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When Veblen meets Krugman: social network and city dynamics.

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Abstract

The present paper explores the role of the social structure of suburban areas on city dynamics. We focus on relative concerns in the form of conspicuous consumption and introduce them into a standard economic geography model a la Krugman. We show that the level of social integration within the suburban areas of cities and the level of economic integration across cities are crucial in determining the city sizes.

An interesting case arises with moderate trade costs when relatively small shares of income are devoted to the consumption of the differentiated good: if classes of workers are segregated (as in homogenous suburban areas), relative concerns tend to generate disperse, medium size, cities; when workers of different classes socially interact, relative concerns contribute to foster socially integrated megalopolises. This result shows that keeping-up-with-the-Joneses motives may generate counterintuitive results when agents are able to chose their location.

Keywords: agglomeration, conspicuous consumption, city dynamics, migration, network effects, economic geography.

J.E.L. Classification: F12, F15, F20, F22, R23

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1 Introduction

The understanding of the forces shaping economic geography has mainly focused on the production of goods and services, on their exchange via markets and on the flow of inputs, particularly labour. Clearly, location decisions of agents depend on the perceived well being, and a recurrent theme here is that, in addition to own consumption and leisure, well being appears to depend on the consumption of others with whom agents interact and compare themselves. The aim of the present paper is to shed some light on the role played by this type of social interaction on city dynamics and migration decisions, and the interplay between agglomeration and dispersion forces, eventually responsible of the spacial imbalances. In doing so the paper is a contribution to what Duranton (2008) considers as one of the major challenges for spatial economics: the development of a 'theory of proximity' that explains why direct interactions between economic agents matter and how.

In constructing a theory of city dynamics, both the factors determining migration and the effects of migration on these variables have to be considered. Migration flows have traditionally been explained by models that focus on the role of economic differences, such as that of income levels, between countries of origin and of destination.[1] As surveyed by Radu (2008), there is now compelling evidence that migration decisions are influenced "by the actual or intentional migration choices in one's peer group (endogenous effects) or by the group's specific characteristics (contextual effects)". In particular, agents evaluate their income not only in absolute terms but also in comparison with others and this affects their migration decisions substantially, as shown by Stark and Taylor (1991) and by a large subsequent literature.[2] The other side of the coin, is that migration affects these variables. Firstly, it directly affects the composition of the peer group and its characteristics. Secondly, the flow of workers affects the size and composition of the labour force, then affecting the wage and incomes of the agents. Departing from most of the existing body of research on migration, we integrate the production and the demand side of the economy and look at the general equilibrium effects of social comparisons, and are thus able to address the full role of social interaction on city dynamics. Secondly, our work explores the role of the finer city organization, particularly the level of segregation among its citizen.

Main assumptions and their empirical relevance

The main structure of the model follows the "footloose entrepreneur" model proposed by Forslid and Ottaviano (2003).[3] Two goods are produced and ex-

[1] See Borjas (1994) for a discussion on several aspects of the economics of immigration.

[2] For example, Quinn (2006) shows "relative deprivation" is a significative factor in domestic migration decisions among Mexicans.

[3] We choose the model by Forslid and Ottaviano (2003) because of its analytical tractability and because it turns out to be isomorphic to the core-periphery model by Krugman (1991).

changed, an agricultural good and a modern manufactured good, which is composed by N different varieties. The agricultural good is produced by immobile unskilled workers. Each of the N of varieties of the manufactured good is produced by a monopolistically competitive firm with increasing returns to scale, employing both unskilled and skilled workers. As in the footloose model, skilled workers are interregionally mobile and can be thought as self-employed entrepreneurs. Finally, the transport of goods across cities might be costly. The main innovation of the paper is the introduction of neighborhood effects affecting the demand of the modern good. Indeed, the model considers economies in which cities, or regions, are composed of suburban areas in which agents socially interact via relative concerns that are channelled by consumption of the modern good.

The assumptions embedded in the supply side of the model described above are standard in the economic geography literature and have been extensively discussed.[4]Regarding the demand side, we adopt the description of the social structure developed in Ghiglino and Goyal (2010), modified to consider the case in which one of the goods is composed by a continuum of differentiated varieties. This framework takes into account the influential empirical literature[5] showing that relative concerns are important within "neighbours", where the neighbourhood generating the reference group, are friends, family, colleagues, etc...[6]

More specifically, the model makes three important assumptions regarding the relative concerns: 1) agents care about the consumption of a manufactured complex good by the agents in the "neighborhood", or reference group; 2) this effect is increasing in the size of the reference group, rather than solely on the average; 3) the size of the reference group increases with the size of the city. Although there is no unambiguous direct evidence that these properties hold, we believe there is strong indirect evidence in support of these assumptions.

The first assumption is directly related to the existence of conspicuous consumption, which is a well established fact. A significant recent paper is Kuhn et al. (2011) that analyses the effect of lottery gains. As expected, the winners typically increase their consumption in superior goods. The effect of an unexpected lottery gain of a value of eight month average household income (18500€) is primarily on own consumption of durable consumable goods, as cars. More important the study shows an effect on the consumption pattern of neighbours of the winning household. This effect mainly concerns cars, see Table 6 in Kuhn

[4]See Fujita, Krugman and Venables (1999), Fujita and Thisse (2002) and Baldwin, Forslid, Martin, Ottaviano and Robert-Nicoud (2003) for a literature review on new economic geography models.

[5]There is also a vast theoretical literature on relative concerns. Among the many, see e.g., Abel (1990), Frey and Stutzer (2002), Hopkins and Kornienko (2004), Arrow and Dasgupta (2007) and Ghiglino and Goyal (2010).

[6]See, for example, Blanchflower and Oswald (2004), Layard (2005), Luttmer (2005) and Kuhn, Kooreman, Soetevent, Kapten (2011).

et al. (2011), and confirms the intuition that the good generating conspicuous consumption has to be visible and repeatedly seen. The effect on the probability that the immediate neighbour buys a car within 6 month is above 7%, equivalent to an estimated own effects of winning €10,000.

In another important paper, Luttmer (2005) shows that the well being of agents depends on the well being of the reference group, mainly represented by neighbours, friends and relatives. Interestingly, the effect depends on the number of encounters with these "neighbours" (see Table 5 in Luttmer (2005)). In particular, specifications 3a-3d in Table 5 show that the effect of neighbours is stronger for those who socialise more frequently with neighbours, but this is not true for friends, family or colleagues. In the case of neighbours, the coefficient β of the effect of neighbours income on own happiness in the regression, doubles when going from those who have infrequent social contacts (less than once a month) to those with frequent social contacts (at least once a month), that is, from $\beta = 0.161$ to $\beta = 0.335$. Finally, note that specification 2 in Table 5 confirms the assumption made in the present paper that positive and negative deviations have symmetric effects. Although this study concerns the well being, we may assume that a similar pattern exists for conspicuous consumption, so that the second assumption holds.

Note that both the evidence from Kuhn et al. (2011) and from Luttmer (2005) indicate that contacts with direct neighbours are important. On the other hand, in our model, the reference group is the city, which by the way fits well with the notion of agglomeration in New Economic Geography. It might be argued that because of this, the empirical evidence on neighbourhoods is not helpful to asses the relevance of social interaction across the city. As discussed in Section 4, the main results are scale invariant so that this is not an issue.

Finally, several recent studies show that the number of social ties increases with the size of a city. Schläpfer et al. (2014) (see also Bettencourt (2013)) study mobile phone data for cities in Portugal and UK. The study finds an increase in the level of social interaction with city size. In fact it finds that the total number of mobile phone communications increases superlinearly with the city size. More precisely, for every doubling of a city's population the average number of reciprocated mobile phone contacts per person grows approximately by 12% to 25%, that is, $< degree > \propto N^{\beta-1}$ with $\beta - 1 = [0.12, 0.25]$. This means that during the observation period (over a year) an average urban habitant in Lisbon (500 000 h.) accumulated more than twice as many reciprocated contacts as an average resident of Lixa (4000 h.), see fig. 1c in Schläpfer et al. (2014). As mentioned by the authors, mobile phone communications provide a good proxy for person-to-person interaction.[7] To conclude, although the result is not direct evidence that conspicuous consumption increases with city size, it constitutes a strong indirect support of the third assumption.

[7] See Saramaki et al. (2014), Eagle et al. (2009) and Wesolowski et al. (2013).

The new forces produced by conspicuous consumption.

The primary intuition is that introducing conspicuous consumption into a standard new economic geography model produces a *new centrifugal force* for mobile agents/workers because the comparison to other's consumption exerts a direct negative externality that pushes them toward the less densely populated region. In other words, the larger the population of a region is, the greater the desire for conspicuous consumption of its inhabitants is: this gives rise to a situation in which individuals "run" after each other in trying to increase their consumptions with respect to others, making some agents worse off and creating a new centrifugal force for those agents who are mobile. In the paper we show that the presence of the conspicuous good effect also adds a *new centripetal force* to the ones at work in the original model, and, as result, it may contradict the intuition.[8] The reason is that when some workers move in a region, the demand for each firm in this region increases not only because of the presence of these workers, but also because all other neighboring workers increase their demand according to the conspicuous good effect. This larger increase in the demand per firm strengthens the "market size" effect in the original model producing a larger increase in operating profits of firms located in the larger region, which can finally translate into higher real wages attracting more workers in the region.

Which of these effects prevails depends on the trade costs between the two regions, because this determines the degree of localization of the effects of relative concerns. Specifically, while the strength of the new centrifugal force power is independent of the trade costs, the strength of the new centripetal force increases with these. The reason is that, when trade costs are high, markets are more segmented and the conspicuous good effect that increase the demand for the differentiated good is stronger in the larger market, tending to destabilize the symmetric equilibrium. On the contrary, when trade costs are low, the conspicuous good effects are less localized: in this case an increase in the demand for the differentiated goods due to relative concerns spills over to the other region (as imports from this region are cheaper) and the centripetal force tends to be weaker.

In the paper, we compare in detail two different scenarios; full social integration in each region and full segregation in each region. Finally, we let agents choose between an integrated region and a region composed of two fully segregated areas.

Integrated social networks.

We begin the detailed analysis by considering a simple structure, in which the economy is composed of two integrated cities that we represent as two *com-*

[8] We recall that in Forslid and Ottaviano (2003) there are two centrifugal forces, which are the so-called market-size effect and the cost-of-living effect, and one centrifugal force, that is the market-crowding effect. We define these forces as the "traditional" or "original" forces.

plete "regional" regular networks.[9] These are endogenously determined and are equal in the case of the symmetric equilibrium and different in partially or fully agglomerated equilibria. Therefore, in partially or fully agglomerated equilibria, the global equilibrium network that is composed by the two regional networks, has a core-periphery structure. Of course, in the case of the symmetric equilibrium this is a regular network with each agent having the same number of links (neighbors). We show that for medium and high trade costs, relative concerns tend to destabilize the symmetric equilibrium and stabilize the agglomeration equilibrium. For low trade costs, relative concerns stabilize the symmetric equilibrium and full-agglomeration equilibrium disappears. This is in stark contrast with the case without relative concerns in which full agglomeration is the unique equilibrium.

Segregated social networks.

We pursue further our analysis and depart from complete regional networks, obtaining new interesting results. Indeed, it is plausible that workers in a given region do not compare themselves with all other agents in that region but rather to a subset of them. We focus on the case in which different ethnic groups, or workers in different sectors of the economy, are associated to different *segregated networks* of relationships and therefore react differently to the interpersonal comparisons, and ultimately affect city dynamics. In this case we find that: (i) with free trade economic activity is dispersed (as in the case of the complete network); (ii) for intermediate trade costs, the agglomeration equilibrium can be either favored or destabilized by relative concerns; (iii) for high trade costs, the symmetric equilibrium is always stable.

Choosing the city.

Finally, we let agents choose between a region with a complete network and a region composed of two fully segregated areas. An interesting pattern emerges: although for very high trade costs interior solutions may survive, in plausible economies there is a strong tendency to observe full agglomeration in the integrated region. At the same time full agglomeration in the segregated region can also coexists for intermediate trade costs.

Related literature.

This work is a contribution to the developing literature which integrates *endogenous social interactions in a spatial model*.[10] Closely related to the present

[9] As it will be specified later, a regional network is complete when in each given region, each worker compares his/her consumption with the average consumption of all the agents in the region weighted by the measure of the comparison group. Moreover, a regional network is *regular* when all agents of a given type occupy equivalent positions in the network and therefore can be treated as identical.

[10] For a review on the literature on social interactions and urban economics see Ioannides (2012, chapter 5).

paper, Helsley and Strange (2007), Mossay and Picard (2011) and Helsley and Zenou (2014) consider a setup in which agents choose their location and interact with others at a cost which is proportional to the distance they have to go through to interact, showing that their market equilibrium outcomes are not optimal because of social externalities.[11] In particular, Helsley and Strange (2007) consider the case in which all interactions take place at the left edge of a strip of land and evaluate whether the equilibrium delivers the first best levels of visit and populations density. Helsley and Zenou (2014) also consider that all interactions take place in a geographical center. They make use of the tools of graph theory to model the interactions within social networks and they show that agents who are more central in the social network, or are located closer to the geographic center of interaction, choose higher levels of interactions in equilibrium. Finally, Mossay and Picard (2011) assume that social interactions are global and find that a single city emerges in equilibrium when agents locate on a line segment, while multiple equilibria with odd numbers of cities emerge when the spatial economy extends along a circumference.

The contributions mentioned above depart from our work as they do not explicitly consider a supply side in which consumption goods are produced and, therefore, lack a general equilibrium analysis. Moreover, they consider closed economies that do not exchange goods among them. Furthermore, the main reason why agents move in these models is only that to interact with other agents, while in our case agents move to the region in which they work and live and, contextually, interact with others in their reference group by comparing with them their level of consumption. Finally, also the nature of the considered interactions is different: while in all the above mentioned works the result of an interaction is beneficial for the interacting agents, in our work conspicuous consumption impose a negative externality given that individuals are negatively affected by an increase in consumption by their neighbors.

This work also makes a contribution to the strand of the literature that analyzes how *individual location choices can lead to segregation*. In his seminal work, Shelling (1971) shows that large differences in terms of location decision may emerge if the preference for interacting with people from the same community is very mild. Indeed, he finds that total segregation persists even if most of the population is tolerant about heterogeneous neighborhood composition. More recently, Mossay and Picard (2013) assume that intra-group interactions are more frequent than inter-group interactions and analyze how agents choose their land consumption and their location. Specifically, they show that integration is never a spatial equilibrium as the two populations they consider never cluster in an integrated city. This result is due to the fact that agents have higher returns when they interact with individuals of their own group.[12] In our work, we show that without having a preconception about the preference for segregation and integration, the simple fact that agent may evaluate relative

[11] The literature includes works without location choice, such.as those by Johnson and Gilles (2000), Galeotti et al. (2006), Carayol et al. (2008).

[12] Similar segregation issues that may arize due to social network are analyzed by de Marti and Zenou (2012) in a framework that does not consider an explicit spatial framework.

concerns may push them to choice an integrated environment as this can allow them to enjoy a higher level of welfare for a wider range of integration levels.

The paper is organized as follows. In Section 2 we introduce the new economic geography model modified to take into account relative concerns. In Section 3 we define our equilibrium and stability concepts. In Section 4 we present our findings and discuss the properties of the equilibria of the model when the network in each region is complete with linear size effects. In Section 5 we analyse the case of segregated networks. In Section 6 we consider a mixed case in which agents choose between two regions, each with a different network. Section 7 concludes. Most of the proofs are contained in the Appendix.

2 The general model

We consider an economy that consists of two regions indexed by $r = 1, 2$, populated by a mass $L_1 + L_2 = 2L$ of immobile unskilled workers, and a mass $H_1 + H_2 = H$ of interregionally mobile skilled workers, each supplying inelastically one unit of his/her specific type of labour. We assume that unskilled workers are evenly distributed between the two regions, with $L_r = L$. Skilled workers can be thought as self-employed entrepreneurs, as in the footloose entrepreneur model by Forslid and Ottaviano (2003), that move freely between the two regions, with their level in each region endogenously determined and given by H_r and $H_v = H - H_r$, with $v = 1, 2$ and $r \neq v$. Wages perceived by skilled and unskilled workers in region r are, respectively, given by w_{H_r} and w_{L_r}.

Two goods are produced and exchanged in the model. The agricultural good is produced under perfect competition with constant returns to scale employing one unit of unskilled labour to obtain one unit of output and it is homogeneous across the two regions and freely exchanged between them. The price p_{Ar} of the agriculture good in region r is chosen as the numeraire. Therefore, the unskilled wage, w_{L_r}, is equal to 1 in both regions ($w_{L_r} = p_{Ar} = 1$). There is also a mass N of varieties of a modern manufactured good: each variety is produced by a monopolistically competitive firm with increasing returns to scale employing both skilled and unskilled workers.

2.1 The demand side

Each individual i located in region r consumes the quantity A_{ir} of the agricultural good, and a mass N of varieties of the manufactured good, with each variety denoted by index s and consumed in quantity $X_{ir}(s)$. The differentiated varieties are aggregated by a constant elasticity of substitution function in X_{ir}. We assume that agents care not only about their consumption of the differentiated good X_{ir}, but also about those of their neighbours X_{-ir} in the same region r. We note $\Lambda_r(i)$ the set of direct neighbours/acquaintances to agent i in region

r, which are also denoted by $-ir$. In other words, we assume that there is a *conspicuous effect* of consumption and that it affects only agents that are both geographically and socially close.

Given these assumptions, and adapting Ghiglino and Goyal (2010) to the continuous case, we assume that workers' preferences are represented by the following utility function

$$U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i))) = A_{ir}^{1-\mu} (\Phi(X_{ir}, X_{-ir}, \Lambda_r(i)))^{\mu} \quad \text{with} \quad 0 < \mu < 1 \tag{1}$$

where

$$X_{-ir} = \int_{\Lambda_r(i)} X_{jr} dj$$

and $\Phi : R \times R \times D \to R$, with X_{jr} denoting the consumption of the differentiated good by individual $j \in \Lambda_r(i)$ in the neighbourhood of individual i in r, and $\Lambda_r(i) \in D$ is an element of the subset D of the real line. The specification of $\Lambda_r(i)$ will be refined when considering specific networks and the size effect.[13]

For fixed set of neighbours, the function Φ is increasing in X_{ir} and decreasing in X_{-ir}. Individual i in region r cares about his/her consumption relative to the average consumption in his/her neighbourhood, that is given by $X_{-ir}/\int_{\Lambda_r(i)} dj$, but this effect might be weighted by a term $S(\Omega(\Lambda_r(i)))$ characterizing the *size of the neighborhood*.[14] We assume that $S()$ is an arbitrary non decreasing function taking values between 0 and $\Omega(\Lambda_r(i))$, with $\Omega(\Lambda_r(i))$ denoting the Lebesgue measure of the set of neighbours. In the paper we focus on the first polar case, the *additive specification* with $S(\Omega(\Lambda_r(i))) = \Omega(\Lambda_r(i))$ when the effect of the size of the neighborhood is linear. In the associated working paper, we also consider the other polar specification, that is, the *average specification* with $S(\Omega(\Lambda_r(i))) = 1$ where only the average consumption of neighbors matters.[15] These considerations are reflected in the following *general formulation*, which is valid when $\Lambda_r(i) \neq \emptyset$:[16]

$$\Phi(X_{ir}, X_{-ir}, \Lambda_r(i)) = X_{ir} + \alpha S(\Omega(\Lambda_r(i))) \left[X_{ir} - \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \right]. \tag{2}$$

[13] Note that this formulation appears to be the interesting case to consider; see Frank (2007) for a discussion on differences in social sensitiveness across goods and Kuhn et al. (2011) for evidence that relative consumption effects are prominent with some goods but not others. It is also natural to assume that if consumption externalities are symmetric for both goods, then the social comparisons will simply wash out and equilibrium will be analogous to the equilibrium in an economy with no consumption externalities (see Arrow and Dasgupta (2007)).

[14] The first systematic analysis of the size effect in relative concerns was done in Ghiglino and Goyal (2010). More recently, Liu et al. (2011) also consider this issue.

[15] We focus on these two polar cases because they help us clarify the types of effects at work; however, we recognize that the intermediate specification with a small size effect may be more realistic. In this paper we have mainly explored the role of relative concerns in location decision and city dynamics when the size of the comparison group matters. In the working paper we explore the somewhat more standard pure average specification and precisely show where the change in specification affects the results. In fact, most of the results are not affected by the change and the broad picture remains unaffected.

[16] If $\Lambda_r(i) = 0$ then obviously $\Phi(X_{ir}, X_{-ir}) = X_{ir}$.

Some remarks are in order here. When the conspicuous effect is absent, that is when $\alpha = 0$, we fall back to the Forslid and Ottaviano (2003) model.[17] When $\alpha > 0$, individuals are negatively affected by the consumption of their neighbors; and $\alpha < 0$ corresponds to a positive externality. We will focus on the case $\alpha > 0$, because it captures the idea that individuals are negatively affected by an increase in consumption by neighbors.

The total mass of produced varieties N, is the sum of the mass of varieties produced in region r, n_r, and the mass of varieties produced in region v, n_v. The horizontally differentiated manufactured good consumed by individual i in region r is given by

$$X_{ir} = \left(\int\limits_{s \in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where $\sigma > 1$ is both the elasticity of demand of any variety and the elasticity of substitution between any two varieties.

Let $p_{kr}(s)$ be the price of a manufactured good s produced in $k = 1, 2$ and sold in r. Given the price for the agricultural good $p_{Ar} = 1$ and his/her income w_{ir}, the problem of a consumer i in region r is to maximize the utility function in (1) subject to the budget constraint

$$A_{ir} + \int\limits_{s \in N} p_{kr}(s) X_{ir}(s) ds = w_{ir} \tag{4}$$

In Appendix 1.A it is shown that then the individual demand in region r for variety s produced in k is given by

$$X_{ikr}(s) = \frac{p_{kr}(s)^{-\sigma}}{p_{X_r}^{1-\sigma}} E_{ir} \tag{5}$$

where $E_{ir} = p_{X_r} X_{ir}$ is the individual expenditure on manufactures in region r and p_{X_r} is the local price index of manufactures in r defined as follows

$$p_{X_r} = \left(\int\limits_{s \in n_r} p_{rr}(s)^{1-\sigma} ds + \int\limits_{s \in n_v} p_{vr}(s)^{1-\sigma} ds \right)^{\frac{1}{1-\sigma}} \tag{6}$$

From the consumer's problem in Appendix 1.A, we also find that the consumer demand for agriculture in region r is

$$A_{ir} = (1-\mu) \left(w_{ir} - p_{X_r} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1 + \alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \right) \tag{7}$$

and that the individual demand of manufactures in region r is

$$X_{ir} = \frac{\mu}{p_{X_r}} \left(w_{ir} + p_{X_r} \frac{1-\mu}{\mu} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1 + \alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \right) \tag{8}$$

[17] Forslid and Ottaviano (2003) assume that $L_r = L/2$ and $H = 1$.

Let us define the *wage net of the conspicuous effect of individual i in region r* as follows

$$W_{\alpha ir} \equiv w_{ir} - p_{X_r} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \tag{9}$$

Notice that $W_{\alpha ir}$ must be positive to ensure the positivity of the demand for the agricultural good in (7).[18] Furthermore, the individual demand for agriculture in region r can be rewritten as follows

$$A_{ir} = (1-\mu) W_{\alpha ir}, \tag{10}$$

while the individual demand of manufactures in region r (8) is

$$X_{ir} = \frac{\mu}{p_{X_r}} W_{\alpha ir} + \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj} \tag{11}$$

Expressions (10) and (11) show that for given prices and wages, the demand in both goods depend linearly on the wage net of the conspicuous consumption effect, $W_{\alpha ir}$. This decomposition highlights the loss in disposable income as agents waste part of their income only to adjust their own consumption to the consumption of their neighbours. In particular, it shows that there is a fall in the demand of the ordinary good. Concerning the differentiated good, we see from (11) that the demand has an additional term that increases with α, for given consumption of neighbours ($\int_{\Lambda_r(i)} X_{jr} dj$). In fact, expression (8) shows that when α increases, this second effect dominates that produced by the reduction of $W_{\alpha ir}$ and, thus, the individual demand X_{ir} increases with α.[19]

To simplify the analysis we restrict our attention to the case in which the network of interactions within cities/regions is *regular* in the sense that all agents of a given type in region r occupy equivalent positions and therefore can be treated as identical. This implies that the consumption of the differentiated good is equal for all skilled in region r, noted then $X_{ir} = X_{H_r}$. The same must be true for the unskilled, so that $X_{ir} = X_{L_r}$. Consider now a skilled agent i in region r. Let $\Lambda_r(i) = \Lambda_r(H)$. The term $\int_{\Lambda_r(i)} X_{jr} dj$ can be rewritten in this case as

$$\int_{\Lambda_r(H)} X_{jr} dj = h_{H_r}(H_r) X_{H_r} + l_{H_r}(H_r) X_{L_r}$$

where $h_{H_r}(H_r)$ is the mass of high skilled neighbours to a high skill agent in r, $l_{H_r}(H_r)$ is the mass of low skilled neighbours to a high skill agent in r. The

[18] When performing the numerical analysis, we choose values of α sufficiently small for the given values of the other parameters that ensure that $W_{\alpha ir}$ is positive.

[19] Moreover, if we define $E_{\Lambda_r(i)} \equiv p_{X_r} \int_{\Lambda_r(i)} X_{jr} dj$ as the *total expenditure on the differentiated good by neighbours* of individual i in region r, we can also point out what follows: for given wages w_{ir} and total expenditures of neighbours of individual i in region r on the differentiated good $E_{\Lambda_r(i)}$, an increase in conspicuous consumption effects (that is an increase in α) reduces the share of wage used by individuals to buy the traditional good, $\frac{A_{ir}}{w_{ir}} = (1-\mu)\left(1 - \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{E_{\Lambda_r(i)}}{w_{ir} \int_{\Lambda_r(i)} dj}\right)$, and, consequently, increases the share of wage used to buy the manufactured good.

mass $\Omega_r(H)$ of neighbours for the skilled agent i in r is then

$$\Omega_r(H) = h_{H_r}(H_r) + l_{H_r}(H_r)$$

Note that these masses depend only on the total mass of skilled agents in region r, because the network is regular and because the unskilled are immobile. If, instead, i is an unskilled agent, the total consumption of his/her neighbours is

$$\int_{\Lambda_r(L)} X_{jr} dj = h_{L_r}(H_r) X_{H_r} + l_{L_r}(H_r) X_{L_r}$$

where $h_{L_r}(H_r)$ is the mass of high skilled neighbours to a low skill agent in region r, $l_{L_r}(H_r)$ is the mass of low skilled neighbours to a low skill agent in region r, and the total mass of neighbours for the unskilled agent i in r is

$$\Omega_r(L) = h_{L_r}(H_r) + l_{L_r}(H_r)$$

It is important to stress that we assume that the network in each region is exogenously given, because agents choose to which region they migrate but do not chose on the type of network that prevails in these regions. Still, typically the masses $h_{H_r}(H_r)$, $h_{L_r}(H_r)$, $l_{H_r}(H_r)$ and $l_{L_r}(H_r)$ are endogenously determined and depend on H_r. In the sequel we simplify notation and drop the explicit dependence on H_r.

The individual demand of the differentiated good by a skilled consumer in region r , X_{H_r}, and that by an unskilled consumer in region r, X_{L_r}, depend on the structure of the network and the size of the neighborhood $S()$. In general, in Appendix 1.B we describe how we obtain the system (40) of two equations that can be solved *for a given structure of the network* and *size of the neighborhood* to find X_{H_r} and X_{L_r}. Clearly, the quantities depend on h_{H_r}, h_{L_r}, l_{H_r} and l_{H_r}. Finally, total demand for the differentiated good in region r, X_r, is given by

$$X_r = X_{H_r} H_r + X_{L_r} L, \tag{12}$$

while the total demand in region r for variety s produced in region k derived from (5) and (12) is

$$X_{kr}(s) = \frac{p_{kr}(s)^{-\sigma}}{p_{X_r}^{-\sigma}} \left(X_{H_r} H_r + X_{L_r} L \right) \tag{13}$$

2.2 The supply side

Each manufacturing variety is produced by a monopolistically competitive firm with increasing returns to scale employing both skilled and unskilled workers. Specifically, to produce $Q_r(s)$ units of variety s the firm located in region r incurs a fixed input requirement of f units of skilled workers independently of the production level, and $\beta Q_r(s)$ units of unskilled workers. The cost function for each firm producing variety s in region r is

$$TC_r(s) = f w_{H_r} + \beta Q_r(s)$$

Given the fixed input requirement, the mass of firms producing in region r is proportional to the mass of its skilled residents with

$$n_r = \frac{H_r}{f} \tag{14}$$

To import one unit of a variety in each given region, $\tau > 1$ units have to be shipped from the other region. Hence, profits for a firm producing variety s in region r are given by the sum of revenues from the domestic (r) and the foreign ($v \neq r$) region net of total cost of production

$$\Pi_r(s) = p_{rr}(s)X_{rr}(s) + p_{rv}(s)X_{rv}(s) - fw_{H_r} - \beta\left[X_{rr}(s) + \tau X_{rv}(s)\right] \tag{15}$$

From the first order condition for the maximization of profits, we obtain that the price set by the firm producing variety s for the domestic market r and the the foreign market v are, respectively,

$$p_{rr}(s) = \frac{\sigma}{\sigma - 1}\beta \quad \text{and} \quad p_{rv}(s) = \frac{\sigma}{\sigma - 1}\beta\tau \tag{16}$$

for every $r, v = 1, 2$ and $r \neq v$.

Using (16), (14) and $H_v = H - H_r$, the price index in (6) becomes

$$p_{X_r} = \frac{\beta\sigma}{\sigma - 1}\left(\frac{1}{f}\right)^{\frac{1}{1-\sigma}}\left[H_r + (H - H_r)\,\phi\right]^{\frac{1}{1-\sigma}} \tag{17}$$

where $\phi = \tau^{1-\sigma} \in [0, 1]$ is a direct measure of the *freeness of trade*, with its value equal to zero when trade costs are prohibitively high ($\tau \to \infty$), and equal to 1 when markets are perfectly integrated ($\tau = 1$). An analogous expression holds for the price index p_{X_v}, that is

$$p_{X_v} = \frac{\beta\sigma}{\sigma - 1}\left(\frac{1}{f}\right)^{\frac{1}{1-\sigma}}\left[(H - H_r) + H_r\phi\right]^{\frac{1}{1-\sigma}} \tag{18}$$

Wages for skilled are derived from the free entry condition, which implies that profits in (15) are equal to zero in equilibrium: this together with (16) implies that the wage is

$$w_{H_r} = \frac{\beta}{(\sigma - 1)\,f}\left[X_{rr}(s) + \tau X_{rv}(s)\right] = \frac{\beta}{(\sigma - 1)\,f}Q_r(s) \tag{19}$$

where $Q_r(s) \equiv X_{rr}(s) + \tau X_{rv}(s)$ is the total production by the firm producing variety s in region r. Finally, we can use (13), (16), (17) and (18) to obtain that the production of the firm producing variety s in r, $Q_r(s)$, is

$$Q_r(s) = f^{\frac{\sigma}{\sigma-1}}\left\{\frac{X_{H_r}H_r + X_{L_r}L}{\left[H_r + (H - H_r)\,\phi\right]^{\frac{\sigma}{\sigma-1}}} + \phi\frac{X_{H_v}(H - H_r) + X_{L_v}L}{\left[(H - H_r) + H_r\phi\right]^{\frac{\sigma}{\sigma-1}}}\right\}$$

3 Mobility decision and equilibrium

The indirect utility function for agent i, as obtained in Appendix 1.A is

$$U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i))) = \eta\left[1 + \alpha S(\Omega(\Lambda_r(i)))\right]^{\mu} \frac{W_{\alpha ir}}{(p_{X_r})^{\mu}} \tag{20}$$

where $\eta \equiv (1-\mu)^{1-\mu} \mu^{\mu}$.

Taking into account that $H_v = H - H_r$ and that $W_{\alpha H_r}$ is the value that $W_{\alpha ir}$ takes for skilled workers in region r, we can use (20) to analyze the location decision of skilled workers. Formally, we need to consider the logarithm of the ratio of the current indirect utility levels in region r with respect to region v, that is

$$V(H_r, \phi, \alpha) \equiv \ln\left[\left(\frac{p_{X_v}}{p_{X_r}}\right)^{\mu}\left(\frac{1 + \alpha S(\Omega_r(H))}{1 + \alpha S(\Omega_v(H))}\right)^{\mu} \frac{W_{\alpha H_r}}{W_{\alpha H_v}}\right] \tag{21}$$

where $\Omega_k(H) = \Omega(\Lambda_k(H))$ is the mass of neighbours to a skilled worker in region k.[20] From (17) and (18) we can see that the value of α does not directly influence the price indexes in the two regions in (21). However, it can have an indirect effect as it brings a change in the mass of skilled workers (and of firms) in the two regions and therefore of H_r. Another indirect effect may come through the wage in (19).

In the model the notion of equilibrium is associated to the absence of migration. We then define a *spatial equilibrium* by $\dot{H}_r = 0$. In fact, the decision of a worker to migrate depends on the value taken by the function V. When $0 < H_r < H$, if $V(H_r, \phi, \alpha) > 0$ then a worker from region v will move to region r while a worker from region r would not move. On the other hand, if $V(H_r, \phi, \alpha) < 0$ a worker from region r would migrate to region v. Consequently, at an interior equilibrium we expect $V(H_r, \phi, \alpha) = 0$. When there is *full agglomeration* in region r, or in region v, that is $H_r = H$ or $H_r = 0$, the dynamics is slightly more subtle. Formally, the process of migration is determined by an equation of motion. Expressing time with t, and following Forslid and Ottaviano (2003), the migration process is summarized as follows

$$\dot{H}_r \equiv dH_r/dt = \begin{cases} V(H_r, \phi, \alpha) \text{ if } 0 < H_r < H \\ \min\{0, V(H_r, \phi, \alpha)\} \text{ if } H_r = H \\ \max\{0, V(H_r, \phi, \alpha)\} \text{ if } H_r = 0 \end{cases} \tag{22}$$

According to the Samuelson's principle only stable equilibria deserve interests as they are the only one surviving an evolving environment. A spatial equilibrium is *stable* if a deviation from the equilibrium, that is a deviation of H_r from its equilibrium value, generates changes in H_r - described by the equation of motion in (22) - that bring the distribution of workers back to the

[20] Alternatively, we could examine the location decision of skilled workers studying the current indirect utility differential, as in Forslid and Ottaviano (2003). However, we choose to work with the logarithm of the ratio of the current indirect utility levels because it gives an expression that is analytically more tractable, which, in any case, identifies the same critical values of ϕ for the sustainability of the different types of equilibria.

original one. Hence, an interior equilibrium ($0 < H_r < H$) is stable only if $\frac{dV}{dH_r}(H_r, \phi, \alpha) \equiv V_{H_r}(H_r, \phi, \alpha) < 0$. A corner configuration ($H_r = 0$ or $H_r = H$) is an equilibrium as soon as $V(0, \phi, \alpha) < 0$ or $V(H, \phi, \alpha) > 0$. Therefore, a corner equilibrium is generically stable.

The function $V(H_r, \phi, \alpha)$ explicitly, but also implicitly through the wages, depends on H_r and on the conspicuous effect. Indeed, substituting (17) and (18) into (21) we obtain that

$$V(H_r, \phi, \alpha) \equiv \ln \left\{ \left[\frac{(H - H_r) + H_r\phi}{H_r + (H - H_r)\phi} \right]^{\frac{\mu}{1-\sigma}} \left(\frac{1 + \alpha S(\Omega_r(H))}{1 + \alpha S(\Omega_v(H))} \right)^{\mu} \frac{W_{\alpha H_r}}{W_{\alpha H_v}} \right\} \quad (23)$$

4 The complete network

The benchmark economy we investigate is one in which in each given region, each worker compares his/her consumption with the weighted average consumption of all the agents in the region weighted by the measure of the comparison group. Formally, when the region is *complete (or integrated),* we have that $h_{H_r}(H_r) = h_{L_r}(H_r) = H_r$ and $l_{H_r}(H_r) = l_{L_r}(H_r) = L$, and $S(\Omega_k(H))) = \Omega_k(H)$. The assumption that the comparison group is the whole city is adopted for simplicity and has no qualitative implications. Indeed, we could instead assume that the reference neighbourhood is a fraction of the city. As the network is complete, this small neighbourhood would have exactly the same average as the whole city but its size would be affected. This would only scale down the weight, and the results would be unaffected.[21]

As described in Appendix 1.B, equations (7) and (8) can be used to show that the individual demand of the differentiated good by a skilled workers in region r is

$$X_{H_r} = \frac{\mu}{p_{X_r}} \frac{w_{H_r} + \alpha(L + H_r w_{H_r}) + \alpha\mu(w_{H_r} - 1)L}{1 + \alpha\mu(L + H_r)} \quad (24)$$

and that by an unskilled workers living in the same region is

$$X_{L_r} = \frac{\mu}{p_{X_r}} \frac{1 + \alpha(L + H_r w_{H_r}) - \alpha\mu(w_{H_r} - 1)H_r}{1 + \alpha\mu(L + H_r)} \quad (25)$$

Moreover, from (12)-(13), (16)-(17), (24) and (25) we find that for a firm producing variety s in region r, the aggregate local demand is

$$X_{rr}(s) = \mu \frac{(\sigma - 1)(L + H_r w_{H_r})}{\sigma\beta(n_r + n_v\phi)} \frac{1 + \alpha\Omega_r}{1 + \alpha\mu\Omega_r} \quad (26)$$

[21] Of course the numerical examples would be affected.

while the aggregate foreign demand in region v is

$$X_{rv}(s) = \mu \frac{(\sigma - 1)\,\tau^{-\sigma}\,(L + H_v w_{H_v})}{\sigma\beta\,(n_v + n_r\phi)} \frac{1 + \alpha\Omega_v}{1 + \alpha\mu\Omega_v} \tag{27}$$

with $r, v = 1, 2$ and $r \neq v$, and with the total number of neighbors in region r and v respectively given by $\Omega_r = H_r + L$ and $\Omega_v = H_v + L$.

4.1 The new forces produced by conspicuous consumption.

In a new economic geography model there are different contrasting forces that shape the final distribution of the economic activity in the space, and those that tend to promote geographical concentration (centripetal forces) are contrasted by those that tend to oppose it (centrifugal forces). In the seminal paper by Krugman (1991) and in Forslid and Ottaviano (2003), centripetal forces are generated in the presence of mobile skilled workers by the tendency of monopolistic firms to locate their production in the market that has a larger dimension ('market size effect') where the larger income originates an associated larger local expenditure, and then firms export their production from this location to the smaller market. Centripetal forces are generated also by the fact that consumers tends to move to the region in which they are able to consume more local varieties because goods tend to be cheaper in the region where there are more firms as in this location consumers import a narrower range of products avoiding more of the incidence of trade costs ('cost of living effect'). Centrifugal forces are, instead, generated by the fact that firms tend to locate in a country in which there are relatively few competitors ('market crowding effect'). We claim that the introduction of relative concerns in this work strengthens the traditional market size effect in the original model and introduce a new centrifugal force.

A first insight on the effect produced on centripetal forces by the introduction of relative concerns can be gained from equation (26). Indeed, taking the derivative of $X_{rr}(s)$ with respect to α, we find that for a given distribution of workers (and consequently of firms) and for given wages, the demand $X_{rr}(s)$ increases with α. Moreover, for fixed wages, price indexes and α, an increase in H_r and in Ω_r, also increases the demand $X_{rr}(s)$, the strength of this effect being increasing in α. This property is due to the fact that the migration of skilled workers to a region induces the workers in the region to increase their demand, larger values of α generating larger increases.[22] The increase in the demand tends to increase the operating profits of firms producing in the same region

[22] To have an insight of what happens, we consider the following thought experiment. For given values of α, of the wages and of the price indexes - which imply a given value of $(n_r + n_v\phi)$, we evaluate the following expression

$$\frac{\partial X_{rr}(s)}{\partial H_r} - \left(\left.\frac{\partial X_{rr}(s)}{\partial H_r}\right|_{\alpha=0} \right) =$$

and, consequently, this tends to increase the wages received by skilled workers employed in the region, a centripetal force. Trade costs play here an important role. Indeed, if we consider the symmetric equilibrium for given total expenditures and price indexes, we find that the extent of the increase of w_{H_r} due to increases in H_r is larger when ϕ is low, and is reduced as ϕ increases.[23] This is due to the fact that the introduction of relative concerns, which increase the conspicuous demand for the differentiated goods, produces effects that are more localized when trade costs are high (low ϕ), rising more sharply the demand for local firms than that for imported goods.

Hence, relative concerns strengthen the traditional centripetal force identified as the *market-size effect* in Forslid and Ottaviano (2003), who observe that in the region hosting more firms (and skilled workers) there are additional operating profits and consequently additional skilled income, a fraction of which is spent on local manufactures, increasing local expenditures and increasing demand per firm for a given price index. This is also what happens in the present paper where the increase in the mass of skilled neighbors in a region induces a positive effect on demand per local firm, which rises skilled real wage that in turn, in the presence of relative concerns, results in a stronger increase in the demand per firm and thus magnifies the traditional "market size" effect in the original core-periphery model. Hence, we can state:

Proposition 1 *Relative concerns produce a new centripetal force as, at a given price index, they magnify the increase in the demand per firm in the region hosting more skilled workers magnifying the increase in the skilled wage in the larger region. The strength of this force increases with trade costs.*

Conspicuous consumption has also a more direct effect on individual welfare. Indeed, comparing the expression for $V(H_r, \phi, \alpha)$ in (23) when $\alpha > 0$ with the case $\alpha = 0$, we see that the introduction of relative concerns brings in the factor

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$$= \frac{\partial\left(\mu\frac{(\sigma-1)\left(L+H_r w_{H_r}\right)}{\sigma\beta(n_r+n_v\phi)}\frac{1+\alpha(H_r+L)}{1+\alpha\mu(H_r+L)}\right)}{\partial H_r} - \left(\left.\frac{\partial\left(\mu\frac{(\sigma-1)\left(L+H_r w_{H_r}\right)}{\sigma\beta(n_r+n_v\phi)}\right)}{\partial H_r}\right|_{\alpha=0}\right) =$$

$$= \alpha\mu(1-\mu)(\sigma-1)\frac{L+2w_{H_r}H_r+Lw_{H_r}+w_{H_r}\alpha\mu(L+H_r)^2}{\sigma\beta(L\alpha\mu+\alpha\mu H_r+1)^2(n_r+\phi n_v)} > 0$$

[23] Indeed, at the symmetric equilibrium (with $H_r = H/2$ and $\frac{\left(L+H_r w_{H_r}\right)}{H_r+(H-H_r)\phi} = \frac{\left(L+H_v w_{H_v}\right)}{(H-H_r)+H_r\phi}$), for given total expenditures and price indexes (that imply given values of $(L+H_r w_{H_r})$ and $H_r + (H-H_r)\phi$), the sign of $\frac{\partial w_{H_r}}{\partial H_r}$ with

$$w_{H_r} = \frac{\mu}{\sigma}\frac{\left(L+H_r w_{H_r}\right)}{H_r+(H-H_r)\phi}\left[\frac{1+\alpha(H_r+L)}{1+\alpha\mu(H_r+L)} + \phi\frac{1+\alpha(H-H_r+L)}{1+\alpha\mu(H-H_r+L)}\right]$$ is equal to the sign of $$\frac{\partial\left(\frac{1+\alpha(H_r+L)}{1+\alpha\mu(H_r+L)}+\phi\frac{1+\alpha(H-H_r+L)}{1+\alpha\mu(H-H_r+L)}\right)}{\partial H_r} = \alpha(1-\mu)\frac{(H\alpha\mu+2L\alpha\mu+2)^2(1-\phi)}{4(L\alpha\mu+\alpha\mu H_r+1)^2(H\alpha\mu+L\alpha\mu-\alpha\mu H_r+1)^2} > 0$$ which is positive when $\alpha > 0$. Let us notice that the extent of this increase is larger when ϕ is low, and that it becomes smaller when ϕ increases.

$\frac{1+\alpha\Omega_r(H)}{1+\alpha\Omega_v(H)} = \frac{1+\alpha\Omega_r}{1+\alpha\Omega_v} = \frac{1+\alpha\Omega_r}{1+\alpha(H-\Omega_r+2L)}$ that is increasing in Ω_r. Furthermore, relative concerns also modify the third factor, as it affects the wage net of relative concerns (see equation (9)). To analyze this effect, notice that the conspicuous effect in $W_{\alpha H_r}$ can be rewritten using (24) and (25). Indeed, we can substitute the total amount of consumption by the neighbours of skilled workers $\int_{\Omega_r(H)} X_{jr}dj = X_{H_r}H_r + X_{L_r}L$ in the definition of $W_{\alpha H_r}$ in (9) and rewrite the wage of skilled workers net of the conspicuous effect. The following expression is obtained;[24]

$$W_{\alpha H_r} = w_{H_r} - \alpha\mu\frac{L + H_r w_{H_r}}{1 + \alpha\mu\left(L + H_r\right)} \tag{28}$$

Expression (28) shows that for a given skilled wage, $W_{\alpha H_r}$ tends to decrease when H_r increases.[25] This identifies the existence of a negative effect on $W_{\alpha H_r}$ produced by the increase in the mass of skilled workers in a region (i.e., H_r) in the presence of conspicuous good consumption. Thus, the presence of this negative effect tends to push skilled workers back to the region of origin. Indeed, the increase in the mass of skilled workers in a region increases the mass of neighbors to both types of workers. This increases the consumption in the manufacturing goods, inducing a welfare loss on both skilled and unskilled workers, but only for the mobile skilled agents this can induce them to relocate toward the other region. Hence, we can state the following:

Proposition 2 *Relative concerns generate a new centrifugal force for mobile skilled agents as they can relocate towards the region hosting less skilled workers where the welfare loss from conspicuous consumption is smaller.*

We can now focus our attention on the existence and stability of equilibria in this model.

4.2 The existence and stability of equilibria

The model can generate several types of equilibria, that is, situations in which agents are not willing to migrate. As in the standard core-periphery model, it can be shown that the *symmetric equilibrium*, with $H_r = H_v = H/2$ and all variables assuming the same value in both regions, always exists as an interior

[24] An analogous expression can be obtained for the wage of unskilled workers net of the conspicuous effect $W_{\alpha L_r}$. Specifically, we can substitute the total amount of consumptions by the neighbours of unskilled workers $\int_{\Omega_r(L)} X_{jr}dj = X_{H_r}H_r + X_{L_r}L$ in the definition of $W_{\alpha L_r}$ in (9) and obtain that

$$W_{\alpha L_r} \equiv 1 - \alpha\mu\frac{L+H_r w_{H_r}}{1+\alpha\mu(L+H_r)}$$

This expression is important to check the positivity of agricultural demand by unskilled workers in (7).

[25] This is true when the skilled wage is above a minimum level smaller than 1, that is for $w_{H_r} > L\alpha\mu/(1+L\alpha\mu)$.

equilibrium. However, its stability depends on the underlying parameters.[26] In Appendix 2.A we show that the following Proposition holds for $H < 2\frac{(1+L\alpha)}{\alpha(\sigma-1)}$:[27]

Proposition 3 *For low and high trade costs, the symmetric equilibrium is always stable while full agglomeration is never an equilibrium. On the other hand, for intermediate values of trade costs, the full agglomeration equilibrium exists while the symmetric equilibrium is unstable.*[28]

To illustrate the relationship between the strength of relative concerns (α), the openness to trade (ϕ) and the existence and stability of the equilibria, we consider a numerical simulation of the model. The analysis is presented in the Appendix (2.A). A summary of the results is presented here as bifurcation diagrams reported in Figure 1.[29]

In Figure 1 the spatial equilibrium distribution of workers and firms is drawn as a function of the freeness of trade ϕ, $\phi \in [0,1]$, and for different values of α. The bold continuous lines represent stable equilibria (as pointed out by the arrows), while the bold discontinuous lines represent unstable asymmetric equilibria. Specifically: the first diagram in case a) is drawn for $\alpha = 0$ and replicates the bifurcation diagram in Forslid and Ottaviano (2003, p. 237); the second diagram in case b) is obtained with $\alpha = 0.02$; the third graphic in case c) shows how the bifurcation diagram appears for $\alpha = 0.04$. As in Forslid and Ottaviano, ϕ_b and ϕ_s respectively denote the symmetry breaking point and the sustain point.[30] Moreover, we define ϕ_d as the "dispersion point", given that the symmetric equilibrium becomes stable as soon as $\phi > \phi_d$, and ϕ_u as the "unsustain point", because full agglomeration disappears after the freeness of trade has increased beyond this new critical level. An important regularity for its implication, is that when α increases, all the four critical points ϕ_s, ϕ_b, ϕ_d and ϕ_u decrease progressively. The bifurcation diagrams therefore shifts on the left as α increases.[31]

From the bifurcation diagrams (and Table 1 in Appendix 2.A) we see that when $\alpha = 0.02$, $\phi_s < \phi_b < \phi_d < \phi_u$. In this case the symmetric equilibrium is stable for low and for high levels of integration ϕ. Full agglomeration is an equilibrium (and it is stable as it is a corner equilibrium) for intermediate values

[26] We assume that the no black hole condition, that is $\mu < \sigma - 1$, holds. This condition rules out the case in which the symmetric equilibrium is never stable when relative concerns are absent.

[27] In Appendix 2.A we point out that $H < 2\frac{(1+L\alpha)}{\alpha(\sigma-1)}$ is required to have a positive value of the wage net of the conspicuous effect for skilled workers in r in the symmetric equilibrium, that is $W_{\alpha H_r}(H/2) > 0$.

[28] Note that the intervals of costs associated to full agglomeration equilibria and to unstable symmetric equilibria may be different, as seen in the discussion below.

[29] This numerical analysis is performed with $L = 4$, $H = 10$, $\sigma = 2$ and $\mu = 0.11$.

[30] Specifically, the symmetry breaking point ϕ_b is such that the symmetric equilibrium is stable for $\phi \in (0, \phi_b)$, and the sustain point ϕ_s is the critical value of ϕ such that full agglomeration is an equilibrium for $\phi > \phi_s$.

[31] Notice that the same type of graphic in figure 1.*c* is obtained for other values of alpha, that is $\alpha = 0.06$, $\alpha = 0.08$, $\alpha = 0.10$, $\alpha = 0.12$, $\alpha = 0.14$ and $\alpha = 0.16$, with the corresponding critical values given in Table 1 in Appendix 2.A. that are such that $\phi_b < \phi_s < \phi_d < \phi_u$.

of ϕ while no partial agglomeration is a stable equilibrium. When $\alpha = 0.04$, we find that $\phi_b < \phi_s < \phi_d < \phi_u$. Here the symmetric equilibrium is stable for low and for high levels of integration ϕ. Full agglomeration is an equilibrium for intermediate values of ϕ and stable partial agglomeration equilibria exist for $\phi \in (\phi_b, \phi_s)$. To summarize, for low trade costs, relative concerns tends to destabilize the full agglomeration equilibrium and stabilize the symmetric equilibrium. For medium and high trade costs, relative concerns tends to destabilize the symmetric equilibrium and stabilize the agglomeration equilibrium.

Insert Figure 1 about here

Fig. 1 Bifurcation diagrams

We give some intuition on the forces generating the above results. When trade costs are small, relative concerns tend to stabilize the symmetric equilibrium while they tend to destabilize it when trade costs are intermediate. This is due to the fact that the new centrifugal force created by relative concerns has a power that is independent of trade costs. On the other hand, conspicuous consumption strengthens forward linkages, and thus creates a centripetal force whose power increases with trade costs. Indeed, the effects of introducing relative concerns, which increase the conspicuous demand for the differentiated goods, are more localized when trade costs are high (low ϕ), rising more sharply the interior demand. This force destabilizes the symmetric equilibrium as it increases the new centripetal market size effect in the standard model. On the contrary, when trade costs are low, the increase in the demand for differentiated goods spills over to the other region and the dispersion forces again dominate.

Figure 1 also shows an interesting property: when $\alpha \geq 0.04$, stable asymmetric equilibria are possible and there exists a *pitchfork* pattern with a continuous, and easily reversible, transition from symmetry to agglomeration. This is not a traditional property either of the seminal core-periphery model by Krugman (1991) or of the Forslid and Ottaviano (2003) footloose entrepreneur model. Indeed, these models exhibit catastrophic agglomeration and locational hysteresis with a tomahawk pattern, as can be seen from the bifurcation diagram obtained with $\alpha = 0$. In these models, once trade freeness has increased beyond the "break point", ϕ_b, all the mobile manufacturing sector catastrophically fully agglomerates in one region. If then trade costs increase again, they do not restore the symmetric equilibrium until ϕ falls below the "sustain point", ϕ_s, which lies at a strictly lower level of trade freeness than the break point ϕ_b.

Hence, the presence of the conspicuous good effect is responsible of a location pattern which is new, and sometimes replaces, the original tomahawk diagram identified by the literature on the core-periphery model as the new centrifugal force created by conspicuous consumption is able to stabilize the symmetric equilibrium for low trade costs and to avoid the catastrophic agglomeration in the original core-periphery model allowing for a gradual agglomeration of the manufacturing sector once trade freeness increases beyond the break point.[32]

[32] For other models in which catastrophic agglomeration is replaced with gradual and partial agglomeration processes, see for e.g. Helpman (1998), Tabuchi (1998), Ludema and Wooton

5 Segregated social networks

So far we assumed that the comparison group was the entire population in the region of residence. However, it might be argued that more realistically interactions occur within a smaller group. For the sake of tractability we analyze the case in which skilled workers are affected only by the consumption of other skilled workers and unskilled workers only by that of other unskilled workers. In Appendix 2.B we prove the following result.

Proposition 4 *In the case of segregated networks full agglomeration is an equilibrium only for intermediate values of openness to trade* ϕ.

Unfortunately, further results for the symmetric equilibrium can only be obtained numerically and are reported in Appendix 2.B. The analysis shows the effects in the case of the complete and the segregated network differ when μ, that is, the share of income devoted to differentiated good consumption net of the conspicuous effect, is large. Specifically, we observe that:[33]

1. For low trade costs (large ϕ), relative concerns stabilize the symmetric equilibrium in both types of networks.
2. In the segregated network, for medium and high trade costs (intermediate and small ϕ).
   (a) When the expenditure on the differentiated good is relatively large, that is μ large, conspicuous consumption tends to destabilize the symmetric equilibrium.
   (b) When μ is relatively small, the resulting force is centrifugal, stabilizing the symmetric equilibrium. Indeed, the centrifugal force is large because skilled workers only compare with other skilled workers while the centripetal market size effect is less important because the unskilled are unaffected by changes in the consumption of skilled in the region.

Finally, comparing these results with those presented in Section 4, we obtain the interesting following result

Property 5 *Suppose that trade costs are moderate. When relatively small shares of income are devoted to the consumption of the differentiated good: if classes of workers are segregated (as in homogenous suburban areas), relative concerns tend to generate disperse, medium size, cities; when workers of different classes socially interact as described by the case of complete networks, relative concerns contribute to foster socially integrated megalopolises.*

(1999), Ottaviano et al. (2002), Tabuchi and Thisse (2002), Murata (2003), Pflüger (2004), Nocco (2009), Berliant and Kung (2009), Pflüger and Suedekum (2011) and Ottaviano (2012).

[33] Not surprisingly, the masses H and L are both relevant for the stability of equilibria in the case of the complete network, while only H is relevant for the segregated network. Note that they are not relevant in the model with no relative concerns of Forslid and Ottaviano (2003).

6 Choosing the right city

So far we have given no choice to the workers on the type of social network they can integrate. We now focus on the case in which skilled workers can choose between two regions: one characterized by an integrated, or complete, network and the other by two segregated networks. Let r be the region with an integrated network and v the region with two segregated networks, one composed only of skilled workers and the other one composed only of unskilled workers.

In Appendix 2.C we compute the log of the indirect utility levels in the mixed case, noted $V^{mix}(H_r, \phi, \alpha)$. Not surprisingly, we find that $V^{mix}(H/2, \phi, \alpha)$ is different than 0 provided $\alpha > 0$, so that the symmetric configuration is not an equilibrium with relative concerns. Interior equilibria with partial agglomeration may however exist. We have performed a numerical analysis for various values of the parameters of the model. The analysis shows that two configurations may arise. These are illustrated in Figure 2.a and 2.b, which are obtained for $\alpha = 0.08$, $\sigma = 2$, $H = 10$, $L = 4$ and $\mu = 0.11$ [Fig. 2.a] and for $L = 20$ and $\mu = 0.4$ [Fig. 2.b].[34]

Insert Figure 2 about here

Fig. 2 Choosing the right city

Fig. 2.b illustrates the pattern followed by population density as a function of the level of freeness of trade. Starting from high trade costs (i.e., low values of ϕ), as these decrease, the population density in the integrated region r increases, and full aggregation is reached in fact for a relatively high value of the costs. The intuition is as follows. First, there is a strong market size force pushing the skilled workers toward the integrated region and this tendency is reinforced by the direct comparison with the low average consumption of the neighbours (indeed, the unskilled wage is lower than the skilled wage). On the other hand, in the segregated region the direct comparison does not produce any utility benefit to the agents as these are homogeneous (the skilled compare their consumption with the skilled). Surprisingly, for intermediate trade costs another equilibrium may coexist in which all workers move toward the segregated city. Indeed, a very populated segregated city might be initially more attractive than a deserted integrated city, because of the very small size of the market in the integrated city. Clearly, this may only happen for intermediate trade costs, as otherwise either the market size effect is too small (with small trade costs) or the attraction of the integrated city too strong.

In Fig. 2.a the effect is reversed. In this case, as a response to the scarcity of the unskilled population, the unskilled wage is larger than the skilled wage. Consequently, the conspicuous consumption of the unskilled may be large, pushing up the average consumption of the manufactured good in the integrated network. This mechanisms strengthens the centrifugal force acting on the mobile

[34] We notice that for the two cases represented in Figure 2.a and 2.b the condition $H < 2\frac{(1+L\alpha)}{\alpha(\sigma-1)}$, which is assumed in Proposition 3, holds.

skilled workers. The role played by the share of expenditure in the differentiated good is also highlighted by this example, as here the reduction in μ weakens the centripetal market size effect. To conclude, the skilled prefer the move to a segregated city, a type of deprived but happy Chelsea borough.

In light of the above analysis, we conjecture that in general, when agents can choose between migrating to regions with different networks of interpersonal comparisons, there exists an interval of values of trade costs such that there exists an asymmetric interior equilibrium and intervals for which the full agglomeration equilibrium exists and is stable. This interesting results shows that full agglomeration tends to occur in the city with integrated social networks, at least in our model in which skilled workers are more mobile than unskilled. However, even in this case a stable equilibrium with full agglomeration in the segregated region can exist. Note that for high trade costs, there exists an asymmetric interior equilibrium, which can be in one of the two regions depending on the parameters of the model.

To summarise, we can state the following.

Property 6 *When unskilled workers are sufficiently abundant and a sizable part of the income is devoted to the consumption of the differentiated good, for high trade costs, there exists an asymmetric interior equilibrium, which eventually becomes full agglomeration in the integrated region as trade costs decrease to intermediate and low levels. Furthermore, for an open interval of the trade cost, there also exists a stable full agglomeration equilibrium in the segregated region.*

7 Conclusion

The goal of the paper is to explore the role of relative concerns in location decision and city dynamics. In core-periphery models of economic geography, full agglomeration is typically the outcome when trade costs vanish. We have shown that relative concerns generates a powerful centrifugal force that can stop this process. However, the notion of trade costs should not be defined too narrowly and they merely represent a wide variety of obstacles to economic integration. As a result, these trade costs are difficult to evaluate but surely exceed the physical transportation costs and import tariffs. We therefore analyse the model also with intermediate trade costs. Surprisingly, when trade costs are intermediate, relative concerns also generate a new centripetal force favoring full agglomeration. The driving force is the market size effect, that is, a centripetal force produced by the increase in per firm domestic demand of the conspicuous good, which rises profits and wages. The direction of the resultant force is however highly sensitive to the network structure.

In particular, for a given value of trade costs, relative concerns might be such that full agglomeration emerges when agents in the city are fully integrated while a stable symmetric equilibrium emerges when the cities are segregated in homogeneous but separated areas. In general, it appears that the centrifugal force of relative concern is stronger in segregated organisations.

An interesting pattern emerges when agents can choose between regions with different organisations. Although for some costs, interior solutions may survive, there is a strong tendency to observe full agglomeration in the integrated region, at least when the endowment of unskilled workers is relatively large. Indeed, in this case skilled workers prefer to compare their consumption level with the average consumption of neighbours in the integrated region because it is smaller than the average consumption of neighbours in the segregated region. To sum up, for intermediate shipping costs and relative concerns, workers tends to migrate to regions in which the different types of agents affect each others behavior.

The previous analysis has interesting policy implications. Indeed, the planner can, by favoring one type of city organisation rather than another, affect the migration patterns and the type of agglomeration arising in equilibrium. However, the optimal type of city structure will depends on the exact welfare function of the planner, an exercise which is rarely consensual.

A related recent development of new economic geography has seen the introduction of urban structure. In this literature workers that choose to live in a certain region becomes urban residents and consume land, while firms do not consume land. Workers commute to a regional central business district (CBD) in which jobs and varieties of the differentiated good are available.[35] These models bridge the gap between the two polar cases considered in the literature by the traditional new economic geography models, which ignore urban structure altogether, and the literature on the system-of-cities developed by Henderson (1974), which considers commuting costs and housing space consumption. Assuming that commuting costs are negligible and that urban residents do not consume land, we simplify the analysis and focus our attention on the effects of conspicuous consumption on the spatial distribution of workers between different cities. However, we recognize that it would be interesting to investigate the interaction of conspicuous consumption effects on the urban structure of cities when urban costs are considered. We leave this for future research.

[35] See, for instance, Tabuchi (1998), Tabuchi and Thisse (2006) and Gaigné and Thisse (2014). Basically, the urban structure of these models disappears when commuting costs are equal to zero.

APPENDIX 1.

1.A. Demand side and price index

In this Appendix we compute the consumer's demand for the differentiated good X_{ir} and for the agricultural good A_{ir} in region r, the indirect utility function $U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i)))$, the individual demand in region r for variety v ($X_{ir}(v)$) and we define the price index p_{X_r}.

Each individual i in region r solves the program

$$Max_{A_{ir},X_{ir}} U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i))) = A_{ir}^{1-\mu} \left(\Phi(X_{ir}, X_{-ir}, \Lambda_r(i))\right)^{\mu}$$

$$s.t.\ A_{ir} + \int\limits_{s\in N} p_r(s) X_{ir}(s) ds = w_{ir}$$

with

$$\Phi(X_{ir}, X_{-ir}, \Lambda_r(i)) = X_{ir} + \alpha S(\Omega(\Lambda_r(i))) \left[X_{ir} - \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj}\right]$$

As $\int_{\Lambda_r(i)} dj = \Omega(\Lambda_r(i))$ and $X_{ir} = \left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}}$, the Lagrangean function is

$$\begin{aligned} & L \\ = \; & A_{ir}^{1-\mu}\left[\left[1+\alpha S(\Omega(\Lambda_r(i)))\right]\left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}} - \alpha S(\Omega(\Lambda_r(i)))\frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))}\right]^{\mu} \\ & +\lambda\left(w_{ir} - A_{ir} - \int\limits_{s\in N} p_r(s) X_{ir}(s) ds\right) \end{aligned}$$

with the first order conditions with respect to A_{ir}, the consumption of variety s, $X_{ir}(s)$, and of variety v, $X_{ir}(v)$, and of λ, respectively given by

$$(1-\mu) A_{ir}^{-\mu} \Phi^{\mu} = \lambda \tag{29}$$

$$\mu A_{ir}^{1-\mu} \Phi^{\mu-1} \left[1+\alpha S(\Omega(\Lambda_r(i)))\right] (x)^{\frac{\sigma}{\sigma-1}-1} X_{ir}(s)^{\frac{\sigma-1}{\sigma}-1} = \lambda p_r(s) \tag{30}$$

$$\mu A_{ir}^{1-\mu} \Phi^{\mu-1} \left[1+\alpha S(\Omega(\Lambda_r(i)))\right] (x)^{\frac{\sigma}{\sigma-1}-1} X_{ir}(v)^{\frac{\sigma-1}{\sigma}-1} = \lambda p_r(v) \tag{31}$$

$$w_{ir} = A_{ir} + \int\limits_{s\in N} p_r(s) X_{ir}(s) ds \tag{32}$$

with $x = \int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}} ds$. Considering the ratio of (30) and (31), we obtain:

$$\frac{X_{ir}(s)^{-\frac{1}{\sigma}}}{X_{ir}(v)^{-\frac{1}{\sigma}}} = \frac{p_r(s)}{p_r(v)} \tag{33}$$

Let us notice that from (33) we obtain that

$$X_{ir}(s) = \left(\frac{p_r(v)}{p_r(s)}\right)^{\sigma} X_{ir}(v)$$

which can be substituted into the definition of $X_{ir} \equiv \left(\int\limits_{s\in N} X_{ir}(s)^{\frac{\sigma-1}{\sigma}} ds\right)^{\frac{\sigma}{\sigma-1}}$ to get

$$X_{ir}(v) = \frac{p_r(v)^{-\sigma}}{\left(\int\limits_{s\in N} (p_r(s))^{1-\sigma} ds\right)^{\frac{\sigma}{\sigma-1}}} X_{ir} \tag{34}$$

From (29), (31) and (34) we get that

$$A_{ir} = \frac{\left[\left[1+\alpha S(\Omega(\Lambda_r(i)))\right] X_{ir} - \alpha S(\Omega(\Lambda_r(i))) \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))}\right] (1-\mu)\, p_{X_r}}{\mu\left[1+\alpha S(\Omega(\Lambda_r(i)))\right]} \tag{35}$$

where $p_{X_r} \equiv \left(\int\limits_{s\in N} (p_r(s))^{1-\sigma} ds\right)^{\frac{1}{1-\sigma}}$ is the price index of manufactured varieties. Then we substitute (35) into (32) and we use also (34) to obtain the consumer's demand for the differentiated good in region r

$$X_{ir} = \frac{\mu}{p_{X_r}} \left(w_{ir} + p_{X_r} \frac{1-\mu}{\mu} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))}\right) \tag{36}$$

Previous expression can be substituted into (35) to obtain the consumer's demand for the agricultural good in region r, that is

$$A_{ir} = (1-\mu)\left(w_{ir} - p_{X_r} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))}\right) \tag{37}$$

Then, substituting (36) and (37) into

$$U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i))) = A_{ir}^{1-\mu} \left(\Phi(X_{ir}, X_{-ir}, \Lambda_r(i))\right)^{\mu}$$

we obtain the indirect utility function:

$$U(A_{ir}, \Phi(X_{ir}, X_{-ir}, \Lambda_r(i))) = \frac{(1-\mu)^{1-\mu} \mu^{\mu} \left(1+\alpha S(\Omega(\Lambda_r(i)))\right)^{\mu}}{(p_{X_r})^{\mu}}$$
$$\left(w_{ir} - p_{X_r} \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))} \frac{\int_{\Lambda_r(i)} X_{jr} dj}{\Omega(\Lambda_r(i))}\right)$$

Expression (34) can be used to derive an expression for the expenditure in manufacturing, E_{ir}, that is

$$E_{ir} = \int\limits_{s\in N} p_r(s) X_{ir}(s) ds = p_{X_r} X_{ir}$$

This expression can be substituted into (34) to get the individual demand in region r for variety v

$$X_{ir}(v) = \frac{p_r(v)^{-\sigma}}{p_{X_r}^{1-\sigma}} E_{ir}$$

1.B. Individual demand of the differentiated good by workers in region r.

In this appendix we show how we derive the individual demand by the skilled and by the unskilled consumers in region r. As $\int_{\Lambda_r(i)} dj = \Omega(\Lambda_r(i))$, the demands for each *skilled* individual i in region r as given by (7) and (8) can be rewritten as

$$\begin{aligned} A_{H_r} &= (1-\mu)\left[w_{H_r} - p_{X_r} \tfrac{\alpha S(\Omega_r(H))}{1+\alpha S(\Omega_r(H))} \tfrac{(h_{H_r} X_{H_r} + l_{H_r} X_{L_r})}{\Omega_r(H)}\right], \\ X_{H_r} &= \tfrac{\mu}{p_{X_r}} \left[w_{H_r} + p_{X_r} \tfrac{1-\mu}{\mu} \tfrac{\alpha S(\Omega_r(H))}{1+\alpha S(\Omega_r(H))} \tfrac{(h_{H_r} X_{H_r} + l_{H_r} X_{L_r})}{\Omega_r(H)}\right]; \end{aligned} \tag{38}$$

while for each *unskilled* individual i in region r the demands are

$$\begin{aligned} A_{L_r} &= (1-\mu)\left[w_{L_r} - p_{X_r} \tfrac{\alpha S(\Omega_r(L))}{1+\alpha S(\Omega_r(L))} \tfrac{(h_{L_r} X_{H_r} + l_{L_r} X_{L_r})}{\Omega_r(L)}\right], \\ X_{L_r} &= \frac{\mu}{p_{X_r}} \left[w_{L_r} + p_{X_r} \tfrac{1-\mu}{\mu} \tfrac{\alpha S(\Omega_r(L))}{1+\alpha S(\Omega_r(L))} \tfrac{(h_{L_r} X_{H_r} + l_{L_r} X_{L_r})}{\Omega_r(L)}\right]. \end{aligned} \tag{39}$$

Considering the second equations in (38) and in (39), we obtain a system of two equations in the two unknowns X_{H_r} and X_{L_r} given by

$$\left\{ \begin{array}{c} X_{H_r} = w_{H_r} \frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu) S(\Omega_r(H))}{1+\alpha S(\Omega_r(H))} \frac{(h_{H_r} X_{H_r} + l_{H_r} X_{L_r})}{\Omega_r(H)} \\ X_{L_r} = \frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu) S(\Omega_r(L))}{1+\alpha S(\Omega_r(L))} \frac{(h_{L_r} X_{H_r} + l_{L_r} X_{L_r})}{\Omega_r(L)} \end{array} \right. \tag{40}$$

where we used the fact that $w_{L_r} = 1$ and where $\Omega_r(H) = h_{H_r} + l_{H_r}$, $\Omega_r(L) = h_{L_r} + l_{L_r}$.

APPENDIX 2.

In the case of the additive specification, we know that $S(\Omega(\Lambda_r(i))) = \Omega(\Lambda_r(i))$. Hence, with $\Omega_r(H) = h_{H_r} + l_{H_r}$ and $\Omega_r(L) = h_{L_r} + l_{L_r}$, the system in (40) becomes

$$\begin{cases} \frac{1+\alpha(h_{H_r}+l_{H_r})-\alpha(1-\mu)h_{H_r}}{1+\alpha(h_{H_r}+l_{H_r})} X_{H_r} = w_{H_r}\frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu)}{1+\alpha(h_{H_r}+l_{H_r})} l_{H_r} X_{L_r} \\ \frac{1+\alpha(h_{L_r}+l_{L_r})-\alpha(1-\mu)l_{L_r}}{1+\alpha(h_{L_r}+l_{L_r})} X_{L_r} = \frac{\mu}{p_{X_r}} + \frac{\alpha(1-\mu)}{1+\alpha(h_{L_r}+l_{L_r})} h_{L_r} X_{H_r} \end{cases} \quad (41)$$

This system is solved to obtain the individual demands of the differentiated good by the skilled workers and by the unskilled workers in region r, i.e., X_{H_r} and X_{L_r}.

2.A. Complete networks

Let us now consider the complete network and derive the expression for skilled wages as a function of H_r. Substituting $X_{rr}(s)$ and $X_{rv}(s)$ from (26) and (27) into (19) and making use of (14), the wage paid to skilled workers in region r must satisfy the following equation

$$w_{H_r} = \frac{\mu}{\sigma}\left[\frac{(L+H_r w_{H_r})}{H_r+(H-H_r)\phi}\frac{1+\alpha\Omega_r}{1+\alpha\mu\Omega_r} + \phi\frac{(L+H_v w_{H_v})}{(H-H_r)+H_r\phi}\frac{1+\alpha\Omega_v}{1+\alpha\mu\Omega_v}\right] \quad (42)$$

and an analogous expression holds for w_{H_v}. Hence, we get a system of two linear equations in w_{H_r} and w_{H_v} that can be solved to obtain the two regional wages for skilled workers as an explicit function of a given distribution of workers, H_r and H_v, between the two regions, and we find that the wage paid in region r to skilled workers is

$$w_{H_r} = \mu L \frac{\sigma A(H_v+\phi H_r)+\phi\sigma B(H_r+\phi H_v)+H_v\mu BA(\phi-1)(\phi+1)}{\phi\sigma^2(H_r^2+H_v^2)-\phi\mu\sigma(AH_r^2+BH_v^2)-H_vH_r[\mu\sigma(A+B)-\sigma^2(\phi^2+1)+\mu^2BA(\phi-1)(\phi+1)]} \quad (43)$$

where $A = \frac{\alpha\Omega_r+1}{\alpha\mu\Omega_r+1}$ and $B = \frac{\alpha\Omega_v+1}{\alpha\mu\Omega_v+1}$.[36] This expression shows that the wage depends on the value of α, and A and B represent a measure of the effects produced by the proximity of neighbors, respectively, in r (where the number of neighbours is given by $\Omega_r = H_r + L$) and in v (where the number of neighbours is given by $\Omega_v = H_v + L$).

The wage of skilled workers in (43) evaluated in the case in which they are evenly distributed in the two regions is given by the following expression

$$w_{H/2} = \frac{2\mu L}{H}\frac{\alpha\Omega_s+1}{\sigma-\mu+\mu\alpha\Omega_s(\sigma-1)}$$

where $\Omega_s \equiv (H/2+L)$ is the number of neighbours in each region at the symmetric equilibrium.[37] It can be readily shown that the wage (w_{H_r}) increases

[36] An analogous expression to (43) holds for w_{H_v}.

[37] Specifically, we observe that $A(H_r = H_v = H/2) = B(H_r = H_v = H/2) = \frac{\alpha\Omega_s+1}{\alpha\mu\Omega_s+1}$.

in the symmetric equilibrium with α. Finally, evaluating the wage of skilled workers in (43) when they are all in region r we get that

$$w_{H_r} = \frac{\mu L}{H} \frac{2 + \alpha (H + 2L) (\mu + 1) + 2\alpha^2 \mu L(H + L)}{(\mu\alpha L + 1) [\sigma - \mu + \alpha\mu (H + L) (\sigma - 1)]}$$

In what follows we show that Proposition 3 in Section 4.2 holds.

Proof of Proposition 3. First, consider the derivative of $V(H_r, \phi, \alpha)$ evaluated at the symmetric equilibrium. It can be shown using the expressions for skilled wages derived for the symmetric equilibrium in this Appendix that

$$V_{H_r}(H/2, \phi, \alpha) \tag{44}$$

$$= \frac{4\left(f_2\phi^2 + f_1\phi + f_0\right)}{H(\phi + 1)(\sigma - 1)[\alpha (H + 2L) + 2]\{2 + \alpha [2L - H(\sigma - 1)]\}} * \frac{1}{\{2[\sigma - \mu + \phi(\sigma + \mu)] + \mu\alpha (H + 2L)[\sigma - 1 + \phi(\sigma + 1)]\}} \tag{45}$$

where the coefficients f_0, f_1 and f_2 are functions of μ, σ, L, H and α. We know that all factors in the denominator are positive, given that $\sigma > 1 > \mu$ and that the term $\{2 + \alpha [2L - H(\sigma - 1)]\}$ is positive when $W_{\alpha H_r}$ evaluated at $H_r = H/2$ is positive.[38] Thus, the sign of $V_{H_r}(H/2, \phi, \alpha)$ depends on the sign of the expression $F \equiv f_2\phi^2 + f_1\phi + f_0$ in the numerator, and the symmetric equilibrium is stable when $F < 0$ and unstable when $F > 0$. However, we know that when $\alpha = 0$, $F = a_0 = 8(1 - \phi)(2\phi\sigma\mu - \mu\phi - \phi\sigma + \phi\sigma^2 + \mu^2\phi - \mu^2 + \sigma - \sigma^2 + 2\sigma\mu - \mu)$. In this case, a_0 is the relevant term in determining the sign of $V_{H_r}(H/2, \phi, \alpha)$, and, as in Forslid and Ottaviano (2003), the symmetry breaking point $\phi_b^{FO} = \frac{(\sigma-1-\mu)(\sigma-\mu)}{(\mu+\sigma)(\mu+\sigma-1)} < 1$ is such that the symmetric equilibrium is stable only for $\phi \in (0, \phi_b^{FO})$.[39] Note that for $\phi = 1$, $F = a_0 = 0$. As α becomes positive and rises, F becomes negative, that is $F = 4\mu H\alpha^2\sigma(\sigma-1)(1-\mu)(2L+H)\{\alpha[H(\sigma - 1) - 2L] - 2\} < 0$,[40] and the symmetric equilibrium remains stable for $\phi = 1$. By continuity the result holds for high levels of integration (i.e., large ϕ) and the symmetric equilibrium is always stable for low trade costs. Moreover, for prohibitively high trade costs, when $\phi = 0$, $F = f_0$ that is negative for $\alpha = 0$ as in this case $f_0 = 8(\mu+1-\sigma))(\sigma-\mu) = F < 0$; by continuity the result holds for positive and sufficiently small values of α, with $f_0 = 8(\mu+1-\sigma))(\sigma-\mu)+a_1\alpha+a_2\alpha^2+a_3\alpha^3 = F < 0$ where the coefficients a_1, a_2 and a_3 are functions of μ, σ, L and H. This proves the results in Proposition 3 on the symmetric equilibrium.

We now focus our attention on equilibria in which all skilled workers move to one region. Note that, provided these *full agglomeration equilibria* exist, they

[38] This requires that the relative mass of skilled workers with respect to the mass of unskilled is not relatively too high, that is, $H < 2\frac{(1+L\alpha)}{\alpha(\sigma-1)}$.

[39] We assume that the no black hole condition, that is $\mu < \sigma - 1$, holds. This condition rules out the case in which the symmetric equilibrium is never stable.

[40] We recall that we assume $H < 2\frac{(1+L\alpha)}{\alpha(\sigma-1)}$ to have $W_{\alpha H_r}(H/2) > 0$.

are stable, and in what follows we show that the final part of Proposition 3 holds.

The value of $V(H_r, \phi, \alpha)$ when all skilled workers are located in region r is given by

$$V(H, \phi, \alpha) = \ln \left\{ \frac{g_0 \{2 + \alpha [2L - H(\sigma - 1)]\} \phi^{1 - \frac{\mu}{\sigma - 1}}}{d_2 \phi^2 + d_1 \phi + d_0} \right\} \tag{46}$$

where

$$\begin{aligned} g_0 &= \sigma (1 + \alpha \mu L) \left[\tfrac{\alpha (H+L)+1}{\alpha L+1} \right]^{\mu} > 1 \\ d_2 &= [1 + \alpha (H + L)][\mu \alpha L (\sigma + 1) + \sigma + \mu] > 0; \\ d_1 &= -H \alpha \sigma [\mu \alpha (H + L) (\sigma - 1) + \sigma - \mu] < 0; \\ d_0 &= (1 + \alpha L) [\mu \alpha (H + L) (\sigma - 1) + \sigma - \mu] > 0 \end{aligned}$$

When $\alpha = 0$, expression (46) becomes

$$V(H, \phi) = \ln \left(\frac{2 \sigma \phi^{1 + \frac{\mu}{1 - \sigma}}}{\sigma (1 + \phi^2) - \mu (1 - \phi^2)} \right) \tag{47}$$

Equation (47) generates the sustain point ϕ_s, i.e., the value of ϕ such that full agglomeration is an equilibrium for $\phi > \phi_s$, as in Forslid and Ottaviano (2003).[41] When α is positive, we know that $g_0 > 1$ and that expression $\phi^{1+\frac{\mu}{1-\sigma}}$ is increasing in $\phi \in [0, 1]$ from 0 (if $\phi = 0$) to 1 (if $\phi = 1$).[42] Given the sign of the parameters d_2, d_1 and d_0, expression $d_2\phi^2 + d_1\phi + d_0$ in the denominator of (46) is an upward opening parabola in ϕ with positive value d_0 when $\phi = 0$.[43] Moreover, we know that, when workers are all agglomerated in r, expression $\{2 + \alpha [2L - H(\sigma - 1)]\}$ in the numerator must be positive to have $W_{\alpha H_r} > 0$.[44] Finally, the parabola in the denominator $(d_2\phi^2 + d_1\phi + d_0)$ must be positive in order to have $W_{\alpha H_v} > 0$. Figure 3 presents the two possible scenarios for this parabola by means of the two continuous curves when α is positive and sufficiently small. In Figure 3 the numerator $N = g_0 \{2 + \alpha [2L - H(\sigma - 1)]\} \phi^{1-\frac{\mu}{\sigma-1}}$ is represented by the dotted line, which characterizes an increasing function in ϕ taking the value 0 when $\phi = 0$ and $g_0 \{2 + \alpha [2L - H(\sigma - 1)]\} > 0$ when $\phi = 1$.[45] The lower parabola can be excluded with $\alpha > 0$ as it implies that with $\phi = 1$ the full agglomeration is an

[41] See expression (25) at page 236 in Forslid and Ottaviano (2003).

[42] We recall that we assume the no black hole condition corresponding to the case of $\alpha = 0$ holds, that is $\mu < \sigma - 1$.

[43] Moreover, its minimum value is attained at $\phi = -\frac{d_1}{2d_2} > 0$ when $\alpha > 0$ (and at $\phi = 0$ when $\alpha = 0$), so that the parabola has a negative slope in $\phi = 0$ only if α is positive.

[44] This requires that $H < 2\frac{(1+L\alpha)}{\alpha(\sigma-1)}$.

[45] When $\alpha = 0$: the numerator is still an increasing function in ϕ taking the value 0 when $\phi = 0$ but it is smaller for any other value of ϕ; the parabola is always increasing for $\phi \in [0, 1]$, it intersects the vertical axis at a lower value of d_0, that is $\sigma - \mu$, and it is equal to the numerator (2σ) when $\phi = 1$.

equilibrium, which contradicts direct computation.[46] The only relevant case is then the other parabola $D = d_2\phi^2 + d_1\phi + d_0$ (that intersect the dotted curve in two critical points ϕ_s and ϕ_u).[47] In this case, full agglomeration in region r is an equilibrium only for intermediate values of ϕ (when the higher parabola lies below the dotted curve) for $\phi \in (\phi_s, \phi_u)$ as $V(H, \phi, \alpha)$ is positive. This proves the results in Proposition 3 on full agglomeration equilibria.
QED

Insert Figure 3 about here

Fig. 3 Full agglomeration equilibrium

A general overview: the bifurcation diagram.

To explore further the relationship between the strength of relative concerns (α), the openness to trade (ϕ) and the existence and stability of the equilibria we need to focus on numerical simulations of the model. First, let us perform the numerical analysis on a model in which $L = 20$, $H = 10$, $\sigma = 2$ and $\mu = 0.4$ (Figure 4.a) or $\mu = 0.11$ (Figure 4.b).[48] We know that the sign of $V_{H_r}(H/2, \phi, \alpha)$ in (44) depends on the sign of the expression $F \equiv f_2\phi^2 + f_1\phi + f_0$, and the symmetric equilibrium is stable when $F < 0$ and unstable when $F > 0$. The solid curves represent $F = a_0$ as a function of ϕ when $\alpha = 0$, as in Forslid and Ottaviano (2003). They show that in this case the symmetric equilibrium is stable only if $F < 0$, that is, only if $\phi < \phi_b^{FO}$. Assume now the existence of relative concerns, $\alpha > 0$. Figure 4.a and Figure 4.b represents the function F

[46] Indeed, we know that when $\phi = 1$ the denominator is

$$d_2\phi^2 + d_1\phi + d_0 = \sigma\left(H\alpha\mu + L\alpha\mu + 1\right)\left(H\alpha + 2L\alpha - H\alpha\sigma + 2\right)$$

while the numerator is

$$g_0\left\{2 + \alpha\left[2L - H(\sigma - 1)\right]\right\}\phi^{1-\frac{\mu}{\sigma-1}} = \sigma\left(1 + \alpha\mu L\right)\left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu}\left\{2 + \alpha\left[2L - H(\sigma - 1)\right]\right\}$$

Hence when $\phi = 1$, the argument of the logarithm in (46) is given by $\left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu}\frac{(1+\alpha\mu L)}{(H\alpha\mu+L\alpha\mu+1)}$, which is equal to 1 when $\alpha = 0$. With a positive value of α, we can show that $\left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu}\frac{(1+\alpha\mu L)}{(H\alpha\mu+L\alpha\mu+1)} < 1$ and, thus, agglomeration is never an equilibrium when $\phi = 1$.

Proof. $\left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu}\frac{(1+\alpha\mu L)}{(H\alpha\mu+L\alpha\mu+1)} < 1$ requires that $\left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu} < \frac{H\alpha\mu+L\alpha\mu+1}{1+\alpha\mu L}$. We know that these two expressions, defined respectively as $LHS = \left[\frac{\alpha(H+L)+1}{\alpha L+1}\right]^{\mu}$ and $RHS = \frac{H\alpha\mu+L\alpha\mu+1}{1+\alpha\mu L}$, are both equal to 1 when $\mu = 0$, and that they both increase in the range $\mu \in [0,1]$ and assume the same value $\frac{\alpha(H+L)+1}{(1+\alpha L)}$ when $\mu = 1$. However, since $\forall\mu \in (0,1)$ the $\frac{\partial^2 LHS}{\partial\mu^2} > 0$ and $\frac{\partial^2 RHS}{\partial\mu^2} < 0$, the LHS is convex in μ and the RHS is concave in μ, which implies that $LHS < RHS$. Q.E.D.

[47] The two critical points ϕ_s and ϕ_u in Figure 3 correspond to those obtained in Table 1 for $\alpha = 0.16$ when $L = 4$, $H = 10$, $\sigma = 2$ and $\mu = 0.11$.

[48] The values of the parameters for which curves in Figure 4.$a - b$ are drawn are equal to those used to derive Figure 5.$a - b$.

for $\alpha = 0.02, 0.04, 0.06, 0.08$. They show that with relative concerns, as ϕ rises the symmetric equilibrium becomes stable as soon as $\phi > \phi_d$, where ϕ_d is the "dispersion point" (so-called as the symmetric equilibrium is stable for $\phi > \phi_d$). Hence, the symmetric equilibrium is stable both for small values of the freeness of trade ϕ, such that $\phi < \phi_b$, and for high levels of integration, such that ϕ is above the new critical level ϕ_d.

Then, we can use expression (46) to find the other two critical points ϕ_s and ϕ_u for the equilibrium with full agglomeration. Full agglomeration in region r is an equilibrium only for $\phi \in (\phi_s, \phi_u)$. As ϕ rises from 0 and reaches the sustain point, ϕ_s, full agglomeration becomes a stable equilibrium for $\phi > \phi_s$, as in the case $\alpha = 0$. However, this equilibrium again disappears with $\alpha > 0$ after trade costs have decreased beyond a new critical level, that we name "unsustain point", ϕ_u.

Insert Figure 4 about here

Fig.4 Stability of the symmetric equilibrium for the complete network: the plot of F

Table 1 shows the values of ϕ_s, ϕ_b, ϕ_u and ϕ_d obtained numerically for the model with $L = 4$, $H = 10$, $\sigma = 2$ and $\mu = 0.11$.[49] The analysis in Table 1 is summarized by the bifurcation diagrams reported in Figure 1 in the text of the paper.

TABLE 1.
CRITICAL POINTS FOR DIFFERENT VALUES OF α.

	ϕ_s		ϕ_b	ϕ_d		ϕ_u
$\alpha = 0$	0.7167	$<$	0.7181	/		/
$\alpha = 0.02$	0.6347	$<$	0.6350	0.9891	$<$	0.9892
$\alpha = 0.04$	0.5850	$\boxed{>}$	0.5845	0.9731	$<$	0.9736
$\alpha = 0.06$	0.5504	$>$	0.5495	0.9589	$<$	0.9599
$\alpha = 0.08$	0.5245	$>$	0.5235	0.9475	$<$	0.9489
$\alpha = 0.1$	0.5043	$>$	0.5032	0.9386	$<$	0.9403
$\alpha = 0.12$	0.4877	$>$	0.4867	0.9317	$<$	0.9338
$\alpha = 0.14$	0.4739	$>$	0.4731	0.9265	$<$	0.9289
$\alpha = 0.16$	0.4622	$>$	0.4616	0.9227	$<$	0.9254

2.B. Segregated networks

In the case of the segregated network the system in (41) becomes

$$\begin{cases} \frac{1+\alpha H_r - \alpha(1-\mu)H_r}{1+\alpha H_r} X_{H_r} = w_{H_r} \frac{\mu}{p_{X_r}} \\ \frac{1+\alpha L - \alpha(1-\mu)L}{1+\alpha L} X_{L_r} = \frac{\mu}{p_{X_r}} \end{cases} \tag{48}$$

[49] The choice of a smaller value of L and $\mu = 0.11$ allows for the asymmetric stable equilibria when $\alpha > 0.04$.

which can be solved for X_{H_r} and X_{L_r} to find respectively that

$$\begin{aligned} X_{H_r} &= \mu \frac{w_{H_r}}{p_{X_r}} \frac{1+\alpha H_r}{1+\alpha\mu H_r} \quad \text{and} \\ X_{L_r} &= \mu \frac{1}{p_{X_r}} \frac{1+\alpha L}{1+\alpha\mu L}. \end{aligned} \tag{49}$$

Making use of these solutions, we can rewrite the total demand in region r for variety s produced in region k in (13) as follows

$$X_{kr}(s) = \mu \frac{p_{kr}(s)^{-\sigma}}{p_{X_r}^{1-\sigma}} \left(w_{H_r} \frac{1+\alpha H_r}{1+\alpha\mu H_r} H_r + \frac{1+\alpha L}{1+\alpha\mu L} L \right) \tag{50}$$

where the price indices and the prices are respectively given by (17), (18) and (16). Making use of (50), the wage in (19) can be rewritten as follows

$$w_{H_r} = \frac{\mu}{\sigma} \left[\frac{(A_r w_{H_r} + A_L)}{H_r + (H - H_r)\phi} + \frac{\phi (A_v w_{H_v} + A_L)}{(H - H_r) + H_r \phi} \right]$$

where $A_r \equiv \frac{1+\alpha H_r}{1+\alpha\mu H_r} H_r$, $A_v \equiv \frac{1+\alpha(H-H_r)}{1+\alpha\mu(H-H_r)} (H - H_r)$ and $A_L \equiv \frac{1+\alpha L}{1+\alpha\mu L} L$. Previous expression can be considered together with the analogous expression obtained for w_{H_v} to get a system of two equations in two unknowns w_{H_r} and w_{H_v}, that can be solved to find the two skilled regional wages given by

$$w_{H_r} = \frac{\mu A_L \left[2\sigma\phi H_r + \sigma H_v \left(1+\phi^2\right) - A_v \mu (1-\phi)(\phi+1) \right]}{D_s} \tag{51}$$

and

$$w_{H_v} = \frac{\mu A_L \left[2\sigma\phi H_v + \sigma H_r \left(\phi^2+1\right) - A_r \mu (1-\phi)(\phi+1) \right]}{D_s} \tag{52}$$

with the denominator of both wages given by $D_s \equiv \sigma^2 (H_v + \phi H_r)(H_r + \phi H_v)$ $- A_r \sigma\mu (H_v + \phi H_r) - \sigma\mu A_v (H_r + \phi H_v) + \mu^2 (1-\phi)(1+\phi) A_r A_v$.

Then, $W_{\alpha ir}$ in (9) can be written for skilled workers in region r as

$$W_{\alpha H_r} = \frac{w_{H_r}}{1+\alpha\mu H_r},$$

while for unskilled it is given by

$$W_{\alpha L_r} = \frac{1}{1+L\alpha\mu}$$

Hence, we can rewrite (23) as follows

$$V(H_r, \phi, \alpha) = \ln \left\{ \left[\frac{(H - H_r) + H_r \phi}{H_r + (H - H_r)\phi} \right]^{\frac{\mu}{1-\sigma}} \left[\frac{1+\alpha H_r}{1+\alpha (H - H_r)} \right]^{\mu} \frac{\frac{w_{H_r}}{1+\alpha\mu H_r}}{\frac{w_{H_v}}{1+\alpha\mu(H-H_r)}} \right\} \tag{53}$$

where the two regional wages for skilled workers w_{H_r} and w_{H_v} can be substituted respectively from (51) and (52).

We first focus on the equilibrium with full agglomeration and we show that what stated in Proposition 4 in Section 5 for segregated networks holds.

Proof of Proposition 4 Expression (53) evaluated when *all* skilled workers are located in region r (i.e., $H_r = H$ and $H_v = 0$) is

$$V^s(H,\phi,\alpha) = \ln \left\{ \frac{2\sigma\left(1+\alpha H\right)^{\mu}\phi^{1+\frac{\mu}{1-\sigma}}}{\mu H\left[\sigma - 1 + \phi^2\left(\sigma+1\right)\right]\alpha + \left[\sigma - \mu + \phi^2\left(\sigma+\mu\right)\right]} \right\}$$

The value of $V^s(H,\phi,\alpha)$ in this case does not depend on L. On the one hand, when $\phi \to 0$ (that is, in autarky), $V^s(H,0,\alpha) \to -\infty$ and full agglomeration is not an equilibrium. On the other hand, with complete integration, $\phi = 1$, we find that $V^s(H,\phi,\alpha) = \ln \frac{(1+\alpha H)^{\mu}}{(1+H\alpha\mu)}$, which is negative, provided $\alpha > 0$, because $\frac{\partial\left(\frac{(1+\alpha H)^{\mu}}{(1+H\alpha\mu)}\right)}{\partial\alpha} = -\mu\alpha H^2 \frac{(1-\mu)}{H\alpha+1}\frac{(H\alpha+1)^{\mu}}{(H\alpha\mu+1)^2} < 0$ and full agglomeration is not an equilibrium either with complete integration. Moreover, when α is positive and sufficiently small full agglomeration is an equilibrium only for intermediate values of openness to trade as the numerator, which is a concave function in ϕ that increases from 0 when $\phi = 0$ to $2\sigma\left(1+\alpha H\right)^{\mu}$ when $\phi = 1$, and the denominator, which is a convex parabola in ϕ that increases from its minimum value $\mu H\left(\sigma-1\right)\alpha + \sigma - \mu > 0$ when $\phi = 0$ to $2\sigma\left(1+H\alpha\mu\right)$ when $\phi = 1$, intersect twice in ϕ_s and ϕ_u, both in the range $(0,1)$.[50] **QED**

We now consider the symmetric equilibrium and we show that we can derive the results of the numerical analysis mentioned in the final part of Section 5 for the symmetric equilibrium. The derivative of $V\left(H_r,\phi,\alpha\right)$ in (53) with respect to H_r evaluated at the symmetric equilibrium is given by the expression

$$V^s_{H_r}\left(H/2,\phi,\alpha\right) = \frac{4\left(k_2\phi^2 + k_1\phi + k_0\right)}{H(1+\phi)(\sigma-1)(2+\alpha H)\left(h_1\phi + h_0\right)}$$

where the coefficients k_2, k_1, k_0, h_1 and h_0 are functions of μ, σ, H and α and they do not depend on L.[51] As the denominator of $V^s_{H_r}\left(H/2,\phi,\alpha\right)$ is positive,

[50] This result can be compared with the case in which $\alpha = 0$ that implies that: the numerator is still an increasing function in ϕ taking the value 0 when $\phi = 0$ but it is smaller for any other value of $\phi \in [0,1]$; the parabola is always increasing for $\phi \in [0,1]$, it intersects the vertical axis at a lower value, that is $\sigma - \mu$, and it is equal to the numerator (2σ) when $\phi = 1$.

[51] Specifically,

$$\begin{aligned}
k_2 &= \alpha^2 H^2 \mu\left(\sigma+1\right)\left[\sigma\left(\mu-2\right) + 2\left(1-\mu\right)\right] - 2\alpha H\left[\sigma\left(\sigma+\sigma\mu-1\right) + 3\mu\left(\sigma+\mu-1\right)\right] \\
&\quad -4\left(\sigma+\mu-1\right)\left(\sigma+\mu\right); \\
k_1 &= 2\alpha^2 H^2 \mu^2\left[\sigma\left(\sigma-1\right)+1\right] + 4\alpha H\left[\sigma\left(1+\mu\right)\left(\sigma-1\right) + 2\mu^2\right] + 8\left[\sigma\left(\sigma-1\right)+\mu^2\right] > 0; \\
k_0 &= \alpha^2 H^2 \mu\left(\sigma-1\right)\left[\sigma\mu - 2\left(\sigma-1\right)\right] - 2\alpha H\left[\mu\left(\mu+3\right) + \sigma^2\left(\mu+1\right) - \sigma\left(5\mu+1\right)\right] \\
&\quad -4\left(\sigma-\mu-1\right)\left(\sigma-\mu\right); \\
h_1 &= 2\left(\sigma+\mu\right) + \mu\alpha\left(\sigma+1\right)H > 0; \\
h_0 &= 2\left(\sigma-\mu\right) + \mu\alpha\left(\sigma-1\right)H > 0.
\end{aligned}$$

the sign of $V^s_{H_r}(H/2, \phi, \alpha)$ depends on that of the parabola $G \equiv k_2\phi^2 + k_1\phi + k_0$, with the symmetric equilibrium stable (vs unstable) only when G is negative (vs positive). We focus our analysis on the simulated model with $H = 10$ and $\sigma = 2$. Figure 5.a (on the left) represents the value of G as a function of ϕ in the case of $\alpha = 0$ (solid curve), $\alpha = 0.02$ (dash dot), $\alpha = 0.04$ (dash), $\alpha = 0.06$ (dot) and $\alpha = 0.08$ (long dash) when $\mu = 0.4$.

Insert Figure 5 about here

Fig.5 Stability of the symmetric equilibrium for the segregated network: the plot of G

Figure 5.b (on the right) represents G for the same values except that now $\mu = 0.11$, where μ is proportional to the share of income, net of the conspicuous effect, devoted to acquire the differentiated good.[52] The numerical analysis shows that, the introduction of weak relative concerns tends to stabilize the symmetric equilibrium for high levels of integration ϕ. Stability always holds for $\phi = 1$ as then $G = 4\alpha^2 H^2 \mu\sigma(\sigma - 1)(\mu - 1) < 0$. The size of the interval of ϕ on which stability holds is increasing in α and decreasing with μ. Figure 5.a also shows that for intermediate levels of integration ϕ the symmetric equilibrium is destabilized when when μ is large. Indeed, in this case the agglomerative effect they produce is stronger than the dispersion effect. On the other hand, Figure 5.b shows that when μ is relatively low the symmetric equilibrium can be stabilized for low and intermediate values of economic integration ϕ provided α is sufficiently large.

2.C. The mixed case

Let us consider the case of the additive specification. Then when region r has a complete (integrated) network, we know that the aggregate demand in r of variety s produced in r, $X_{rr}(s)$, is given by (26), while the aggregate demand in r of variety s produced in v can be obtained from (27) and it is given by

$$X_{vr}(s) = \mu \frac{(\sigma - 1)\,\tau^{-\sigma}}{\sigma\beta\,(n_r + n_v\phi)} \frac{(L + H_r w_{H_r})\,(L\alpha + \alpha H_r + 1)}{1 + \alpha\mu\,(L + H_r)}$$

On the other hand, given that region v has two segregated networks, we know that the aggregate demands in v of variety s produced respectively in v and in r can be obtained from (50). Hence, the wage in (19) in region r in the mixed case can be rewritten as follows

$$w_{H_r} = \frac{\mu}{\sigma}\left[\frac{(L + H_r w_{H_r})}{(H_r + H_v\phi)} A + \phi\frac{(w_{H_v} A_v + A_L)}{(H_v + H_r\phi)}\right]$$

[52] Specifically, from (4), (9) and (10), we know that $\mu = \frac{p_{X_r}\left(X_{ir} - \frac{\alpha S(\Omega(\Lambda_r(i)))}{1+\alpha S(\Omega(\Lambda_r(i)))}\frac{\int_{\Lambda_r(i)} X_{jr} dj}{\int_{\Lambda_r(i)} dj}\right)}{W_{\alpha ir}}$.

while that obtained in region v is given by

$$w_{H_v} = \frac{\mu}{\sigma}\left[\frac{(w_{H_v}A_v + A_L)}{(H_v + H_r\phi)} + \frac{\phi\,(L + H_r w_{H_r})}{(H_r + H_v\phi)}A\right]$$

We use these last two equations to find the wages of skilled workers in the two regions, which are respectively

$$w_{H_r} = \mu\frac{\sigma\phi^2 A_L H_v - AL\mu A_v + AL\sigma H_v + \sigma\phi A_L H_r + AL\mu\phi^2 A_v + AL\sigma\phi H_r}{\sigma^2 H_r H_v + \sigma^2\phi H_r^2 + \sigma^2\phi H_v^2 + A\mu^2 A_v H_r + \sigma^2\phi^2 H_r H_v - \sigma\mu A_v H_r - \sigma\mu\phi A_v H_v - A\sigma\mu\phi H_r^2 - A\mu^2\phi^2 A_v H_r - A\sigma\mu H_r H_v}$$

and

$$w_{H_v} = \mu\frac{\sigma A_L H_r - A\mu A_L H_r + \sigma\phi A_L H_v + AL\sigma\phi^2 H_r + A\mu\phi^2 A_L H_r + AL\sigma\phi H_v}{\sigma^2 H_r H_v + \sigma^2\phi H_r^2 + \sigma^2\phi H_v^2 + A\mu^2 A_v H_r + \sigma^2\phi^2 H_r H_v - \sigma\mu A_v H_r - \sigma\mu\phi A_v H_v - A\sigma\mu\phi H_r^2 - A\mu^2\phi^2 A_v H_r - A\sigma\mu H_r H_v}$$

where the two denominators in the two equations are equal.

Then, we find that with $W_{\alpha Hr}$ for skilled workers in the integrated region r given by

$$W_{\alpha Hr} \equiv w_{H_r} - \alpha\mu\frac{L + H_r w_{H_r}}{L\alpha\mu + \alpha\mu H_r + 1}$$

and $W_{\alpha Hv}$ for skilled workers in the segregated region v

$$W_{\alpha Hv} = \frac{w_{H_v}}{\alpha\mu H_v + 1},$$

the log of the indirect utility levels $V\,(H_r, \phi, \alpha)$ in the mixed case is given by

$$V^{mix}\,(H_r, \phi, \alpha) = \ln\left[\left(\frac{p_{X_v}}{p_{X_r}}\right)^{\mu}\left(\frac{1+\alpha(L+H_r)}{1+\alpha H_v}\right)^{\mu}\frac{w_{H_r} - \alpha\mu\frac{L+H_r w_{H_r}}{L\alpha\mu+\alpha\mu H_r+1}}{\frac{w_{H_v}}{\alpha\mu H_v+1}}\right] \qquad (54)$$

Evaluating $V^{mix}\,(H_r, \phi, \alpha)$ in (54) when *all* skilled workers are located in the integrated region r we obtain the following expression[53]

$$V^{mix}\,(H, \phi, \alpha) = \ln\frac{(1+\alpha\mu L)\{2+\alpha[2L-H(\sigma-1)]\}[\alpha(H+L)+1]^{\mu}\sigma\phi^{1-\frac{\mu}{\sigma-1}}}{\{[L\alpha\mu(\sigma+1)+\mu+\sigma][1+\alpha(H+L)]\}\phi^2+(L\alpha+1)[\alpha\mu(\sigma-1)(H+L)+\sigma-\mu]}$$

From the inspection of $V^{mix}\,(H, \phi, \alpha)$ we know that agglomeration in r can be an equilibrium only for high or intermediate ϕ. When $\phi = 1$, the argument of the logarithm in $V^{mix}\,(H, 1, \alpha)$ is equal to 1 when $\alpha = 0$. With a positive value of α, we can show that the argument of the logarithm in $V^{mix}\,(H, 1, \alpha)$ is smaller than 1 and, thus, agglomeration is an equilibrium when $\phi = 1$, only for relatively large value of μ provided that the number of unskilled workers is relatively large with respect to that of skilled workers, that is $H < 2L/(\sigma-1)$.[54] More generally,

[53] Where $2 + \alpha\,[2L - H(\sigma - 1)]$ in the numerator has to be positive to have a positive value of $W_{\alpha Hr}$ when $H_r = H$. All the other factors in the numerator and denominator are positive.

[54] Proof. The argument of the logarithm in $V^{mix}\,(H, 1, \alpha)$ is smaller than 1 if $\frac{[H\alpha+2L\alpha+2+\alpha\mu(H+2L+2L^2\alpha+2HL\alpha)]}{(1+\alpha\mu L)\{2+\alpha[2L-H(\sigma-1)]\}} < [\alpha(H + L) + 1]^{\mu}$. We know that these two expressions, defined as $LHS = \frac{[H\alpha+2L\alpha+2+\alpha\mu(H+2L+2L^2\alpha+2HL\alpha)]}{(1+\alpha\mu L)\{2+\alpha[2L-H(\sigma-1)]\}}$ and $RHS = [\alpha(H + L) + 1]^{\mu}$, are respectively equal to $\frac{H\alpha+2L\alpha+2}{H\alpha+2L\alpha-H\alpha\sigma+2} > 1$ and to 1 when $\mu = 0$, and that they both increase in the range $\mu \in [0, 1]$ and assume respectively the value $2\frac{H\alpha+L\alpha+1}{H\alpha+2L\alpha-H\alpha\sigma+2}$ and $H\alpha + L\alpha + 1$ when $\mu = 1$. If $H > 2L/(\sigma - 1)$ we find always that $LHS > RHS$ for $\mu \in [0, 1]$ and therefore full agglomeration in r with $\phi = 1$ is never an equilibrium. However, if $H < 2L/(\sigma - 1)$ full agglomeration in r with $\phi = 1$ is an equilibrium for relatively large value of μ that ensure that $LHS < RHS$. Q.E.D.

we know that in this last case full agglomeration in r is an equilibrium also for all values of $\phi \in (\phi_s, 1)$.

Instead, evaluating $V^{mix}(H_r, \phi, \alpha)$ in (54) when all skilled workers are located in the segregated region v, we find that

$$\begin{aligned} & V^{mix}(0, \phi, \alpha) \\ = \quad & \ln \frac{\left\{(L\alpha+1)[H\alpha\mu(\sigma+1)+\sigma+\mu]\phi^2 - H\sigma\alpha[(\sigma-1)\alpha\mu H+\sigma-\mu]\phi+(L\alpha+1)[H\alpha\mu(\sigma-1)+\sigma-\mu]\right\}}{2\sigma(L\alpha+1)^{1-\mu}(\alpha H+1)^{\mu}\phi^{1-\frac{\mu}{\sigma-1}}} \end{aligned}$$

From the inspection of $V^{mix}(0, \phi, \alpha)$ we know that agglomeration in v can be an equilibrium only for high or intermediate ϕ. When $\phi = 1$, the argument of the logarithm in $V^{mix}(0, 1, \alpha)$ is equal to 1 when $\alpha = 0$.

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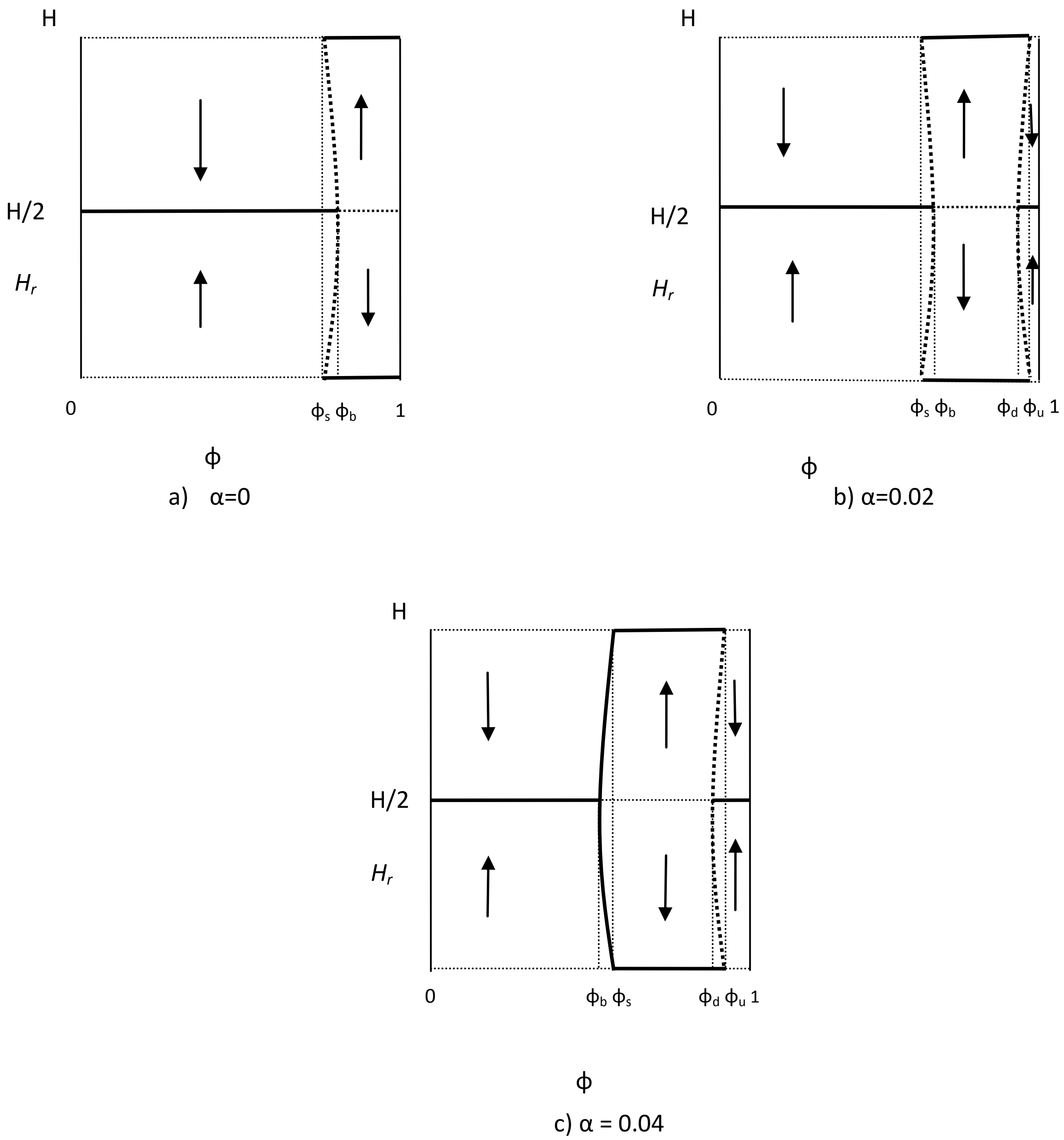
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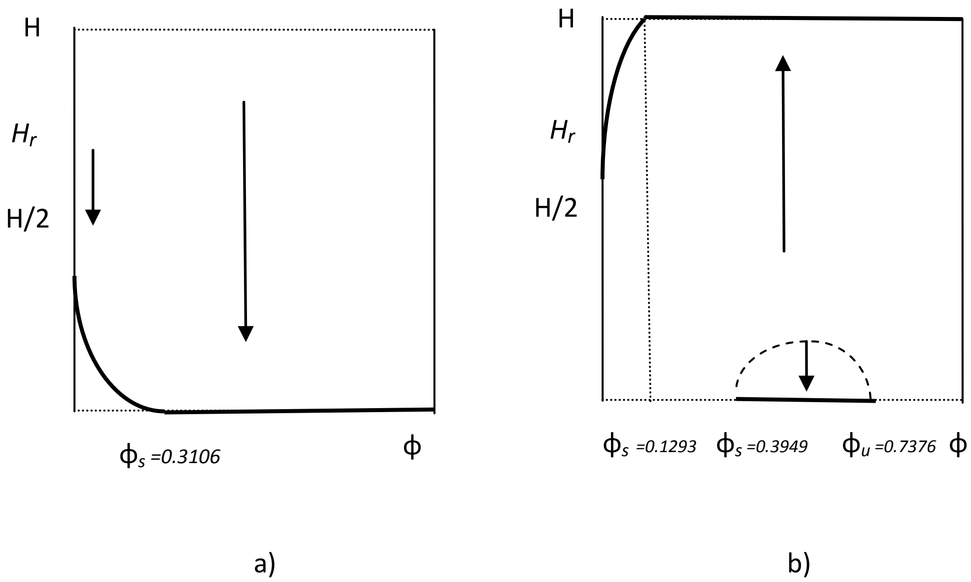
c) α = 0.04

Fig. 1



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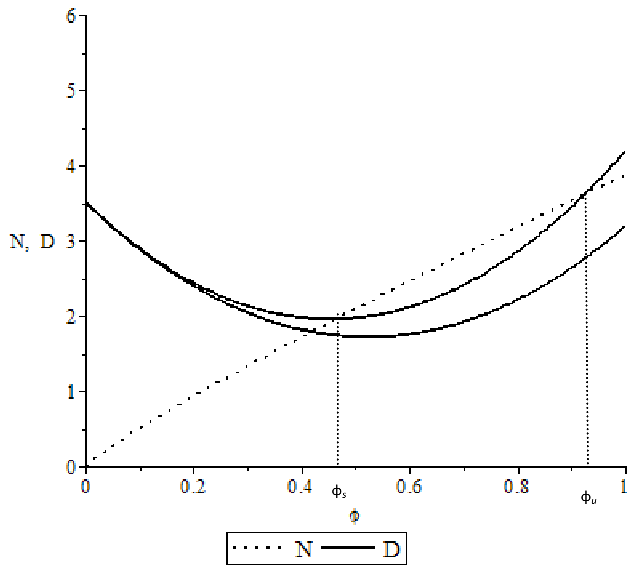

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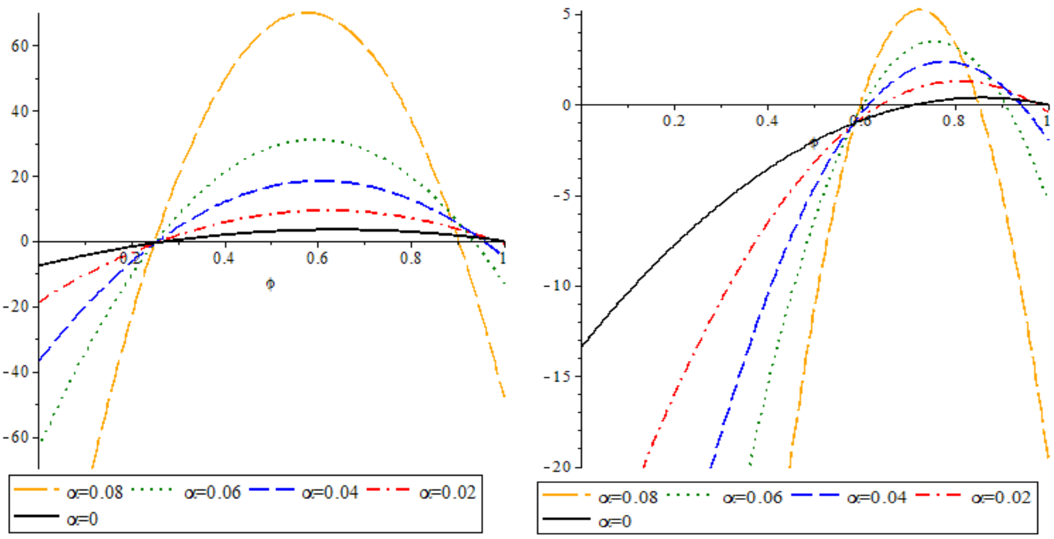

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-10
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-20
0.2
0.4
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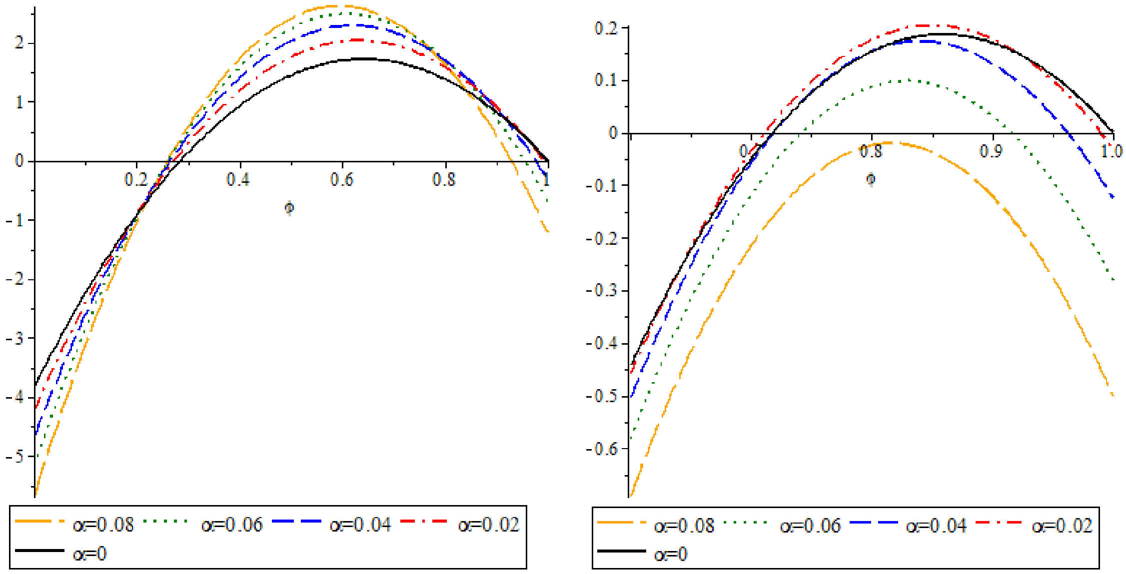
α=0

a) b)

Fig. 4



a) b)

Fig. 5