zemlin (MTZ) fashion (Toth & Vigo, 2014).

To formally describe our mathematical model, let $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ be a directed graph. Here, $\mathcal{V} = \{0\} \cup \mathcal{V}^c \cup \mathcal{V}^p$ is the set of vertices, for which 0, $\mathcal{V}^c$, and $\mathcal{V}^p$ represent the vans depot, the set of customers, and the set of PT stops, respectively. Moreover, $\mathcal{A} = \mathcal{A}^v \cup \mathcal{A}^d$ is the arc set, where $\mathcal{A}^v$ contains the arcs that can be traversed by vans, whereas $\mathcal{A}^d$ is the set of arcs that can be traversed by delivery robots.

With each vertex $i \in \mathcal{V}^c$ we associate a demand d_i , and for each $i \in \mathcal{V}$ we denote as $\delta^-(i)$ the set of vertices from which vertex i can be reached (i.e., $\delta^-(i) = \{j \in \mathcal{V} : (j, i) \in \mathcal{A}\}$), and as $\delta^+(i)$ the set of vertices that can be reached from vertex i (i.e., $\delta^+(i) = \{j \in \mathcal{V} : (i, j) \in \mathcal{A}\}$).

Each arc $(i, j) \in \mathcal{A}$ is associated with a length l_{ij} and a cost c_{ij} . This cost depends on the type of vehicles traversing the arc. Given a cost coefficients per unit of distance obtained as the sum of a fuel/energy cost per unit of distance and a vehicle cost per unit of distance, denoted as λ^v for vans and λ^d for ADRs, the cost of each arc (i, j) will be $c_{ij} = \lambda^v l_{ij}$ if $(i, j) \in \mathcal{A}^v$, or $c_{ij} = \lambda^d l_{ij}$ if $(i, j) \in \mathcal{A}^d$.

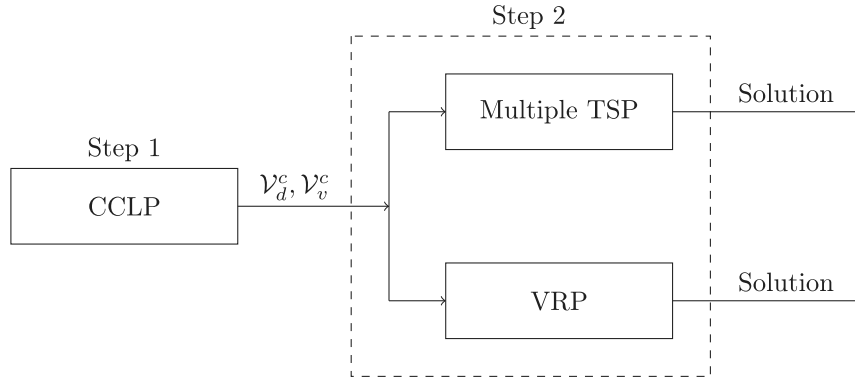


Fig. 3. Scheme of the links between the two phases of the matheuristic.

To account for the peculiarities of the different vehicles, we introduce the following additional notation: q^d , the capacity of an ADR; q^v , the capacity of a van; L^d , the maximum distance that can be covered by an ADR within a single route. Concerning customer service times, we assume that they are negligible.

Finally, with respect to ADRs, let $\mathcal{R}^d$ be the set of their feasible routes, which are non-dominated by other ADR routes. To formally define the concept of domination between ADR routes, we represent a route $r \in \mathcal{R}^d$ as $r = [r(0), r(1), r(2), \dots, r(|\mathcal{N}_r|)]$, where $\mathcal{N}_r$ is the set of customers visited by route r , $r(0) \in \mathcal{V}^p$, and $r(1), \dots, r(|\mathcal{N}_r|) \in \mathcal{V}^c$. To easily consider the association between a customer $i \in \mathcal{V}^c$ and a route $r \in \mathcal{R}^d$, we introduce a binary parameter a_{ir} equal to 1 if and only if customer $i \in \mathcal{V}^c$ is included in route $r \in \mathcal{R}^d$. We highlight that a route $r \in \mathcal{R}^d$ is feasible if $\sum_{h=0, \dots, |\mathcal{N}_r|-1} l_{r(h)r(h+1)} + l_{r(|\mathcal{N}_r|)r(0)} \leq L^d$ and $\sum_{i \in \mathcal{V}^c} d_i a_{ir} \leq q^d$. Moreover, let $b_r = \sum_{h=0, \dots, |\mathcal{N}_r|-1} c_{r(h)r(h+1)} + c_{r(|\mathcal{N}_r|)r(0)}$ be the cost of route $r \in \mathcal{R}^d$. Then, a route r is said to be dominated by a route r' if $\mathcal{N}_r \subseteq \mathcal{N}_{r'}$ and $b_{r'} \leq b_r$, and at least one of the two relationships is strict.

The decision variables are:

- x_{ij} , $(i, j) \in \mathcal{A}^v$, equal to 1 if the arc (i, j) is activated, 0 otherwise;
- s_r , $r \in \mathcal{R}^d$, equal to 1 if route r is selected, 0 otherwise;
- z_i , $i \in \mathcal{V}^c$, equal to 1 if customer i is served by an ADR, 0 if it is served by a van;
- u_i , $i \in \mathcal{V}^c$, total demand already delivered by a van when visiting customer i .

Thus, our mathematical formulation for the LRPVR is:

$$\text{Minimize } \sum_{r \in \mathcal{R}^d} b_r s_r + \sum_{(i,j) \in \mathcal{A}^v} c_{ij} x_{ij} \quad (1)$$

s.t.

$$\sum_{r \in \mathcal{R}^d} a_{ir} s_r = z_i \quad \forall i \in \mathcal{V}^c \quad (2)$$

$$\sum_{i \in \delta^-(j)} x_{ij} = 1 - z_j \quad \forall j \in \mathcal{V}^c \quad (3)$$

$$\sum_{j \in \delta^+(i)} x_{ij} = 1 - z_i \quad \forall i \in \mathcal{V}^c \quad (4)$$

$$\sum_{i \in \delta^+(0)} x_{0i} \leq \sum_{i \in \mathcal{V}^c} (1 - z_i) \quad (5)$$

$$u_i - u_j + q^v x_{ij} \leq q^v - d_j \quad \forall (i, j) \in \mathcal{A}^v \quad (6)$$

$$d_i \leq u_i \leq q^v \quad \forall i \in \mathcal{V}^c \quad (7)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in \mathcal{A}^v \quad (8)$$

$$s_r \in \{0, 1\} \quad \forall r \in \mathcal{R}^d \quad (9)$$

$$z_i \in \{0, 1\} \quad \forall i \in \mathcal{V}^c. \quad (10)$$

Objective function (1) aims to minimize the sum of the costs of both ADR and van routes. Constraints (2) model the set-partitioning constraints and specify that each customer $i \in \mathcal{V}^c$ that is served by an

ADR (i.e., $z_i = 1$) must be contained in exactly one ADR route. Then, the remaining constraints model the routing of the customers serviced by vans, i.e., customers $i \in \mathcal{V}^c$ for which $z_i = 0$. In particular, constraints (3) and (4) state that, in a van route, each customer vertex is connected to two other vertices, which are its predecessor and successor. Constraint (5) ensures that at most $\sum_{i \in \mathcal{V}^c} (1 - z_i)$ van routes are constructed, i.e., at most one for each customer not served by an ADR. Then, constraints (6) are the MTZ subtour-elimination constraints for VRPs, whereas (7) represent capacity constraints. Finally, constraints (8)–(10) define the domain of the variables.

This problem is NP-hard since it is a generalization of the multi-depot capacitated vehicle routing problem with length constraints (Contardo & Martinelli, 2014). As a result, the mathematical formulation is only suitable for solving small-sized instances of the problem, and heuristic approaches are needed to tackle instances of practical size in a reasonable amount of time.

4. A hybrid matheuristic

By running preliminary experiments on formulation (1)–(10), it turns out that the most complicated task is solving the two-index formulation for determining van routes. Thus, the basic idea of our matheuristic approach is to solve the LRPVR in two steps. In step 1, we solve a hybrid set-partitioning and location-based model in which the ADR routes are determined, as in Section 3, using a set-partitioning approach, whereas for van routes we use a formulation inspired by the Location Based Heuristic (LBH) of Bramel and Simchi-Levi (1995), which is proven to be asymptotically optimal for any distribution of customer demands and locations. LBH is based on formulating the routing problem as a location problem known as the Capacitated Concentrator Location Problem (CCLP). Then, in step 2, the solution to the CCLP is transformed into a solution to the routing problem, by solving either several Traveling Salesman Problem (TSPs), or a single VRP. In Fig. 3, we report a scheme of the links between the two phases of the matheuristic, whose details are reported in the following.

4.1. Step 1: solving an approximation of the routing problem

The goal of the first step is solving a CCLP, whose output is a tree connecting the van depot to a number of vertices (called *concentrators*, chosen among the customer vertices) from which star-like structures reach the remaining customers (Fig. 4). The cost of each van route is approximated as the cost of the simple tour that starts at the depot, goes to the concentrator and then back to the depot, plus the costs of inserting customers assigned to the concentrator into such a simple tour. In the CCLP, each customer is a potential concentrator, and the cost of selecting $j \in \mathcal{V}^c$ as a concentrator is approximated as $2c_{0j}$. Then, the approximated cost of associating customer $i \in \mathcal{V}^c$ with concentrator $j \in \mathcal{V}^c$ is determined as in the Seed Tours Heuristic (Bramel & Simchi-Levi, 1995), i.e., $c_{ij} = c_{0j} + c_{ji} + c_{i0} - c_{0j} - c_{j0} = c_{ji} + c_{i0} - c_{j0}$. Thus,

Table 1

Results for the instances with $|\mathcal{V}^c| = 50$ customers.

Instance	% cust.	SOLVER						MATH-mTSP						MATH-VRP						ALNS						
		no ADRs	z^*	Opt. gap	TIME	100 $ \mathcal{V}_g^c / \mathcal{V}^c $	N^d	N^e	z^*	DEV	TIME	TIME_1	100 $ \mathcal{V}_g^c / \mathcal{V}^c $	N^d	N^e	z^*	DEV	TIME	TIME	N^e	z^*	DEV	TIME	100 $ \mathcal{V}_g^c / \mathcal{V}^c $	N^d	N^e
50_1	8.00%		59.00	0.00%		240.14	36.00%	12	1	64.20	8.82%	0.88	0.06	52.00%	15	2	63.58	7.77%	20.38	2	59.57	0.97%	49.00	34.00%	12	1
50_2	16.00%		54.95	0.00%		2776.20	44.00%	10	1	58.47	6.40%	3.53	0.06	74.00%	15	1	58.47	6.40%	2.66	1	55.12	0.30%	297.00	48.00%	9	1
50_3	10.00%		48.99	0.00%		1123.55	40.00%	10	1	50.17	2.41%	3.99	0.07	74.00%	17	1	50.17	2.41%	2.98	1	50.25	2.57%	63.00	74.00%	16	1
50_4	10.00%		45.74	2.92%		3600.00	40.00%	9	1	45.80	0.12%	3.13	0.09	68.00%	14	1	45.80	0.12%	5.29	1	45.42	-0.70%	444.00	60.00%	13	1
50_5	16.00%		55.40	6.51%		3600.00	24.00%	8	1	59.78	7.91%	0.20	0.04	56.00%	12	3	56.06	1.19%	9.28	3	55.32	-0.14%	90.00	42.00%	11	1
50_6	16.00%		54.16	0.00%		2217.57	36.00%	9	1	55.53	2.53%	0.94	0.04	42.00%	10	2	54.69	0.98%	20.31	2	54.44	0.52%	23.00	38.00%	10	1
50_7	14.00%		53.86	0.00%		106.17	46.00%	11	1	58.81	9.18%	2.06	0.05	38.00%	9	3	55.21	2.50%	10.11	3	53.86	0.00%	1.00	46.00%	11	1
50_8	24.00%		58.17	7.44%		3600.00	32.00%	10	1	63.90	9.84%	3.74	0.03	42.00%	11	3	61.51	5.74%	179.33	3	58.67	0.85%	59.00	32.00%	10	1
50_9	12.00%		45.52	11.85%		3600.00	46.00%	12	1	47.84	5.11%	0.95	0.08	64.00%	14	2	47.59	4.56%	6.93	2	45.85	0.74%	10.00	54.00%	13	1
50_10	18.00%		46.56	16.71%		3600.00	24.00%	8	1	50.38	8.21%	1.35	0.07	62.00%	15	2	48.93	5.08%	3600.00	2	47.55	2.13%	18.00	34.00%	10	1
Average			52.23	4.54%		2,446.37	36.80%	9.90	1	55.49	6.05%	2.08	0.06	57.20%	13.20	2	54.20	3.68%	385.73	2	52.60	0.72%	105.40	46.20%	11.50	1

Table 2

Results for the instances with $|\mathcal{V}^c| = 100$ customers.

Instance	% cust.	SOLVER			MATH-mTSP							MATH-VRP							ALNS							
		no ADRs	z^*	Opt. gap	TIME	100 $ \mathcal{V}_g^c / \mathcal{V}^c $	N^d	N^e	z^*	DEV	TIME	TIME_1	100 $ \mathcal{V}_g^c / \mathcal{V}^c $	N^d	N^e	z^*	DEV	TIME	TIME	N^e	z^*	DEV	TIME	100 $ \mathcal{V}_g^c / \mathcal{V}^c $	N^d	N^e
100_1	8.00%	73.98	5.89%		3600.00	27.00%	14	2	81.40	10.02%	3.95	0.33	39.00%	18	3	80.12	8.31%	1600.52	3	78.57	6.21%	233.00	50.00%	23	1	
100_2	9.00%	69.15	6.34%		3600.00	45.00%	20	2	74.91	8.34%	188.08	0.35	53.00%	22	3	72.28	4.53%	3600.00	3	69.62	0.68%	82.00	50.00%	21	1	
100_3	11.00%	70.66	8.43%		3600.00	37.00%	16	2	73.74	4.35%	94.98	0.36	56.00%	23	2	72.59	2.73%	3600.00	2	71.47	1.14%	375.00	50.00%	20	1	
100_4	11.00%	65.81	7.02%		3600.00	39.00%	16	2	70.33	6.86%	46.22	1.08	56.00%	22	3	68.68	4.36%	3600.00	3	67.69	2.86%	498.00	50.00%	20	1	
100_5	9.00%	81.23	5.33%		3600.00	29.00%	14	2	90.64	11.58%	7.19	0.34	52.00%	24	3	86.21	6.12%	1460.71	3	82.79	1.92%	624.00	50.00%	22	1	
100_6	6.00%	76.65	4.97%		3600.00	19.00%	12	2	79.89	4.24%	3.74	0.58	45.00%	21	3	78.90	2.94%	243.18	3	85.61	11.69%	254.00	71.00%	29	1	
100_7	8.00%	79.15	2.68%		3600.00	29.00%	13	2	85.63	8.19%	45.05	0.26	39.00%	17	3	82.07	3.69%	3600.00	3	79.83	0.86%	655.00	50.00%	20	1	
100_8	13.00%	68.35	1.33%		3600.00	56.00%	25	1	72.52	6.10%	8.69	0.30	66.00%	29	2	72.05	5.42%	3600.00	2	68.94	0.86%	36.00	54.00%	23	1	
100_9	13.00%	79.25	10.34%		3600.00	18.00%	9	2	87.40	10.29%	64.94	0.40	37.00%	16	4	81.48	2.81%	141.36	4	82.29	3.83%	137.00	50.00%	21	1	
100_10	6.00%	67.15	3.48%		3600.00	50.00%	22	1	70.18	4.51%	44.93	0.78	53.00%	23	3	68.13	1.47%	3600.00	3	68.40	1.86%	3,333.00	50.00%	22	1	
Average		73.14	5.58%		3600.00	34.90%	16.10	1.80	78.66	7.45%	50.78	0.48	49.60%	21.50	2.90	76.25	4.24%	2504.58	2.90	75.52	3.19%	736.10	52.50%	22.10	1	

Table 3

Results for the instances with $|\mathcal{V}^c| = 200$ customers.

Instance	% cust.	SOLVER			MATH-mTSP							MATH-VRP				ALNS									
		no ADRs	z^*	Opt. gap	TIME	100 $ \mathcal{V}_g^c / \mathcal{V}^c $	N^d	N^e	z^*	DEV	TIME	TIME ₁	100 $ \mathcal{V}_g^c / \mathcal{V}^c $	N^d	N^e	z^*	DEV	TIME	N^e	z^*	DEV	TIME	100 $ \mathcal{V}_g^c / \mathcal{V}^c $	N^d	N^e
200_1	13.00%	123.09	24.12%		3600.00	50.50%	40	2	115.30	-6.33%	902.28	4.09	27.00%	21	5	119.06	-3.28%	3600.00	5	141.94	15.31%	3,036.00	74.50%	52	2
200_2	10.50%	125.40	21.62%		3600.00	70.00%	53	3	119.36	-4.82%	620.67	4.15	24.50%	21	6	129.09	2.94%	3600.00	6	129.09	2.94%	3,559.00	75.00%	54	1
200_3	11.00%	117.16	22.24%		3600.00	20.50%	18	4	110.29	-5.86%	1,086.27	3.19	35.00%	27	5	115.44	-1.47%	3600.00	5	116.72	-0.37%	664	50.00%	37	2
200_4	10.50%	137.40	31.64%		3600.00	32.50%	26	3	122.98	-10.49%	1,176.28	4.38	30.50%	23	6	116.91	-14.91%	3600.00	6	121.83	-11.33%	902	50.50%	37	2
200_5	11.50%	125.34	24.98%		3600.00	75.00%	54	1	115.35	-7.97%	732.32	3.04	38.00%	30	5	114.24	-8.86%	3600.00	5	129.13	3.02%	1,202.00	75.00%	53	1
200_6	12.50%	135.62	25.19%		3600.00	21.50%	20	4	133.84	-1.31%	250.97	3.90	31.50%	24	6	147.95	9.09%	3600.00	6	130.57	-3.73%	3047	50.00%	35	2
200_7	11.50%	121.60	15.19%		3600.00	51.50%	39	2	131.56	8.19%	1,428.87	3.40	24.00%	18	6	136.86	12.55%	3600.00	6	135.17	11.16%	2,366.00	50.00%	36	2
200_8	9.50%	107.18	20.56%		3600.00	50.50%	41	2	113.24	5.66%	1,799.35	3.12	16.50%	14	6	136.74	27.59%	3600.00	6	123.89	15.59%	2,144.00	74.50%	54	2
200_9	12.00%	122.97	23.28%		3600.00	51.50%	40	2	119.59	-2.74%	896.04	3.93	18.50%	14	5	158.03	28.52%	3600.00	5	135.35	10.07%	3,420.00	75.00%	54	1
200_10	11.50%	127.88	25.42%		3600.00	33.00%	26	3	128.39	0.40%	1,080.64	2.92	37.00%	30	7	118.69	-7.19%	3600.00	7	117.17	-8.37%	3,460.00	50.00%	39	2
Average		124.36	23.42%		3600.00	45.65%	35.70	2.60	120.99	-2.53%	997.37	3.61	28.25%	22.20	5.70	129.30	4.50%	3600.00	5.70	128.09	3.43%	2,380.00	62.45%	45.10	1.70

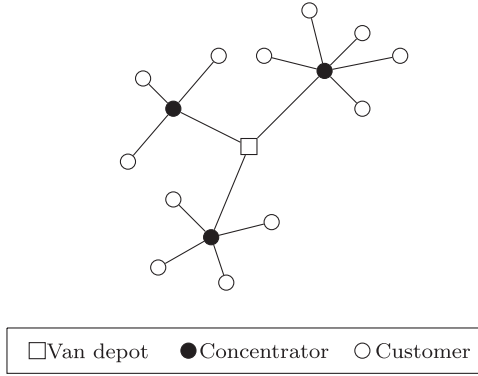


Fig. 4. Output of the CCLP model.

Source: Adapted from [Bramel and Simchi-Levi \(1995\)](#).

the approximated cost of servicing all the customers associated with concentrator $j \in \mathcal{V}^c$ is $\sum_{i \in \mathcal{V}^c, i \neq j} \bar{c}_{ij} x_{ij}$, where x_{ij} ($i, j \in \mathcal{V}^c$) are decision variables equal to 1 if customer i is linked with concentrator j , and 0 otherwise.

The other decision variables of the optimization model we solve in step 1 are:

- s_r , $r \in \mathcal{R}^d$, equal to 1 if ADR route r is selected, 0 otherwise;
- z_i , $i \in \mathcal{V}^c$, equal to 1 if customer i is served by an ADR, 0 if it is served by a van;
- y_j , $j \in \mathcal{V}^c$, equal to 1 if j is a concentrator, 0 otherwise.

Thus, the mathematical model is:

$$\text{Minimize } \sum_{r \in \mathcal{R}^d} b_r s_r + 2 \sum_{j \in \mathcal{V}^c} c_{0j} y_j + \sum_{i, j \in \mathcal{V}^c, i \neq j} \bar{c}_{ij} x_{ij} \quad (11)$$

s.t.

$$\sum_{r \in \mathcal{R}^d} a_{ir} s_r = z_i \quad \forall i \in \mathcal{V}^c \quad (12)$$

$$\sum_{j \in \mathcal{V}^c} x_{ij} = 1 - z_i \quad \forall i \in \mathcal{V}^c \quad (13)$$

$$x_{ij} \leq y_j \quad \forall i, j \in \mathcal{V}^c, i \neq j \quad (14)$$

$$x_{jj} = y_j \quad \forall j \in \mathcal{V}^c \quad (15)$$

$$\sum_{i \in \mathcal{V}^c, i \neq j} d_i x_{ij} \leq q^v \quad \forall j \in \mathcal{V}^c \quad (16)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in \mathcal{V}^c, i \neq j \quad (17)$$

$$y_j \in \{0, 1\} \quad \forall j \in \mathcal{V}^c \quad (18)$$

$$s_r \in \{0, 1\} \quad \forall r \in \mathcal{R}^d \quad (19)$$

$$z_i \in \{0, 1\} \quad \forall i \in \mathcal{V}^c \quad (20)$$

Table 4

Results for the instances with $|\mathcal{V}^c| = 300$ customers.

Instance	% cust.	SOLVER				MATH-mTSP								MATH-VRP				ALNS						
		no ADRs	z^*	Opt. gap	TIME	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d	N^c	z^*	DEV	TIME	TIME ₁	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d	N^c	z^*	DEV	TIME	N^c	z^*	DEV	TIME	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d
300_1	12.67%	268.94	53.49%	3600.00	71.67%	78	4	160.02	-40.50%	1,754.39	23.99	13.67%	16	8	372.15	38.38%	3600.00	8	183.74	-31.68%	3,592.00	66.67%	69	2
300_2	11.00%	250.41	53.51%	3600.00	74.67%	86	3	151.30	-39.58%	2,033.03	21.36	24.67%	28	7	209.14	-16.48%	3600.00	7	185.45	-25.94%	2,977.00	66.67%	70	2
300_3	10.67%	229.40	51.35%	3600.00	80.00%	90	3	159.75	-30.36%	1,061.35	17.04	11.00%	12	9	626.79	173.23%	3600.00	9	162.86	-29.00%	3,439.00	51.33%	54	3
300_4	12.33%	209.81	46.56%	3600.00	73.33%	82	3	153.23	-26.97%	1,406.86	21.05	16.67%	19	8	348.87	66.28%	3600.00	8	162.24	-22.67%	2,724.00	50.00%	53	3
300_5	8.67%	272.95	58.63%	3600.00	87.67%	97	3	149.52	-45.22%	1,013.99	24.60	14.33%	17	7	357.15	30.85%	3600.00	7	167.61	-38.59%	3,338.00	50.00%	54	3
300_6	9.33%	220.97	42.97%	3600.00	78.00%	86	2	155.03	-29.84%	1,152.32	22.88	13.00%	15	9	312.47	41.41%	3600.00	9	173.60	-21.44%	3,220.00	66.67%	70	2
300_7	8.67%	220.92	46.52%	3600.00	79.33%	90	2	143.67	-34.97%	919.64	18.62	27.00%	31	7	182.48	-17.40%	3600.00	7	171.94	-22.17%	1,772.00	66.67%	69	2
300_8	10.00%	223.68	44.10%	3600.00	74.67%	85	3	148.89	-33.44%	1,350.34	21.74	17.00%	20	7	279.34	24.88%	3600.00	7	166.11	-25.74%	2,308.00	50.00%	53	3
300_9	9.67%	207.33	39.30%	3600.00	85.67%	95	3	130.06	-37.27%	2,455.31	24.36	24.00%	28	6	202.35	-2.40%	3600.00	6	152.54	-26.42%	2,966.00	62.00%	65	3
300_10	13.00%	214.79	38.71%	3600.00	74.33%	83	2	149.84	-30.24%	2,026.74	19.83	12.33%	14	7	301.55	40.40%	3600.00	7	172.95	-19.48%	1,578.00	50.00%	54	3
Average		231.92	47.51%	3600.00	77.93%	87.20	2.80	150.13	-34.84%	1,517.40	21.55	17.37%	20.00	7.50	319.23	37.92%	3600.00	7.50	169.90	-26.31%	2,791.40	58.00%	61.10	2.60

Table 5

Results for the instances with $|\mathcal{V}^c| = 400$ customers.

Instance	% cust.	SOLVER						MATH-mTSP						MATH-VRP						ALNS					
		no ADRs	z^*	Opt. gap	TIME	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d	N^c	z^*	DEV	TIME	TIME ₁	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d	N^c	z^*	DEV	TIME	N^c	z^*	DEV	TIME	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d	N^c
400_1	10.50%	347.86	61.63%	3600.00	88.00%	130	6	171.94	-50.57%	2,180.44	107.07	13.25%	20	8	1247.52	258.63%	3600.00	8	208.64	-40.02%	3,462.00	50.00%	69	4	
400_2	10.00%	365.73	65.21%	3600.00	89.75%	140	6	174.93	-52.17%	2,517.74	73.63	15.00%	22	9	401.26	9.72%	3600.00	9	214.90	-41.24%	1,708.00	62.50%	88	3	
400_3	10.75%	339.90	61.15%	3600.00	85.50%	133	4	181.53	-46.59%	1,829.33	67.64	11.75%	18	9	442.09	30.06%	3600.00	9	228.72	-32.71%	3,568.00	75.00%	105	2	
400_4	10.75%	386.60	64.53%	3600.00	88.00%	131	7	180.51	-53.31%	2,608.01	53.12	9.25%	14	9	1298.47	235.87%	3600.00	9	218.29	-43.53%	3,376.00	57.75%	79	4	
400_5	10.25%	333.42	64.05%	3600.00	87.75%	132	5	171.31	-48.62%	2,383.24	83.35	14.75%	23	9	332.26	-0.35%	3600.00	9	206.32	-38.12%	3,486.00	50.00%	71	4	
400_6	10.00%	334.83	60.55%	3600.00	87.75%	129	4	181.26	-45.87%	2,353.57	132.24	10.00%	15	10	387.47	15.72%	3600.00	10	233.50	-30.26%	3,483.00	70.00%	97	3	
400_7	7.75%	347.64	60.99%	3600.00	90.00%	137	4	177.65	-48.90%	2,731.92	122.71	15.75%	26	8	329.86	-5.11%	3600.00	8	220.27	-36.64%	3,193.00	62.50%	87	3	
400_8	9.50%	299.78	57.08%	3600.00	80.75%	122	2	174.93	-41.65%	2,426.11	91.19	18.50%	26	7	477.65	59.33%	3600.00	7	214.75	-28.37%	3,532.00	62.25%	86	4	
400_9	9.25%	369.39	66.32%	3600.00	87.75%	134	8	178.54	-51.67%	2,117.06	140.13	19.25%	31	8	1136.72	207.73%	3600.00	8	229.76	-37.80%	2,504.00	74.75%	104	3	
400_10	11.00%	355.01	63.99%	3600.00	87.50%	134	6	171.06	-51.82%	2,167.04	158.93	10.50%	17	9	363.88	2.50%	3600.00	9	225.87	-36.38%	2,359.00	62.50%	87	3	
Average		348.02	62.55%	3600.00	87.28%	132.20	5.20	176.37	-49.12%	2,331.45	103.00	13.80%	21.20	8.60	641.72	81.41%	3600.00	8.60	220.10	-36.51%	3,067.10	62.73%	87.30	3.30	

Table 6

Results for the instances with $|\mathcal{V}^c| = 500$ customers.

Instance	% cust.	SOLVER					MATH-mTSP					MATH-VRP					ALNS							
		no ADRs	z^*	Opt. gap	TIME	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d	N^c	z^*	DEV	TIME	TIME ₁	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d	N^c	z^*	DEV	TIME	N^c	z^*	DEV	TIME	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d
500_1	10.40%	379.26	65.07%	3600.00	83.40%	153	8	191.37	-49.54%	1,834.40	115.34	12.40%	23	12	1707.05	350.11%	3422.31	12	244.39	-35.56%	3600.00	60.00%	102	4
500_2	10.60%	414.92	64.64%	3600.00	80.60%	151	9	201.01	-51.55%	3,284.76	257.10	9.60%	18	10	414.01	-0.22%	3600.00	10	278.54	-32.87%	3,212.00	60.00%	104	4
500_3	9.60%	383.09	64.17%	3600.00	85.20%	160	7	195.75	-48.90%	2,870.65	249.69	16.00%	29	11	1148.14	199.70%	3600.00	12	247.94	-35.28%	3,212.00	60.00%	104	4
500_4	9.40%	394.49	63.16%	3600.00	77.80%	143	10	200.53	-49.17%	2,479.68	198.53	14.00%	26	10	392.42	-0.53%	3600.00	10	250.20	-36.58%	2,424.00	60.00%	105	4
500_5	10.60%	412.96	64.69%	3600.00	81.00%	152	11	202.24	-51.03%	3,013.16	221.29	20.40%	37	9	433.78	5.04%	3600.00	9	264.53	-35.94%	3,139.00	70.00%	118	3
500_6	8.40%	361.33	60.02%	3600.00	83.20%	152	6	196.99	-45.48%	2,710.69	350.00	23.80%	43	9	426.73	18.10%	3600.00	9	236.22	-34.63%	3,574.00	60.00%	106	4
500_7	10.80%	407.85	63.71%	3600.00	83.00%	155	8	201.30	-50.64%	2,289.09	211.54	13.20%	25	11	426.39	4.55%	3600.00	11	268.39	-34.19%	3,578.00	60.00%	102	4
500_8	10.20%	361.84	60.90%	3600.00	78.80%	149	7	193.08	-46.64%	2,995.31	209.86	14.60%	28	10	413.58	14.30%	3600.00	10	265.28	-26.69%	3,581.00	70.00%	121	3
500_9	9.60%	398.57	63.24%	3600.00	82.00%	150	9	199.51	-49.94%	2,890.64	144.10	7.60%	15	11	1779.42	346.45%	3600.00	11	289.28	-27.42%	2,531.00	80.00%	137	2
500_10	11.20%	434.67	67.82%	3600.00	80.80%	148	12	189.27	-56.46%	3,144.64	284.96	17.40%	32	9	452.05	4.00%	3600.00	9	280.43	-35.48%	3,259.00	60.00%	102	4
Average		394.90	63.74%	3600.00	81.58%	151.30	8.70	197.11	-49.94%	2,751.30	224.24	14.90%	27.60	10.20	759.36	94.15%	3600.00	10.30	262.52	-33.46%	3,195.40	64.00%	110.10	3.60

The aim of objective function (11) is to minimize the sum of the costs of ADR routes and the approximated costs of van routes. Constraints (12) are the set-partitioning constraints, as in (2). Constraints (13) impose that, if variable z_i is equal to 0, then customer $i \in \mathcal{V}^c$ is assigned to exactly one concentrator. Constraints (14) ensure that, when a customer $i \in \mathcal{V}^c$ is assigned to a concentrator $j \in \mathcal{V}^c$ ($j \neq i$), then the corresponding y_j is equal to 1. Moreover, constraints (14) impose that if a customer $j \in \mathcal{V}^c$ is a concentrator, then it is connected to itself. Constraints (16) are capacity constraints, whereas constraints (17)–(20) enforce the domain of the variables.

4.2. Step 2: obtaining a solution to the routing problem

The output of step 1 is a set $\mathcal{V}_d^c \subseteq \mathcal{V}^c$ of customers that are designated to be serviced by ADRs (i.e., $\mathcal{V}_d^c = \{i \in \mathcal{V}^c \mid \exists r \in \mathcal{R}^d : s_r = 1 \text{ and } a_{ir} = 1\}$) and a set $\mathcal{V}_v^c = \mathcal{V}^c \setminus \mathcal{V}_d^c$ of customers to be serviced by vans.

Let $C = \{j \in \mathcal{V}_v^c \mid y_j = 1\}$ be the set of concentrators chosen at the end of step 1, and let $\mathcal{V}_v^c(j) = \{i \in \mathcal{V}_v^c \mid y_j = 1 \text{ and } x_{ij} = 1\}$ be the set of customers associated with concentrator $j \in C$.

In order to transform the solution of the CCLP into a solution of the corresponding routing problem we use two different procedures. In the first one (described in Section 4.2.1), we consider the sets $\mathcal{V}_v^c(j)$, $j \in C$, one at a time and, for each of them, solve a Traveling Salesman Problem (TSP) using the MTZ formulation. In the second approach (reported in Section 4.2.2), we consider all the customers in $\mathcal{V}_v^c$ and solve a VRP using a two-index formulation.

4.2.1. Obtaining the routing solution as multiple TSPs

For a given concentrator $j \in C$, let $\mathcal{V}' = \{0\} \cup \{j\} \cup \mathcal{V}_v^c(j)$ be the set including the van depot, the concentrator j , as well as all the

customers associated with it, and let $N = |\mathcal{V}'|$ be its cardinality. Then, for each concentrator, we solve a TSP using the MTZ formulation (21)–(27). Although it is a well-known classic formulation, we report it for the sake of completeness. We consider a complete directed graph $G' = (\mathcal{V}', \mathcal{A}')$, where the vertex set, without losing generality, is cast as $\mathcal{V}' = \{0, 1, \dots, N-1\}$, and $\mathcal{A}' = \{(i, j) : i, j \in \mathcal{V}' \text{ and } i \neq j\}$ is the set of arcs between the vertices. The decision variables are x_{ij} , $(i, j) \in \mathcal{A}'$, taking value 1 if arc (i, j) is in the tour, 0 otherwise, and u_i , $i \in \mathcal{V}'$, used to exclude subtours. The optimization model is:

$$\text{Minimize } \sum_{(i,j) \in \mathcal{A}'} c_{ij} x_{ij} \quad (21)$$

s.t.

$$\sum_{j \in \mathcal{V}' : (i,j) \in \mathcal{A}'} x_{ij} = 1 \quad \forall i \in \mathcal{V}' \quad (22)$$

$$\sum_{j \in \mathcal{V}' : (j,i) \in \mathcal{A}'} x_{ji} = 1 \quad \forall i \in \mathcal{V}' \quad (23)$$

$$u_i - u_j + N x_{ij} \leq N - 1 \quad \forall i \in \mathcal{V}', j \in \mathcal{V}' : i \neq j \text{ and } i, j > 0 \quad (24)$$

$$u_0 = 1 \quad (25)$$

$$2 \leq u_i \leq N \quad \forall i \in \mathcal{V}' : i > 0 \quad (26)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in \mathcal{A}'. \quad (27)$$

As usual, objective function (21) aims to minimize the total routing cost. Then, constraints (22) and (23) impose that each vertex is visited once. Constraints (24) are used to avoid subtours, and finally constraints (25)–(27) define the feasible values for the variables.

4.2.2. Obtaining the routing solution as a VRP

Table 7Results of the sensitivity analysis with $L^d = 16$ km and varying capacity.

Instance	% cust.	$q^d = 1$						$q^d = 3$					
	no ADRs	z^*	TIME	TIME_1	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d	N^v	z^*	TIME	TIME_1	100 $ \mathcal{V}_d^c / \mathcal{V}^c $	N^d	N^v
500_1	10.40%	198.15	1995.17	24.39	3.40%	17	13	191.37	1834.40	115.34	12.40%	23	12
500_2	10.60%	209.00	2914.68	29.36	1.60%	8	12	201.01	3284.76	257.10	9.60%	18	10
500_3	9.60%	199.29	2540.09	38.74	0.80%	4	13	195.75	2870.65	249.69	16.00%	29	11
500_4	9.40%	212.14	2164.94	33.02	3.80%	19	12	200.53	2479.68	198.53	14.00%	26	10
500_5	10.60%	209.39	2972.91	24.83	3.40%	17	11	202.24	3013.16	221.29	20.40%	37	9
500_6	8.40%	203.80	2650.13	35.67	1.00%	5	12	196.99	2710.69	350.00	23.80%	43	9
500_7	10.80%	210.07	2328.62	31.52	3.40%	17	12	201.30	2289.09	211.54	13.20%	25	11
500_8	10.20%	200.69	2918.05	29.40	2.00%	10	12	193.08	2995.31	209.86	14.60%	28	10
500_9	9.60%	199.88	2772.35	28.03	2.40%	12	11	199.51	2890.64	144.10	7.60%	15	11
500_10	11.20%	196.40	2720.67	34.26	4.00%	20	11	189.27	3144.64	284.96	17.40%	32	9
Average		203.88	2597.76	30.92	2.58%	12.90	11.90	197.11	2751.30	224.24	14.90%	27.60	10.20

Table 8Results of the sensitivity analysis with $L^d = 8$ km and varying capacity.

Instance	% cust.	$q^d = 1$						$q^d = 3$					
	no ADRs	z^*	TIME	TIME_1	$100 \frac{ \mathcal{V}_d^c }{ \mathcal{V}^c }$	N^d	N^v	z^*	TIME	TIME_1	$100 \frac{ \mathcal{V}_d^c }{ \mathcal{V}^c }$	N^d	N^v
500_1	26.80%	199.14	2223.65	44.13	3.40%	17	13	194.64	2062.61	43.47	12.20%	23	12
500_2	32.60%	207.91	3283.52	33.37	1.20%	6	12	201.71	3058.43	73.06	9.20%	18	10
500_3	26.60%	199.07	2926.97	55.63	0.60%	3	13	194.31	2772.73	149.15	11.20%	21	12
500_4	29.80%	212.14	2188.47	41.24	3.80%	19	12	201.37	1986.72	59.87	14.20%	26	10
500_5	26.40%	210.19	2916.69	29.14	3.40%	17	11	205.94	2894.33	71.85	13.60%	25	10
500_6	26.20%	203.71	2415.01	43.69	1.00%	5	12	199.72	2716.37	141.87	22.20%	39	9
500_7	31.20%	208.80	2182.17	36.65	3.40%	17	12	203.20	2423.59	71.47	8.40%	17	12
500_8	33.20%	201.10	2778.46	40.29	2.00%	10	12	198.59	2275.07	75.97	7.60%	14	11
500_9	31.20%	198.36	2611.41	34.09	2.40%	12	11	203.09	2748.02	56.88	7.40%	15	11
500_10	33.60%	197.55	2723.62	41.50	4.00%	20	11	191.37	3322.35	72.22	17.40%	33	9
Average		203.80	2624.99	39.97	2.52%	12.60	11.90	199.39	2626.02	81.58	12.34%	23.10	10.60

we report the two-index formulation for completeness. In this case, we consider a complete directed graph $\mathcal{G}'' = (\mathcal{V}'', \mathcal{A}'')$, where $\mathcal{V}'' = \{0\} \cup \mathcal{V}_v^c$ is the vertex set and $\mathcal{A}'' = \{(i, j) : i, j \in \mathcal{V}'', i \neq j\}$ is the arc set.

The decision variables are x_{ij} , $(i, j) \in \mathcal{A}''$, taking value 1 if arc (i, j) is activated, 0 otherwise, and u_i , $i \in \mathcal{V}''$, indicating the total demand already delivered by a vehicle when visiting vertex i . Then, our mathematical formulation, in which for each $i \in \mathcal{V}''$ the meaning of $\delta^-(i)$ and $\delta^+(i)$ is as in Section 3.2, is:

$$\text{Minimize } \sum_{(i,j) \in \mathcal{A}''} c_{ij} x_{ij} \quad (28)$$

s.t.

$$\sum_{i \in \delta^-(j)} x_{ij} = 1 \quad \forall j \in \mathcal{V}'' \quad (29)$$

$$\sum_{j \in \delta^+(i)} x_{ij} = 1 \quad \forall i \in \mathcal{V}'' \quad (30)$$

$$\sum_{i \in \delta^+(0)} x_{0i} \leq |C| \quad (31)$$

$$u_i - u_j + q^v x_{ij} \leq q^v - d_j \quad \forall (i, j) \in \mathcal{A}'' \quad (32)$$

$$d_i \leq u_i \leq q^v \quad \forall i \in \mathcal{V}'' \quad (33)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in \mathcal{V}'' \quad (34)$$

Objective function (28) minimizes the routing cost. Constraints (29) and (30) impose that each vertex has a single predecessor and a single successor. Constraint (31) states that at most $|C|$ routes can be built. Finally, constraints (32) are used to exclude subtours, and constraints (33) and (34) enforce the domain of the variables.

5. Computational results

In this section, we illustrate and discuss the computational experiments we have performed to assess our matheuristic. All the algorithms

have been coded in Java, while the mathematical models have been coded in OPL and solved using IBM ILOG CPLEX 22.1.0. Moreover, the experiments have been run on a Linux machine with an Intel Core i7 processor clocked at 3 GHz and equipped with 16 GB of RAM. In the following, we first describe the test instances and then we report the details of the benchmark solution approaches. Finally, we present and discuss the results of our tests as well as a sensitivity analysis on some parameters, and derive some additional insights about potential cost and environmental benefits of using the proposed delivery infrastructure.

5.1. Instance generation

To evaluate our approach, we have generated instances resembling the distribution of drugs to pharmacies in the urban area of Rome (Italy), as in De Maio et al. (2024). The delivery points are uniformly sampled from a set of real locations of 751 pharmacies, while the potential depots for ADRs are randomly chosen among the real locations of 75 stops of the Rome underground system. The vans are stationed at a central depot situated roughly at the center of the service area. Using this data, we have randomly generated six different datasets, with 10 instances each, characterized by $|\mathcal{V}^p| = 20$, and $|\mathcal{V}^c| \in \{50, 100, 200, 300, 400, 500\}$. Moreover, we have considered: unit customer demand, delivery robot capacity $q^d = 3$, van capacity $q^v = 50$, and maximum distance covered by an ADR with a full battery $L^d = 16$ km. Finally, regarding costs, the coefficient λ to be used when converting distances into costs is 0.20 €/km for ADRs and 0.31 €/km for vans (details about how such values are obtained are provided in Section 5.5).

5.2. Benchmark approaches

To assess the quality of the solutions generated by our approach, we compare it with: (i) the solution obtained by solving model (1)–(10)

Table 9

Costs and emissions for the different types of vehicles.

Costs (€/km)			Costs (€/vehicle)			Emissions (kg of CO ₂ /km)		
DV	EV	ADR	DV	EV	ADR	DV	EV	ADR
0.156	0.065	0.007	230	250	25	0.200	0.010	0.001

Table 10

Number of vehicles needed and total traveled distance (in km) for the different types of vehicles.

Instance size	Traditional vehicles		ADRs and traditional vehicles			
	N^v	TD^v	N^v	TD^v	N^d	TD^d
50	4.20	213.65	2.00	120.51	13.20	90.64
100	5.20	287.02	2.90	171.77	21.50	127.07
200	7.80	423.22	5.70	304.81	22.20	132.50
300	9.20	500.48	7.50	403.70	20.00	124.91
400	10.70	587.56	8.60	488.85	21.20	124.11
500	12.00	656.98	10.10	537.23	38.00	231.52
Average	8.18	444.82	6.13	337.81	22.68	138.46

with a given time limit, and (ii) the solution obtained by a classical ALNS procedure.

Regarding the latter, we have implemented a straightforward ALNS framework (Pisinger & Ropke, 2010) in which a cheapest-insertion approach (Rosenkrantz et al., 1977) is used to determine an initial solution, which is then improved by a sequence of destroy and repair steps. In particular, as destroy operators we consider: *random routes removal*, that randomly selects h routes to be removed from the current solution ($h = 4$, in our tests); *close customers removal*, in which, after randomly choosing a seed customer, the k requests spatially closest to it are selected, and then all the routes containing such customers are removed from the current solution ($k = 15$, in our tests). On the other hand, we use a single repair operator, consisting of reinserting into the solution all the customers identified during the destroy phase, using the same insertion mechanism employed when constructing the initial solution. As stopping criteria, we consider a maximum number of iterations without improving the current solution (150, in our tests) or a given time limit.

5.3. Analysis of the results

In this section, we analyze the results obtained by: formulation (1)–(10) solved by an off-the-shelf solver (SOLVER, in the following); our matheuristic with the step 2 solved as in Section 4.2.1 (MATH-mTSP, in the following); our matheuristic with the step 2 solved as in Section 4.2.2 (MATH-VRP, in the following); the ALNS of Section 5.2. For each approach we impose a time limit of one hour, with at most 350 s for step 1 of MATH-mTSP and MAT-VRP. The quality of the solution obtained by the various heuristics is assessed by computing, for each instance, the percentage deviation of its objective function value compared to that of SOLVER. The results of this comparison are reported in Tables 1 to 6. Here, the column headings are: “% cust. no ADRs”, percentage of customers that cannot be visited by ADRs, because their distance from the closest PTN station and back exceeds the threshold L^d ; “ z^* ”, best objective function value found by the considered approach, within the given time limit (we report in bold the best value among all the considered approaches); “Opt. gap”, optimality gap as reported by the ILP solver at the end of the time limit, when solving model (1)–(10); “TIME”, total amount of time (in seconds) taken by the considered approach; “ $100|V_d^c|/|V^c|$ ”, percentage of customers serviced by ADRs in the considered solution; “ N^d ”, number of ADR routes in the considered solution; “ N^v ”, number of van routes in the considered solution; “TIME_1”, amount of time (in seconds) taken by Step 1 for the two matheuristics; “DEV”, percentage

deviation of the heuristic objective function value compared to that of SOLVER, obtained as $DEV = 100(z_H^* - z_{SOLVER}^*)/z_{SOLVER}^*$, where z_H^* is the best objective function value found by the considered heuristic. As far as DEV is concerned, a negative value means that the considered heuristic provides better results than SOLVER. Moreover, we observe that columns “ $100|V_d^c|/|V^c|$ ”, “ N^d ”, and “TIME_1” are not reported for MATH-VRP, because such values are the results of Step 1 of the matheuristic, which is shared by both variants.

As expected, as the instance size grows, SOLVER is not able to achieve satisfactory results within the given time limit. Indeed, for five out of 10 instances with 50 customers (Table 1), it is able to find the optimal solution. For three of the remaining five instances, the solution provided by SOLVER is still the best among the four approaches, whereas for the remaining two (instances 50.4 and 50.5) the best solution is obtained by ALNS, although the difference with SOLVER is negligible. In general, for 50 customers the solutions of SOLVER and ALNS are very close, with an average deviation of ALNS with respect to SOLVER of 0.72%. Concerning the two variants of the matheuristic, the average objective function deviation is about 6% for MATH-mTSP, and about 4% for MATH-VRP. When considering instances with 100 customers (Table 2), SOLVER continues to outperform the other approaches, obtaining the best solution for all instances, although none of them is the optimal solution. Again, the results provided by ALNS are not very far from those of SOLVER, with an average percentage deviation of about 3%. The behavior of MATH-mTSP and MATH-VRP is similar to the previous case, with an average deviation of about 7% for the former and about 4% for the latter. The situation begins to change when the instance size increases to 200 (Table 3). In fact, in this case there is not an approach clearly outperforming the others, with MATH-mTSP slightly better. In particular, SOLVER, ALNS, and MATH-VRP obtain the best solution for two instances each, while MATH-mTSP for the remaining four instances. In general, the performance of SOLVER degrades, with an average optimality gap of about 23%. On the other hand, ALNS and MATH-VRP perform worse than SOLVER, attaining an average deviation of 3.43% and 4.50%, respectively, whereas the average deviation of MATH-mTSP is -2.53% . This trend becomes much more consolidated for instances with more than 200 customers. Indeed, for the cases with 300, 400, and 500 customers (Tables 4, 5, and 6, respectively) MATH-mTSP always finds the best solution, suggesting that the approach is very valuable for large instances, representing the most frequent case in the real world. The results provided by SOLVER become worse as the size of the instances increases, with an average optimality gap of about 48%, 63%, and 64% for 300, 400, and 500 customers, respectively. The average deviations are negative for both MATH-mTSP and ALNS, with values ranging from about -50% to about -35% for the former, and from about -36% to about -26% for the latter. On the other hand, the performance of MATH-VRP is poor, with positive average percentage deviation between about 38% and about 95%. In general, the good performance on larger instances of MATH-mTSP compared to MTH-VRP can be justified because solving a number of small TSPs is much easier than solving a relatively large VRP.

Regarding the structure of the solutions, all in all, our matheuristic approach tends to serve by ADRs a lower percentage of customers, especially for larger instances, with respect to the other approaches. As a consequence, the number of delivery robot routes is lower compared to those of SOLVER and of ALNS.

Finally, concerning computing times, MATH-mTSP is faster than all the other approaches for all instance sizes, taking less than the time limit for every instance. Moreover, the first step of the matheuristic turns out to be quite “easy”, needing at most about 224 s, on average, for the largest instances with 500 customers.

5.4. Sensitivity analysis

Here, we investigate how the performance of our approach can vary according to some of the characteristics of the ADRs. To this end,

Table 11
Daily cost (in €) and emission (in kg of CO₂) analysis for the different types of fleet configurations.

Instance size	DV		EV		ADR-DV		ADR-EV	
	Costs	Emissions	Costs	Emissions	Costs	Emissions	Costs	Emissions
50	999.29	42.73	1063.78	2.14	809.45	24.19	838.44	1.30
100	1240.72	57.40	1318.51	2.87	1232.20	34.48	1274.52	1.84
200	1859.94	84.64	1977.30	4.23	1914.47	61.09	2000.64	3.18
300	2193.97	100.10	2332.28	5.00	2288.82	80.86	2401.96	4.16
400	2552.54	117.51	2712.90	5.88	2585.08	97.89	2712.45	5.01
500	2862.36	131.40	3042.38	6.57	3358.42	107.68	3511.37	5.60
Average	1951.47	88.96	2074.52	4.45	2031.41	67.70	2123.23	3.52

we study the sensitivity of the solutions to changes in some of the technological parameters of the delivery robots, namely the capacity q^d and the maximum distance covered with a full battery L^d . In particular, we run MATH-mTSP on the instances with 500 customers by considering a more restrictive setting for q^d and L^d , i.e., $q^d = 1$ and $L^d = 8$ km.

The additional computations are reported in Tables 7 and 8, whose captions are the same as those in the previous tables. More specifically, Table 7 refers to the case in which $L^d = 16$ km, as in Section 5.3. In this case, we do not observe a significant variation of the average objective function value when the capacity is reduced to $q^d = 1$, achieving a value of 203.88, compared to 197.11 with $q^d = 3$ (an increase of about 3%). When we reduce the autonomy of the ADRs to $L^d = 8$ km (Table 8), the performance of the algorithm do not present significant differences for the two values of capacity. In particular, the average objective function values obtained with $q^d = 1$ and $q^d = 3$ are 203.80 and 199.39, respectively (an increase of about 2%). If we cross-compare the results of Tables 7 and 8 for a given value of capacity, we observe that the differences are even smaller. Indeed, with $q^d = 1$ the performance of the algorithm are basically independent from L^d , achieving average values of 203.88 and 203.80 when the autonomy is 16 km and 8 km, respectively. On the other hand, if we consider $q^d = 3$ the difference among the results obtained with $L^d = 16$ km and $L^d = 8$ km is again negligible, with average objective function values of 197.11 and 199.39, respectively. In general, the results of these additional experiments show that our solution is quite robust to changes of the technological parameters of the ADRs.

5.5. Costs and emissions analysis

In this section, we derive some insights related to the potential cost and emission reduction benefits of using ADRs combined with traditional delivery vehicles, compared to delivery by means of traditional vehicles only, being either diesel-powered or electric-powered vans. For this purpose, we run the MATH-mTSP approach in which customer requests must be serviced by: (i) a fleet of ADRs in combination with a fleet of either traditional diesel-fueled vehicles (ADR-DV in the following), or electric-powered vehicles (ADR-EV, in the following), according to the framework considered in this paper; (ii) a fleet of traditional vehicles, either diesel-fueled (DV, in the following), or electric-powered (EV, in the following). In the two latter cases, we neglect the presence of ADRs.

The costs and emissions for the different types of fleet are reported in Table 9. Similarly to De Maio et al. (2024), we consider the Ford Transit 310 L3 H2 2.0L ECOBLUE (Ford Corporate, 2024) for DVs, the Volkswagen E-crafter (Volkswagen, 2024) for EVs, and the delivery robot by Starship Technologies (Starship Technologies, 2024). Concerning the costs of vehicles, they are obtained as the sum of a vehicle and driver cost per day and a fuel/energy cost per km. The former is computed by dividing the vehicle buying cost (equal to 30,000 €, 50,000 €, and 5000 € for DVs, EVs and ADRs, respectively) by the number of days in a life cycle (we consider for all vehicles a life cycle of 200 days per years for 5 years). The staff cost for DVs and EVs is

obtained dividing the driver's annual wage, fixed to 40,000€, by the days in a life cycle, while for ADRs the price is also divided by 10, that is the number of ADRs we assume that a supervisor can control. Then, the fuel/energy cost per km is obtained as follows. For DVs, the fuel cost per km is obtained by considering a cost of 1.90 €/l of diesel fuel and an average consumption of 8.2 l/100 km (mixed driving). For EVs, the energy cost per km is computed by multiplying the energy cost (assumed to be 0.30 €/kWh) and the energy consumption (0.215 kWh/km). The computation is the same for ADRs, but the energy consumption is 0.0247 kWh/km. With respect to the emissions, we have considered the same values reported in De Maio et al. (2024).

The results of our analysis are shown in Tables 10 and 11, in which each row contains the average value over the 10 instances of the same size. In particular, Table 10 reports the number of traditional vehicles and ADRs needed to service all the customers (N^v and N^d , respectively), as well as the total distance traveled by such types of vehicles (TD^v and TD^d , respectively). These results are then used in Table 11 to derive some insights with respect to daily costs and emissions for the different types of vehicles.

The results show that, using ADRs combined with traditional vehicles, the cost remains about the same, whereas the emissions can be highly reduced, compared to a traditional delivery scheme. In particular, the average value of the daily emissions is about 89 kg of CO₂ when using DVs only and could be reduced to about 68 kg of CO₂ partnering them with ADRs (a reduction of about 24%). The emission value could even drop to 3.53 kg of CO₂ if the fleet was composed by EVs and ADRs. In this case, the percentage reduction, compared to DVs, would be of about 96%.

6. Conclusions

In this paper, we have considered a problem in which a heterogeneous fleet of autonomous delivery robots and traditional vehicles is employed in a last-mile distribution setting. To face this problem, we have designed a matheuristic based on the solution of a hybrid set-partitioning and location-based model. In this approach, the ADR and van routes are determined, given their peculiarities, by solving a hybrid set-partitioning and location-based model. In the second step, the solution of the concentrator problem is transformed into a solution to the routing problem, using two alternative strategies. The first one is based on solving a number of TSPs, whereas the second one relies on the solution of a classical VRP. The matheuristic has been compared with both a mathematical formulation for the whole problem and a standard ALNS on a set of instances, resembling a real-world drug distribution problem, with up to 500 requests. The experiments have shown that the proposed approach can provide consistent solution improvements, in particular for the larger instances. Moreover, through a further analysis, we have shown that combining ADRs with traditional vehicles allows us to achieve huge benefits with respect to CO₂ emissions, compared to traditional delivery fleets.

CRedit authorship contribution statement

Gianpaolo Ghiani: Writing – original draft, Validation, Supervision, Methodology, Investigation, Funding acquisition, Data curation, Conceptualization. **Emanuela Guerriero:** Writing – original draft, Validation, Supervision, Methodology, Investigation, Funding acquisition, Data curation, Conceptualization. **Emanuele Manni:** Writing – original draft, Validation, Supervision, Methodology, Investigation, Funding acquisition, Data curation, Conceptualization. **Deborah Pareo:** Writing – original draft, Validation, Methodology, Investigation, Data curation, Conceptualization.

Data availability

Data will be made available on request.

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