Subitizing but not estimation of numerosity requires attentional resources

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The numerosity of small numbers of objects, up to about four, can be rapidly appraised without error, a phenomenon known as *subitizing*. Larger numbers can either be *counted*, accurately but slowly, or *estimated*, rapidly but with errors. There has been some debate as to whether subitizing uses the same or different mechanisms than those of higher numerical ranges and whether it requires attentional resources. We measure subjects' accuracy and precision in making rapid judgments of numerosity for target numbers spanning the subitizing and estimation ranges while manipulating the attentional load, both with a spatial dual task and the "attentional blink" dual-task paradigm. The results of both attentional manipulations were similar. In the high-load attentional condition, Weber fractions were similar in the subitizing (2–4) and estimation (5–7) ranges (10–15%). In the low-load and single-task condition, Weber fractions substantially improved in the subitizing range, becoming nearly error-free, while the estimation range was relatively unaffected. The results show that the mechanisms operating over the subitizing and estimation ranges are not identical. We suggest that pre-attentive estimation mechanisms works at all ranges, but in the subitizing range, attentive mechanisms also come into play.

Keywords: attention, spatial cognition, middle vision

Introduction

Jevons (1871) was the first to note that when making rapid estimates of the number of black beans tossed into a dish, he never made errors for bean numbers up to four, but did for larger numbers, with the standard deviation of the estimates increasing in direct proportion to the physical number of beans (Weber's law). The ability to enumerate quickly and effortlessly numbers up to four has been coined "subitizing" (Kaufman, Lord, Reese, & Volkmann, 1949), from the Latin *subitus* meaning sudden (the root of the common Italian adverb *subito*).

There has been a long-standing debate as to whether enumerating numbers in the subitizing range invokes different processes than for larger ranges of objects. For accurate denomination, or "counting," there is good evidence for the dichotomy: for items up to four, reaction times are quite constant, increasing by at most 40–100 ms per item; for larger numbers the cost of additional items is

250–350, leading to clear changes in curve slope (Atkinson, Campbell, & Francis, 1976; Mandler & Shebo, 1982). Evidence for the dichotomy has also been provided by a PET study (Sathian et al., 1999), but this was not replicated by a more recent, better controlled, functional magnetic resonance imaging study (Piazza, Mechelli, Butterworth, & Price, 2002). Some behavioral studies have also questioned the existence of two distinct processes. For example, Balakrishnan and Ashby (1992) found no evidence of a sharp discontinuity in reaction times between the subitizing and counting ranges: the "mental effort" for enumeration increases with each additional element in the display, both within and beyond the putative subitizing range, with no suggestion of two distinct processes.

Even when subjects do not have the time or opportunity to count the number of objects in the field of view, they can *estimate* numerosity rapidly. Approximate *estimation* of number has been demonstrated in humans (Whalen, Gallistel, & Gelman, 1999), in infants (Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005), in cultural groups with

no word for numbers much above two (Dehaene, Izard, Spelke, & Pica, 2008; Gordon, 2004), in monkeys using a habituation–discrimination paradigm with auditory stimuli (Hauser, Tsao, Garcia, & Spelke, 2003; Sawamura, Shima, & Tanji, 2002), in other mammals (Gallistel, 1990), in birds (Pepperberg, 2006), and even in bees (Dacke & Srinivasan, 2008). After appropriate training, parrots can make a visual number estimation up to six items, and bees up to four. Both are able to generalize this to novel objects. Most recently, number discrimination has been demonstrated in newborns, with a cross-modal matching technique (Izard, Sann, Spelke, & Streri, 2009).

The ability to estimate number correlates strongly with mathematics achievement (Halberda, Mazzocco, & Feigenson, 2008; Piazza et al., in press), suggesting it is strongly linked to other number-based capacities. Recently, it has been shown that the estimation process is strongly subject to *adaptation* (Burr & Ross, 2008) leading the authors to suggest that it is a primary visual sense. Furthermore, there is clear evidence that numerosity estimation is distinct from perception of texture density (Franconeri, Bemis, & Alvarez, 2009; He, Zhang, Zhou, & Chen, 2009; Ross & Burr, 2010).

Estimation of numerosity is rapid and effortless but not errorless. As Jevons (1871) first showed, error increases in direct proportion to the number of items to be estimated, a property known as *Weber's law*. The *Weber fraction*, defined as the just noticeable difference or precision threshold divided by the mean, is usually found to be quite constant over a large range of base numerosities. For example, in a recent study, using rigorous two-alternative forced choice techniques, Ross (2003) reported Weber fractions for adult subjects to be about 0.25 over a wide range of base values (8–60). The value of 0.25—1 in 4—lead Ross to suggest that the precision for estimation may explain the subitizing limit: the quantal leap from the limit 4 to the nearest neighbor is 1, corresponding to the Weber fraction precision limit. Thus, subitizing may be nothing special, merely a consequence of the resolution of estimation mechanisms and the quantal separation at low numbers. Similar ideas have been advanced by Dehaene and Changeux (1993) and Gallistel and Gelman (1992).

Although this idea is appealing, it has not received experimental support. Revkin, Piazza, Izard, Cohen, and Dehaene (2008) explicitly tested the idea by measuring estimation precision for numbers ranging from 1 to 8 (grain of 1) and 10 to 80 (grain of 10). If the same mechanism determined precision over the entire range, Weber fractions for the 1–8 range should be like those of the 10–80 range: but they were not, they were three times lower.

Subitizing tends to be resistant to attempts to disrupt it, and this has lead many to assume that subitizing is *pre-attentive*, or at least makes use of pre-attentive information (Trick & Pylyshyn, 1994). However, a few recent studies suggest that subitizing is in fact vulnerable to manipulations of attentive load. About 200 ms after performing an attentive task, attentive mechanisms are at a low ebb, a phenomenon referred to as the "attentional blink" (Raymond, Shapiro, & Arnell, 1992). During this period, subitizing is highly compromised (Egeth, Leonard, & Palomares, 2008; Juan, Walsh, & McLeod, 2000; Olivers & Watson, 2008; Xu & Liu, 2008). Other studies have shown that during dual tasks, when spatial attention is diverted from the estimation task, subitizing suffers (Railo, Koivisto, Revonsuo, & Hannula, 2008; Vetter, Butterworth, & Bahrami, 2008).

In this study, we take advantage of the fact that manipulations of attention in both space and time can affect subitizing and examine whether it has the same effect on estimation at larger number ranges. The results show that both spatial and temporal attention affects number estimation for low but not high numbers. Furthermore, under conditions of high attentional load, the precision in the subitizing range is reduced to be similar to the estimation range. This suggests that pre-attentive estimation mechanisms can operate over both high and low number ranges: but small numbers, within the subitizing range, can call on an additional attentive mechanism that operates—when attentional resources permit—over a range of up to four items.

Methods

The stimuli were presented in a dimly lit room on a 15-inch Macintosh monitor with 1440×900 resolution at a refresh rate of 60 Hz and mean luminance of 60 cd/m^2. Subjects viewed the stimuli binocularly at a distance of 57 cm from the screen. Stimuli were generated and presented under Matlab 7.6 using PsychToolbox routines (Brainard, 1997).

Attentional blink

Three subjects (2 males, 1 female: mean age 25) with normal or corrected-to-normal vision participated in this study. The technique was to present a stream of 12 white letters in rapid serial visual presentation (RSVP), followed by a cloud of dots, then a random-noise mask. The letters were chosen randomly from the set "A B C D E F G H M N O P", presented on a gray background (see Figure 1). Each letter was presented within a ($5° \times 5°$) matrix for 83 ms (5 frames) with a 33-ms (2-frame) blank gap between consecutive letters. The first target was one of these letters, chosen randomly in each trial and presented in a yellow instead of white, in a position selected to create a specific lag between it and the next target, the dot pattern to be estimated. At the end of the stream, a cloud of dots (T2), varying in number from one to eight, was presented for 130 ms (8 frames) followed immediately by a binary pixel
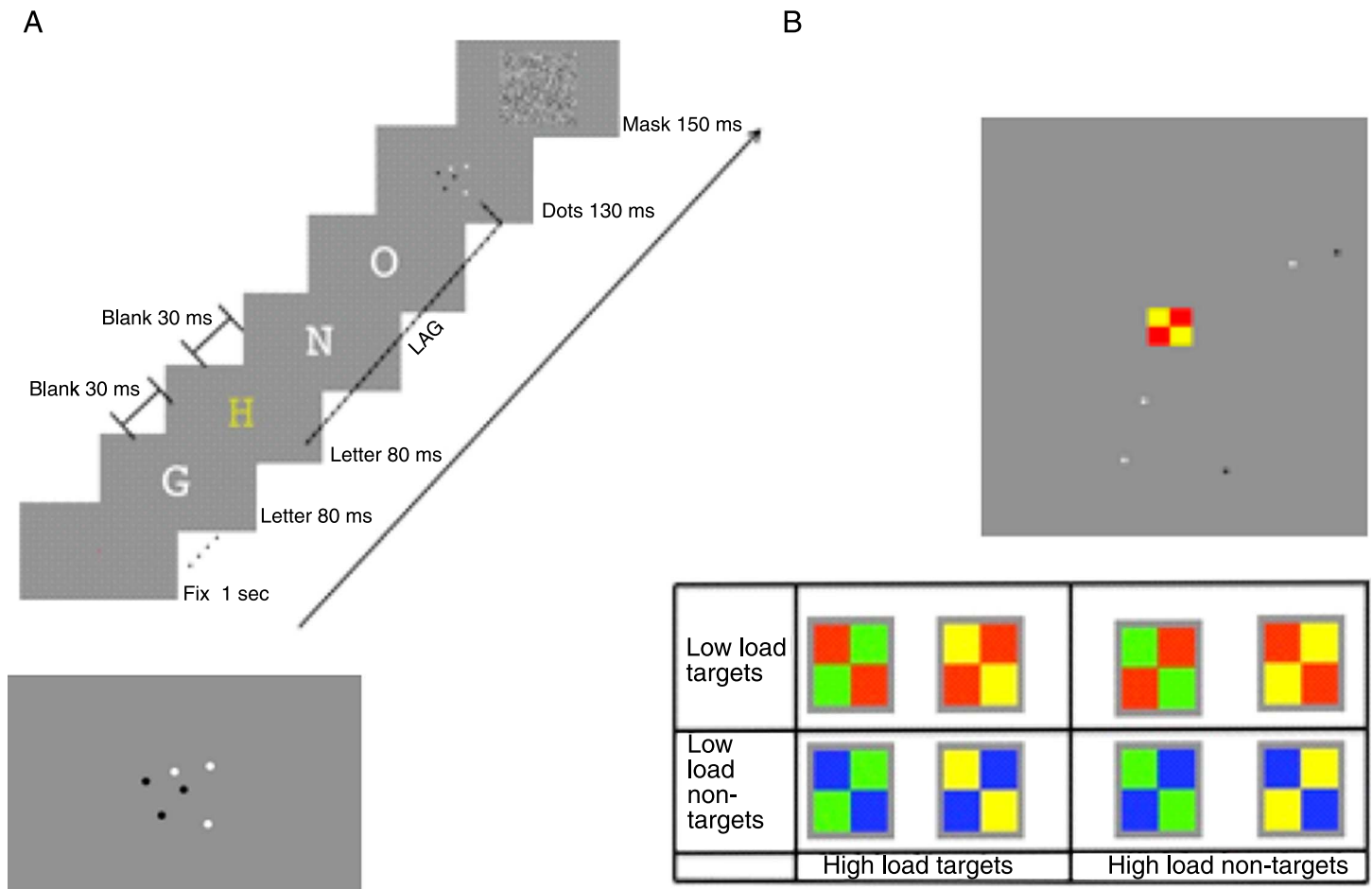
A

B

Figure 1. (A) Attentional blink paradigm. The letter stimuli were presented in RSVP. Each trial began with a fixation point presented for 1 s, followed by an RSVP stream (12 letters), each letter displayed for 80 ms with a 30-ms blank gap. The dot array was displayed for 130 ms and from 110 ms to 880 ms after the target letter (yellow) and was followed by a binary pixel noise mask (150 ms). (B) Spatial attention paradigm. The presentation began with a fixation point for 1 s. Stimuli (dots and colored square) were presented simultaneously for 200 ms, followed by a binary pixel mask (200 ms).

noise mask of 600×600 pixels, randomly black or white, presented for 150 ms. Dots in the target were half-white and half-black so luminance was not a cue to number. Each dot was $0.4°$ in diameter, with position chosen at random within a matrix of $14°$ diameter (Figure 1A).

The task of the subjects was to report first the target letter, then estimate the number of dots that appeared, by mouse-clicking two virtual keyboards that appeared after each trial, the first contained all possible letters, the second the range of numbers from 1 to 8. The important variable was the time lag between the yellow letter and dot stimulus, set at random to be 110, 220, 330, or 880 ms. In separate sessions, subjects were either instructed to ignore the letters and respond only to the number (single-task control); or to respond to both, as mentioned above (experimental attentional blink condition). The response to the number task was considered only if that to the first task was correct (about 90% of trials, constant across lag). In total, we measured 8 levels of numerosity, 4 lags and two response conditions ($8 \times 4 \times 2 = 64$ conditions in all). A total of

2764 trials were run for the control condition (number only) and 3496 for the experimental condition (number plus letter). When plotting the results, the extremes of the range (1 and 8) were discarded, as the subjects were aware of the range, and therefore tended to make fewer errors in the extremes.

Experiment II: Spatial attention

Four subjects (mean age: 24, 1 female, 3 males, different from those of Experiment I) with normal or corrected-to-normal vision participated. The experiment employed a dual-task paradigm (Figure 1B). The stimulus for the primary task was made up of 4 centrally positioned colored squares, each subtending $3°$ of visual angle. The squares could take up eight color combinations, which determined whether the stimulus was a target or not. In the low attentional load condition, the stimulus was a *target* if it contained red squares, irrespective of the spatial arrangement
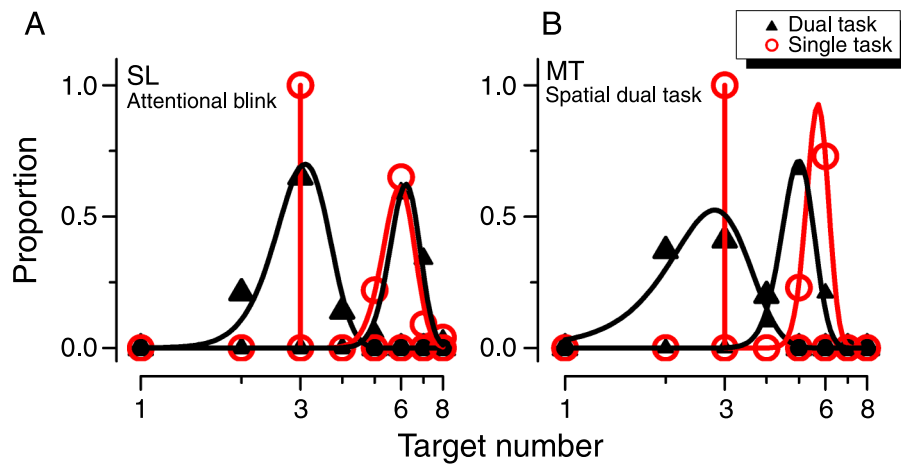
Figure 2. Example of mean probability distribution in the two ranges of numerosity for the two experimental conditions (single task and dual task) in the (A) attentional blink and (B) spatial attention paradigms; $N = 3$ (subitizing) and $N = 6$ (estimation). The increase of the attentional load lead to a considerable increase in the response variability in the subitizing range, while that in the estimation range remained virtually unchanged.

of colors. Under high attentional load conditions, the stimulus was a target if a specific conjunction of color and spatial arrangement was satisfied: two green squares along the right diagonal *or* two yellow squares along the left diagonal. In the no-load condition, the primary stimuli appeared, but subjects could ignore it. The stimulus for the secondary task was a cloud of dots (like those of the other experiment), displayed in random position within an eccentric annulus of 6° inner diameter and 18° of diameter, displayed simultaneously with the primary stimulus. Subjects were required to estimate number of dots in the cloud (which could vary from 1 to 8).

On each trial, the fixation point was presented for 1 s, then the primary and secondary stimuli for 200 ms, followed by the binary pixel noise mask (600 × 600 pixels). Subjects responded with mouse press on a virtual keypad, first to the primary then to the secondary task. Responses to the secondary task were recorded only if those to the primary task were correct. In total, there were three attentional load conditions and 8 numerosities, resulting in 24 conditions per subject. Forty trials were run for each condition, yielding a total of 4000 trials for 4 subjects (Figure 1B).

Data analysis

Data were analyzed separately for each subject. For each subject, the responses were pooled for each condition and numerosity, from which two parameters were estimated: the mean and standard deviation. The standard deviation is the main parameter, providing an estimate of response precision, which, normalized by the number of items in that condition, provides an estimate of the *Weber fraction*, the

standard parameter of precision performance that is often independent of magnitude. The mean estimates systematic biases in judgments, or accuracy, plotted in Figure 6.

Results

Attentional blink

As detailed above, the "attentional blink" is a double-task paradigm where subjects first identify the odd-colored letter in an RSVP stream, then estimate the number of dots in a cloud. Examples of number estimation are shown in Figure 2A, for numerosities 3 and 6, under control conditions (when the letter was presented but ignored: red symbols), and during the peak of the attentional blink (lag 220 ms: black symbols). The distributions of the estimates were well described by a Gaussian, from which the Weber fraction is readily calculated from the standard deviation of the fit. When subjects were not required to perform the dual task, the curves for 3 and 6 were quite different: for numerosity 3 there were no errors (hence a delta function), while for numerosity 6 there were many errors resulting in a distribution with standard deviation of 0.66 (Weber fraction of 0.11).

Spatial dual task

Here subjects performed a double-task paradigm, but for stimuli simultaneously presented. While estimating numerosity of the dot cloud, subjects also performed a central
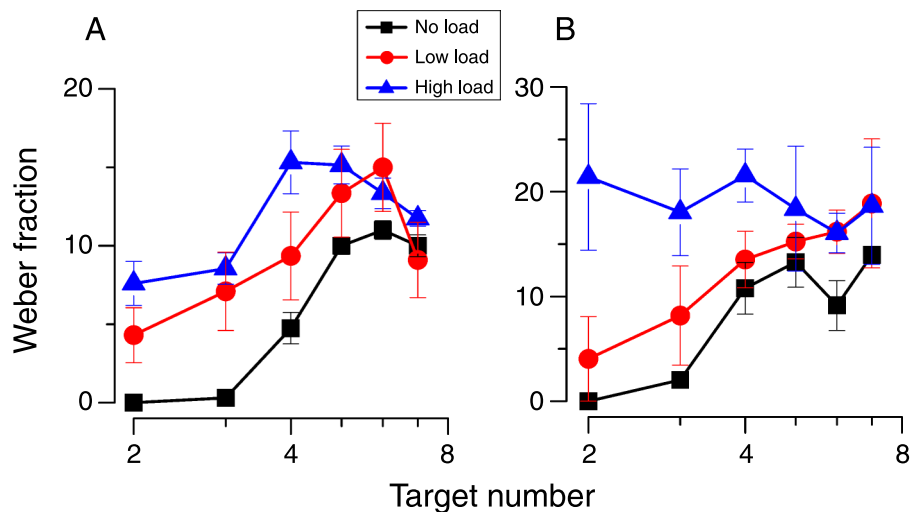
Figure 3. Plot of mean Weber fraction against target number for various attentional loads (see legend) against target number for the (A) attentional blink paradigm and (B) spatially divided attention. For attentional blink paradigm, "No Load" refers to the average of 880-ms lag and all conditions without the double-task; "Low Load" is 110-ms lag; and "High Load" is the average of 330- and 220-ms lags. For both spatial and temporal paradigms, the curves become much flatter at high attentional load.

task, reporting either the presence of a red square (low load), or a conjunction of color and orientation (high load). Figure 2B shows sample distributions of number estimation for target numbers 3 and 6 in high-load and single-task (no-load) conditions. The distributions are very similar to those of Figure 2A. Estimation of three dots was error-free with the single task, while at 6 the estimates formed a clear Gaussian distribution, whose standard deviation yielded a Weber fraction of 0.9. Under high attentional load, this distribution changed little, while that for 3 elements became as broad as that for 6.

Effect of attention on subitizing and estimation

Figure 3 brings the effect of attention out more clearly, plotting Weber fractions (obtained from the standard deviation of the Gaussian fits) against numerosity, for various levels of attentional loads. The results are similar for both paradigms: in the high number range all conditions lead to a similar estimate of Weber fraction around 15%; in the subitizing range, however, the results clearly depend on attentional load, with perfect (or near-perfect) performance at 2 and 3 in the no-load conditions of both experiments.

Figure 4 plots the results another way, separately for the six different numerosities: for the attentional blink experiment, they are plotted as a function of the lag between the two stimuli, for the spatial attention experiment as a function of task complexity. The effect of attentional load is clearly different for different numerosities. At the higher numerosities (5–7), the curves were fairly flat, sitting around 15%, independent of load. However, performance at low numerosities (2–3) clearly depended on task load, reaching near-perfect performance at low and no-load conditions. Performance at 4 was somewhat in between, sitting with the higher number range in the spatial task and lower range in the temporal task. Note that in the subitizing range (<4) the curves in Figure 3A follow the classical attentional blink result, peaking around 200–300 ms, returning to baseline for separations of 900 ms.

Figure 5 shows individual results for the three subjects in the attentional blink experiment (Figure 5A) and four in the spatial dual task (Figure 5B), plotting Weber fractions in the high-load conditions against those in the low-load conditions, separately for the subitizing (2–4) and estimation (5–7) ranges. For the attentional blink, the high load is the average of 220- and 330-ms lag and no load is the average of single task at all lags. The results were very similar for all subjects and both tasks: performance in the higher estimation range was largely independent of load, while in the subitizing range it was strongly dependent on load. The ordinates of all points were very similar for high load, but at low load form two non-overlapping clusters, with near perfect for the subitizing range.

Perceived numerosity and error rate

As mentioned in the Methods section, the numerosity estimation trend is well described by a Gaussian probability distribution, defined by two numbers: the standard deviation, the estimate of response precision, that leads to the Weber fraction when normalized by the target number; and the mean, which estimates the response *accuracy* (a bias away from veridical behavior). Figures 6A and 6C plot the
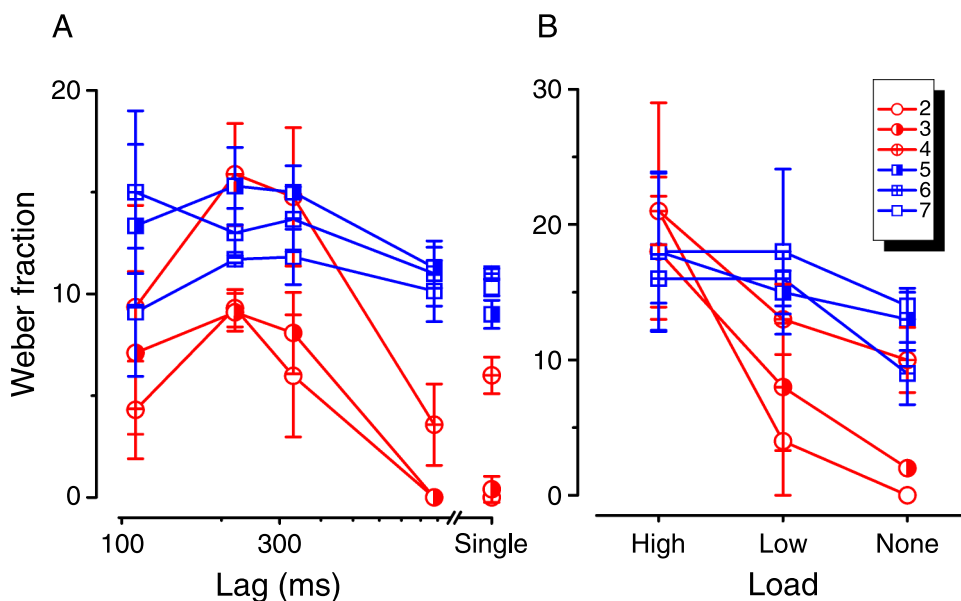
Figure 4. Weber fraction against attentional load, separately for all numbers, in the (A) attentional blink and (B) spatial attention conditions. Large number (5–7) are largely unaffected while small numbers (2–4) show a large effect of attention.

perceived numerosity obtained from the means of the Gaussian fit, averaged over subjects for the two attentional conditions. In general, the perceived numerosity was quite accurate (little bias), tending to follow the actual target number (dashed diagonal). The only systematic deviation from veridicality was in the high-load spatial dual-task condition, where there tended to be an underestimation at the higher numbers.

Finally, Figures 6B and 6D plot "error rate" as a function of target number, to help relate the present results to previous reports, that often express results as error rate. There are two problems with this approach; one is that it

confuses bias and precision, as both lead to errors, but are quite different attributes; the other is that the magnitude of the error is lost. When expressed in this way, the effect of attentional load appears to be larger for higher numerosities, but this is in fact quite misleading.

Discussion

Using two complementary techniques, this study shows that subitizing depends strongly on attentional resources,
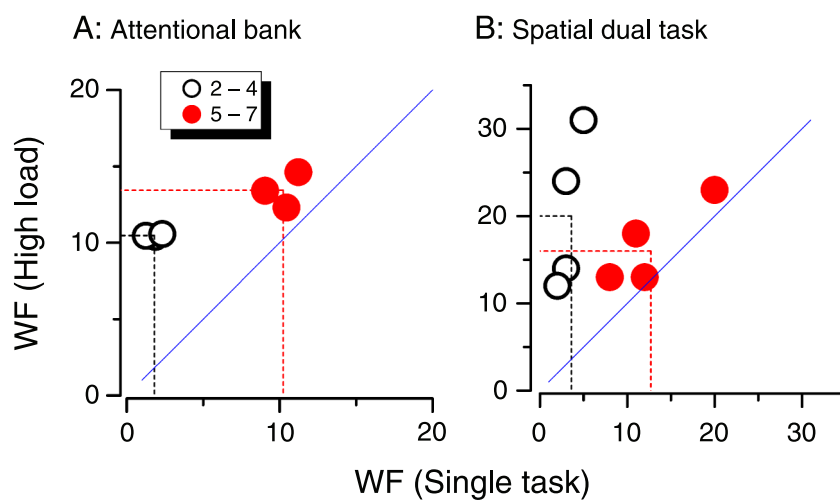

Figure 5. Scatter plot of Weber fractions in high-load against low-load conditions for all subjects in both (A) attentional blink and (B) spatial dual-task conditions, separately for low numbers (2–4) and high numbers (5–7). For the attentional blink, high load was the average of 220 and 330 ms, single task was the average of all delays with no primary task. For all subjects, attentional load affected Weber fraction much more in the subitizing than estimation range.
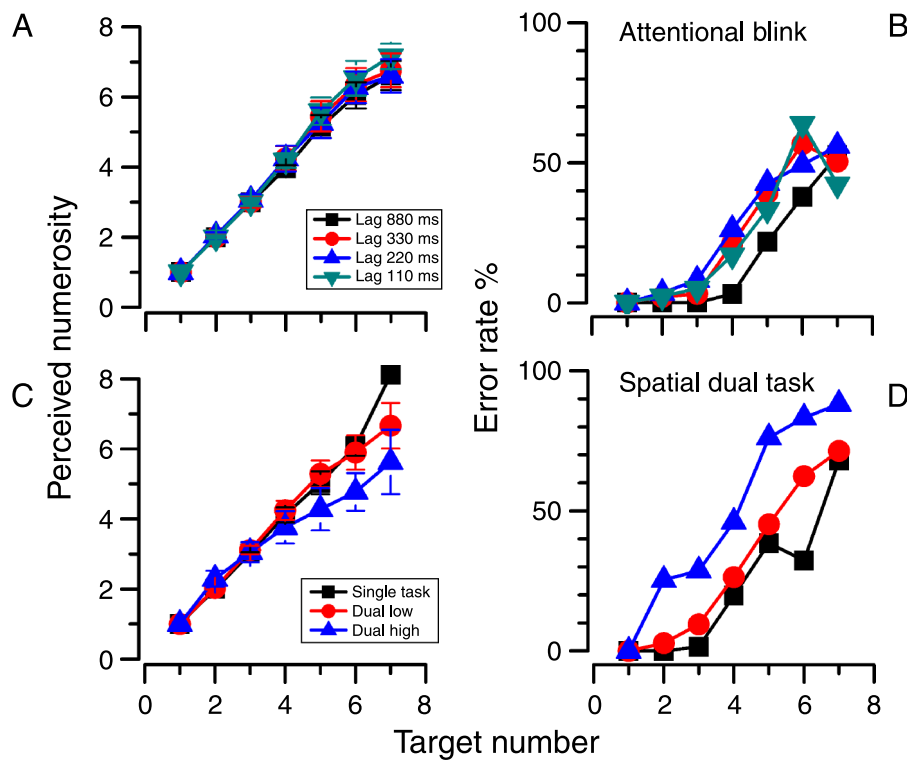
Figure 6. Plot of (A, C) perceived numerosity and (B, D) error rate in the (A, B) attentional blink paradigm and (C, D) dual-task spatial attention. Attentional load had little effect on perceived numerosity. Error rate, as always, is difficult to interpret because it contains errors both in accuracy and precision and does not weight for the amplitude of the error.

while estimation of larger quantities depends far less on attentional load. Under conditions of high attentional demand, both during the attentional blink (200–300 ms after recognition of a target letter) or during an attentionally demanding simultaneous task (detection of a color-orientation conjunction), performance in estimating the number of dots in a cloud remained remarkably constant, around 15% for target numbers ranging from 2 to 8.

It would be difficult to account for these results within the framework of a single mechanism covering the whole range. If this mechanism were attention-dependent at low numbers, it should also be attention-dependent at high numbers. It appears far more plausible that two mechanisms are at work. One possibility is that "density estimation" comes into play in the higher number range. Although we did not control specifically for this possibility, as our previous study (Ross & Burr, 2010) showed that for adult humans density and numerosity activate different processes, it seems likely that the two mechanisms revealed by this study are both involved in number judgment, not density. However, these mechanisms need not be completely separate. A parsimonious explanation could be that estimation mechanisms operate over the entire range, with similar normalized resolution capacity (Weber fraction), but at low numerosities these mechanisms are supplemented by attentional mechanisms, mechanisms that identify and enumerate

very precisely, but have a very low capacity, around four items. A capacity of four items would be consistent, for example, with the capacity to track moving stimuli (Pylyshyn & Storm, 1988), which is heavily dependent on visual attentive mechanisms (Arrighi & Burr, 2009).

This explanation also finds support from recent fMRI studies of neural correlates of visual enumeration under different attentional loads. Ansari, Lyons, van Eimeren, and Xu (2007) have shown that the temporal-parietal junction (rTPJ), an area thought to be involved in stimulus-driven attention (Corbetta & Shulman, 2002), is activated during a comparison task of quantities, but only for small numbers of items, up to 3 or 4. More recently Vetter, Butterworth, and Bahrami (2010) showed that this area responds to small numbers only in conditions of low attentional load. All these studies suggest that this area could be the neural substrate for the attention-assisted boost in performance of estimation in the subitizing range.

Our current results suggest that when this attention-based system is unavailable because of competing demands, the estimation system still functions, providing numerosity estimates for small numbers, but with greatly reduced precision. That the estimation range also spans small numbers is consistent with the single unit physiology (Nieder, Freedman, & Miller, 2002) and behavioral data (Nieder & Miller, 2004) of macaque monkeys, and also

fMRI studies that suggest that the same mechanisms are active for small and large numerosities (Piazza et al., 2002).

Subitizing is often considered to be a pre-attentive process (Trick & Pylyshyn, 1993), or at least to have access to pre-attentive processes (Trick & Pylyshyn, 1994), while enumeration of larger numbers is considered to require attention. This study shows, at least as far as estimation (rather than counting) is concerned, that this distinction is not true. Subitizing was heavily dependent on attentive resources, as previous studies have shown (Egeth et al., 2008; Juan et al., 2000; Olivers & Watson, 2008; Railo et al., 2008; Vetter et al., 2008; Xu & Liu, 2008), while thresholds for numbers outside the subitizing range were completely unaffected by attentional manipulations. Interestingly, the limit of the subitizing range is very similar to that of other attention-related phenomena, such as transfer of information across saccades (Melcher, 2009).

It is interesting that both spatial and temporal manipulations of attention produced similar results, suggesting that the dependence of subitizing on attention is general, not specific to a particular type. It would be interesting to examine the effect of dual attentional tasks in other modalities, such as sound, on visual subitizing (and *vice versa*), as previous studies have shown that vision and audition tap separate attentional resources (Alais, Morrone, & Burr, 2006).

Broadly speaking, our results fit well with other studies of the effects of attention on enumeration. For example, Vetter et al. (2008) showed, with a paradigm very similar to our spatial dual task, that attentional load affected enumeration. They claimed that attention affected equally the subitizing and estimation ranges. However, inspection of their data (their Figure 3D) suggests that although statistically significant, the effects of attentional load were far less in the estimation than the subitizing range. In our hands, the effect of attention in the estimation range (5–7) was very small, and not statistically significant (for both paradigms $p > 0.05$), while the effect of attentional load is strong in subitizing range and is statistically significant (for both paradigms $p < 0.0002$). They also agree in principle with studies showing that the attentional blink and attentional spatial task affects subitizing (Egeth et al., 2008; Juan et al., 2000; Olivers & Watson, 2008; Xu & Liu, 2008). However, it is difficult to see in those studies whether the effect also occurs in the estimation range, as they report error rate rather than precision, that does not estimate performance well.

In our experiments, attentional load caused very little bias in perceived numerosity: *precision* was impaired in dual-task conditions in the subitizing range, but there was very little effect on average perceived numerosity (*accuracy*). Only in the high-load spatial dual task was there a systematic under estimation of numerosity, and there only in the estimation range (where Weber fractions were unaffected by attentional load).

Most studies on numerosity tend to concentrate on two measures, reaction times and percent errors. As the dual task makes reaction times difficult in our paradigm, we concentrated on error rate. However, it is important to distinguish the two forms of error, accuracy and precision. The precision tells us how reliably subjects can make enumeration judgments. Systematic biases or inaccuracies are not related to precision but could reflect other processes. For example, after adapting to fields with large numbers of items, subjects underestimate numerosity, but do so reliably. Therefore looking only at error rate is very uninformative about underlying processes. Another problem with error rate is that the magnitude of the error is lost. For example, confusing 2 with 3 is a 50% error, whereas 10 with 11 is only 10%: yet when scoring error rate, both are scored equally, which leads to an overestimation of the imprecision in the larger range, which can be quite misleading. So while our results agree qualitatively with many previous studies looking at the effect of attention on enumeration, the important difference between the subitizing and estimation ranges is lost in many of those studies.

Two main conclusions can be drawn from the present study: that subitizing and estimation are not identical processes, as they are differently affected by attentional load; and that subitizing, described by many as a pre-attentional process, relies heavily on attentional mechanisms (while estimation mechanisms do not). A parsimonious explanation of the current data would be that estimation processes work over all numerosity ranges, and this is broadly consistent with the animal neurophysiology (Nieder et al., 2002) and human imagining studies (Piazza et al., 2002). However, in the low number range, additional attention-based processes exist, and these have a very limited capacity, around four items. Our results are also in agreement with the recent evidence that the capacity of trans-saccadic perception, measured as the transfer of adaptation aftereffects across gaze shift, is around four items in single-task condition, instead with the addition of visual working memory or counting task this capacity decrease to only one item (Melcher, 2009). When attention is diverted on a demanding task, estimation mechanisms still operate, with lower precision.

References

Alais, D., Morrone, C., & Burr, D. (2006). Separate attentional resources for vision and audition. *Proceedings of the Royal Society of London B, 273,* 1339–1345.

Ansari, D., Lyons, I. M., van Eimeren, L., & Xu, F. (2007). Linking visual attention and number processing in the brain: The role of the temporo-parietal junction in small and large symbolic and nonsymbolic number comparison. *Journal of Cognitive Neuroscience, 19,* 1845–1853.

Arrighi, R., & Burr, D. (2009). Vision and audition do not share attentional resources in sustained tasks [Abstract]. *Journal of Vision, 9*(8):206, 206a, http://www.journalofvision.org/content/9/8/206, doi:10.1167/9.8.206.

Atkinson, J., Campbell, F. W., & Francis, M. R. (1976). The magic number 4 ± 0: A new look at visual numerosity judgements. *Perception, 5,* 327–334.

Balakrishnan, J. D., & Ashby, F. G. (1992). Subitizing: Magical numbers or mere superstition? *Psychological Research, 54,* 80–90.

Brainard, D. H. (1997). The psychophysics toolbox. *Spatial Vision, 10,* 433–436.

Burr, D., & Ross, J. (2008). A visual sense of number. *Current Biology, 18,* 425–428.

Corbetta, M., & Shulman, G. L. (2002). Control of goal-directed and stimulus-driven attention in the brain. *Nature Reviews Neuroscience, 3,* 201–215.

Dacke, M., & Srinivasan, M. V. (2008). Evidence for counting in insects. *Animal Cognitive, 11,* 683–689.

Dehaene, S., & Changeux, X. P. (1993). Development of elementary numerical abilities: A neuronal model. *Journal of Cognitive Neuroscience, 5,* 390–407.

Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science, 320,* 1217–1220.

Egeth, H. E., Leonard, C. J., & Palomares, M. (2008). The role of attention in subitizing: Is the magical number 1? *Visual Cognition, 16,* 463–473.

Franconeri, S. L., Bemis, D. K., & Alvarez, G. A. (2009). Number estimation relies on a set of segmented objects. *Cognition, 113,* 1–13.

Gallistel, C. R. (1990). *The organization of learning* (pp. viii, 648 p.). Cambridge, MA; London: MIT Press.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44,* 43–74.

Gordon, P. (2004). Numerical cognition without words: Evidence from Amazonia. *Science, 306,* 496–499.

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature, 455,* 665–668.

Hauser, M. D., Tsao, F., Garcia, P., & Spelke, E. S. (2003). Evolutionary foundations of number: Spontaneous representation of numerical magnitudes by cotton-top tamarins. *Proceedings of Biological Science, 270,* 1441–1446.

He, L., Zhang, J., Zhou, T., & Chen, L. (2009). Connectedness affects dot numerosity judgment: Implications for configural processing. *Psychonomic Bulletin & Review, 16,* 509–517.

Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. *Proceedings of the National Academy of Sciences of the United States of America, 106,* 10382–10385.

Jevons, W. S. (1871). The power of numerical discrimination. *Nature, 3,* 363–372.

Juan, C.-H., Walsh, V., & McLeod, P. (2000). Preattentive vision and enumeration. *Investigative Ophthalmology & Visual Research, 41,* S39.

Kaufman, E. L., Lord, M. W., Reese, T, and Volkmann, J. (1949). The discrimination of visual number. *American Journal of Psychology, 62,* 496–525.

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General, 111,* 1–22.

Melcher, D. (2009). A shared sensorimotor map for visual memory, counting and trans-saccadic perception [Abstract]. *Journal of Vision, 9*(8):184, 184a, http://www.journalofvision.org/content/9/8/184, doi:10.1167/9.8.184.

Nieder, A., Freedman, D. J., & Miller, E. K. (2002). Representation of the quantity of visual items in the primate prefrontal cortex. *Science, 297,* 1708–1711.

Nieder, A., & Miller, E. K. (2004). Analog numerical representations in rhesus monkeys: Evidence for parallel processing. *Journal of Cognitive Neuroscience, 16,* 889–901.

Olivers, C., & Watson, D. (2008). Subitizing requires attention. *Visual Cognition, 16,* 439–462.

Pepperberg, I. M. (2006). Grey parrot numerical competence: A review. *Animal Cognitive, 9,* 377–391.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., et al. (in press). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition.*

Piazza, M., Mechelli, A., Butterworth, B., & Price, C. J. (2002). Are subitizing and counting implemented as separate or functionally overlapping processes? *Neuroimage, 15,* 435–446.

Burr, Turi, & Anobile

Pylyshyn, Z. W., & Storm, R. W. (1988). Tracking multiple independent targets: Evidence for a parallel tracking mechanism. *Spatial Vision, 3,* 179–197.

Railo, H., Koivisto, M., Revonsuo, A., & Hannula, M. M. (2008). The role of attention in subitizing. *Cognition, 107,* 82–104.

Raymond, J. E., Shapiro, K. L., & Arnell, K. M. (1992). Temporary suppression of visual processing in an RSVP task: An attentional blink? *Journal of Experimental Psychology: Human Perception and Performance, 18,* 849–860.

Revkin, S. K., Piazza, M., Izard, V., Cohen, L., & Dehaene, S. (2008). Does subitizing reflect numerical estimation? *Psychological Science, 19,* 607–614.

Ross, J. (2003). Visual discrimination of number without counting. *Perception, 32,* 867–870.

Ross, J., & Burr, D. C. (2010). Vision senses number directly. *Journal of Vision, 10*(2):10, 1–8, http://www.journalofvision.org/content/10/2/10, doi:10.1167/10.2.10. [PubMed] [Article]

Sathian, K., Simon, T. J., Peterson, S., Patel, G. A., Hoffman, J. M., & Grafton, S. T. (1999). Neural evidence linking visual object enumeration and attention. *Journal of Cognitive Neuroscience, 11,* 36–51.

Sawamura, H., Shima, K., & Tanji, J. (2002). Numerical representation for action in the parietal cortex of the monkey. *Nature, 415,* 918–922.

Trick, L. M., & Pylyshyn, Z. W. (1993). What enumeration studies can show us about spatial attention: Evidence for limited capacity preattentive processing. *Journal of Experimental Psychology: Human Perception and Performance, 19,* 331–351.

Trick, L. M., & Pylyshyn, Z. W. (1994). Why are small and large numbers enumerated differently? A limited-capacity preattentive stage in vision. *Psychological Review, 101,* 80–102.

Vetter, P., Butterworth, B., & Bahrami, B. (2008). Modulating attentional load affects numerosity estimation: Evidence against a pre-attentive subitizing mechanism. *PLoS ONE, 3,* e3269.

Vetter, P., Butterworth, B., & Bahrami, B. (2010). A candidate for the attentional bottleneck: Set-size specific modulation of the right TPJ during attentive enumeration. *Journal of Cognitive Neuroscience,* 1–9.

Whalen, J., Gallistel, C. R., & Gelman, R. (1999). Nonverbal counting in humans: The psychophysics of number representation. *Psychological Science, 10,* 130–137.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74,* B1–B11.

Xu, F., Spelke, E. S., & Goddard, S. (2005). Number sense in human infants. *Developmental Science, 8,* 88–101.

Xu, X., & Liu, C. (2008). Can subitizing survive the attentional blink? An ERP study. *Neuroscience Letters, 440,* 140–144.